Banff International Research Station

for Mathematical Innovation and Discovery

Manifolds with special holonomy and their calibrated submanifolds and connections

Sunday, 2012-Apr-29 to Friday, 2012-May-04

MEALS

◦ Breakfast (Buffet): 07:00–09:00, Sally Borden Building, Monday–Friday
◦ Lunch (Buffet): 11:30–13:30, Sally Borden Building, Monday–Friday
◦ Dinner (Buffet): 17:30–19:30, Sally Borden Building, Sunday–Thursday
◦ Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

∗ *Please remember to scan your meal card at the host/hostess station in the dining room for each meal.* ∗

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday, 2012-Apr-29

16:00	Check-in begins (Front Desk — Professional Development Centre — open 24 hours) Lecture rooms are available after 16:00 (if desired)
17:30 – 19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall Beverages and small assortment of snacks available on a cash honour system.

Monday, 2012-Apr-30

07:00 – 09:00	Breakfast
09:00 – 09:15	Introduction and Welcome by BIRS Station Manager, TCPL
09:15 – 10:00	**Micah Warren** (Princeton University) *"Calibrated geometry in the optimal transportation problem"*
10:00 – 10:45	**Thomas Ivey** (College of Charleston) *"Austere submanifolds in complex projective space"*
10:45 – 11:15	Coffee Break, TCPL
11:15 – 12:00	**Damien Gayet** (Université Lyon I) *"Smoothing moduli spaces of associative submanifolds"*
12:00 – 13:30	Lunch
13:00 – 13:50	Guided Tour of the Banff Centre; meet in the 2nd floor lounge, Corbett Hall
13:50 – 14:00	Group Photo; meet in the foyer of the TransCanada Pipelines Pavilion (TCPL)
14:00 – 14:45	**Yunxia Chen** (Chinese University of Hong Kong) *"Minuscule representation bundles on surfaces with ADE singularities"*
14:45 – 15:15	Coffee Break, TCPL
15:15 – 16:00	**Roger Bielawski** (University of Leeds) *"Pluricomplex geometry and quaternionic manifolds"*
16:00 – 17:30	Informal discussions
17:30 – 19:30	Dinner

Tuesday, 2012-May-01

07:00 – 09:00	Breakfast
09:15 – 10:00	**Rugang Ye** (University of California at Santa Barbara) *"The Laplacian flow"*
10:00 – 10:45	**Benjamin McKay** (University College Cork) *"Some soliton solutions of a flow for G_2-structures"*
10:45 – 11:15	Coffee Break, TCPL
11:15 – 12:00	**Sergey Grigorian** (Stony Brook University) *"Deformations of G_2-structures with torsion"*
12:00 – 13:30	Lunch
14:00 – 14:45	**Thomas Mettler** (Mathematical Sciences Research Institute) *"Holonomy reduction of 2-Segre structures"*
14:45 – 15:15	Coffee Break, TCPL
15:15 – 16:00	**Maurizio Parton** (Universita di Chieti-Pescara) *"*Spin(9)*, complex structures, and vector fields on spheres"*
16:00 – 17:30	Informal discussions
17:30 – 19:30	Dinner

Wednesday, 2012-May-02

07:00 – 09:00	Breakfast
09:15 – 10:00	**Mu-Tao Wang** (Columbia University) *"A separation of variables Ansatz for special Lagrangian submanifolds"*
10:00 – 10:45	**Tommaso Pacini** (Scuola Normale Superiore) *"Gluing constructions for special Lagrangian conifolds in $\mathbb{C}^m$"*
10:45 – 11:15	Coffee Break, TCPL
11:15 – 12:00	**Aaron Smith** (University of Waterloo) *"A theory of multiholomorphic maps"*
12:00 – 13:30	Lunch
13:30 – 17:30	Impromptu talks / Informal discussions / Recreation
17:30 – 19:30	Dinner

Thursday, 2012-May-03

07:00 – 09:00	Breakfast
09:15 – 10:00	**Henrique Sá Earp** (Universidade Estadual de Campinas) *"Perspectives on G_2-instantons"*
10:00 – 10:45	**Derek Harland** (Loughborough University) *"Instantons and Killing spinors"*
10:45 – 11:15	Coffee Break, TCPL
11:15 – 12:00	**Maciej Dunajski** (University of Cambridge) *"G_2 geometry, twistor theory, and cuspidal cubics"*
12:00 – 13:30	Lunch
14:00 – 14:45	**Sergey Cherkis** (University of Arizona) *"Octonions, monopoles, and knots"*
14:45 – 15:15	Coffee Break, TCPL
15:15 – 16:00	**Frederik Witt** (Universität Münster) *"A variational problem for spinors"*
16:00 – 17:30	Informal discussions
17:30 – 19:30	Dinner

Friday, 2012-May-04

07:00 – 09:00	Breakfast
09:00 – 09:45	**Ronan Conlon** (McMaster University) *"A theorem of existence for asymptotically conical Calabi-Yau manifolds"*
09:45 – 10:30	**Alexei Kovalev** (University of Cambridge) *"Asymptotically cylindrical* Spin(7) *manifolds"*
10:30 – 10:45	Coffee Break, TCPL
10:45 – 11:30	**Jason Lotay** (University College London) *"Deforming G_2 conifolds"*
11:30 – 13:30	Lunch
13:30 – 15:00	Informal discussions / Departures

Remember to check out by 12:00 noon. You are welcome to use the facilities (BIRS Coffee Lounge, TCPL, and Reading Room) until 15:00 on Friday, but participants are required to checkout of guest rooms by 12:00.

