

# Neostability Theory

## 29 January - 3 February 2012

### Problems

#### **Problem 1 (Kobi Peterzil)**

Let  $G$  be a definable group in an  $o$ -minimal theory, and  $X \subset G$  a definable subset. Does  $\langle X \rangle$  always contain a definable generic set  $Y$  in the sense that boundedly many translates of  $Y$  cover  $\langle X \rangle$ ? (Equivalently, any definable subset of  $\langle X \rangle$  should be covered by finitely many translates of  $Y$ .)

The answer is positive for vector groups in  $o$ -minimal expansions of the reals.

#### **Problem 2 (Enrique Casanovas)**

Is there a simple  $\omega$ -categorical non-low theory?

Note that it cannot be supersimple nor CM-trivial.

#### **Problem 3 (Martin Ziegler)**

Consider any  $n$ -ary relation  $R$  on  $(\mathbb{C}, +, \cdot)$ . Is there a projective relation  $R'$  (in the sense of descriptive set theory) such that  $(\mathbb{C}, +, \cdot, R) \equiv (\mathbb{C}, +, \cdot, R')$ ?

#### **Problem 4 (John Baldwin)**

Let  $M$  be superstable saturated and  $I \subset M$  indiscernible. Does every permutation of  $I$  extend to an automorphism of  $M$ ?

#### **Problem 5 (Sergeï Starchenko)**

In a dependent theory, consider a formula  $\phi(x, y)$  of dp-rank  $d$  (or  $vc^*$ -density  $d$ ). Suppose the definable family  $\Theta = \{\phi(M, y) : y \models \theta\}$  is  $(d + 1)$ -consistent. Can we partition  $\Theta$  into finitely many definable consistent subfamilies?

Such a partition exists by the fractional Helly number; the problem is to get it definably.

#### **Problem 6 (Frank Wagner)**

Do simple one-based theories have (weak) elimination of hyperimaginaries?

#### **Problem 7 (Itai Ben Yaacov)**

What is the topological complexity for a theory to be simple (or stable, NIP, rosy)?

Answer:  $G_\delta$ .

In a stable theory, is one-basedness a meagre or co-meagre property?

#### **Problem 8 (Ludomir Newelski)**

Is rosyness an absolute property?

#### **Problem 9 (John Goodrick)**

Let  $T$  be strongly dependent and  $T_P$  the theory of saturated elementary pairs of  $T$ . Let  $\phi(x, y)$  be an  $L$ -formula. How is  $\text{dp-rank}_L(\phi)$  related to  $\text{dp-rank}_{L_P}(\phi)$ ?

Note: We should assume that  $T_P$  is also strongly dependent.

#### **Problem 10 (Predrag Tanovic)**

Let  $G$  be a superstable simple group of infinite rank, and  $p$  its generic type. Is there  $n$  such that  $p^{(n)}$  is non-isolated?

Note: No  $\aleph_0$ -categorical stable group is simple.

#### **Problem 11 (Artem Chernikov)**

Let  $M$  be an elementary submodel of  $N$ , and suppose  $p \in S(N)$  divides over  $M$ . It has dependent dividing if there is an instance of a dependent formula in  $p$  which divides over  $M$ . The theory has dependent dividing if all types over models have.

Note: Then  $T$  is  $\text{NTP}_2$ . If  $T$  is simple, dependent dividing equals stable forking.

Do all  $\text{NTP}_2$  theories have dependent dividing?

Is there an unstable class for which stable forking holds?

#### **Problem 12 (Artem Chernikov)**

Let  $A$  be an extension base for non-forking in an  $\text{NTP}_2$  theory. Find  $a$  and  $b$  with the same Lascar strong type over  $A$ , but such that the Lascar distance between them equals 3.