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Sunday, 2012-Apr-29 to Friday, 2012-May-04

ABSTRACTS
(in alphabetical order by speaker surname)

Speaker: **Bielawski, Roger (University of Leeds)**
Title: **Pluricomplex geometry and quaternionic manifolds**

Abstract: I will describe a new type of geometric structure on complex manifolds. It can be viewed as a deformation of a hypercomplex structure, but it also leads to special types of hypercomplex and hyper-Kähler geometry. These structures have both algebro-geometric and differential-geometric descriptions, and there are interesting examples arising from physics. Moreover, a class of pluricomplex manifolds leads to quaternionic-Kähler metrics, generalising the SO(3)-invariant self-dual Einstein examples of Hitchin.

Speaker: **Chen, Yunxia (Chinese University of Hong Kong)**
Title: **Minuscule representation bundles on surfaces with ADE singularities**

Abstract: The minimal resolution of a surface with a simple singularity has a bunch of (-2)-curves as its exceptional locus, whose dual graph is a Dynkin diagram of type ADE. In this talk, we construct minuscule representations of the corresponding Lie algebra using configurations of (-1)-curves. Then we build extension bundles over the resolution using their associated line bundles satisfying: (i) they can be descended to the singular surface and (ii) they carry natural tensorial structures. Using (ii) we construct ADE Lie algebra bundles so that the original vector bundles become minuscule ADE representation bundles over our surface with ADE singularities.

Speaker: **Cherkis, Sergey (University of Arizona)**
Title: **Octonions, monopoles, and knots**

Abstract: Witten's approach to Khovanov homology of knots is based on the five-dimensional system of equations, which we call the Haydys–Witten equations. We formulate a dual seven-dimensional system of equations. It can be formulated on any G_2 holonomy manifold and is a close cousin of the monopole equation of Bogomolny. The octonions play a central role in our view of the Haydys–Witten equations and in the transform relating the five- and seven-dinemsional systems.

Speaker: **Conlon, Ronan (McMaster University)**
Title: **A theorem of existence for asymptotically conical Calabi-Yau manifolds**

Abstract: Asymptotically conical (AC) Calabi-Yau manifolds are Ricci-flat Kähler manifolds that resemble a Ricci-flat Kähler cone at infinity. I will describe an existence theorem for AC Calabi-Yau manifolds which, in particular, yields a refinement of an existence theorem of Tian and Yau for such manifolds. I will also discuss some examples. This is ongoing work with Hans-Joachim Hein.

Speaker: **Dunajski, Maciej (University of Cambridge)**
Title: **G_2 geometry, twistor theory, and cuspidal cubics**

Abstract: We establish a twistor correspondence between seven-parameter families of rational curves in a surface, and certain G_2 structures on moduli spaces of such curves. There are several explicit examples — e.g. the space of all cuspidal cubic curves in $\mathbb{P}^2$ gives rise to a homogenous co-calibrated G_2 structure on $SU(2,1)/U(1)$.

Speaker: **Gayet, Damien (Université Lyon I)**
Title: **Smoothing moduli spaces of associative submanifolds**

Abstract: It is known that deforming a closed associative Y (respectively an associative Y with boundary in a fixed coassociative X) as an associative is an elliptic problem of vanishing index (respectively of index given by the topology of the normal bundle in TX over the boundary of Y). I will explain two ways to ensure smoothness of the moduli space of local associative deformations of Y. The first way is to assume metric conditions on Y and the second is to perturb the G_2 structure in the realm of closed G_2 structures (respectively the boundary X).

Speaker: **Grigorian, Sergey (Stony Brook University)**
Title: **Deformations of G_2-structures with torsion**

Abstract: We consider non-infinitesimal deformations of G_2-structures on 7-dimensional manifolds and derive a closed expression for the torsion of the deformed G_2-structure. We then specialize to the case where the deformation lies in the 7-dimensional representation of G_2 and is hence defined by a vector v. In this case, we explicitly derive the expressions for the different torsion components of the new G_2-structure in terms of the old torsion components and derivatives of v. In particular this gives a set of differential equations for the vector v which have to be satisfied for a transition between G_2-structures with particular torsions. For some specific torsion classes we then explore the solutions of these equations.

Speaker: **Harland, Derek (Loughborough University)**
Title: **Instantons and Killing spinors**

Abstract: I will present some new examples of instantons on manifolds with real Killing spinors and their cones. Examples of manifolds admitting real Killing spinors include nearly Kähler 6-manifolds, nearly parallel G_2-manifolds in dimension 7, Sasaki-Einstein manifolds, and 3-Sasakian manifolds. For each of these classes of manifolds, I will exhibit a connection on the tangent bundle which has reduced holonomy and which solves the appropriate instanton equation. I will also discuss new 1-parameter families of instantons on the cones over real Killing spinor manifolds: these generalise various examples that appeared in the physics literature, and can be lifted to solutions of heterotic supergravity.

Speaker: **Ivey, Thomas (College of Charleston)**
Title: **Austere submanifolds in complex projective space**

Abstract: A submanifold M in Euclidean space $\mathbb{R}^n$ is austere if all odd-degree symmetric polynomials in the eigenvalues of the second fundamental form (in any normal direction) vanish. Harvey and Lawson showed that this condition is necessary and sufficient for the normal bundle of M to be special Lagrangian in $T\mathbb{R}^n \cong \mathbb{C}^n$. A similar result was proved by Karigiannis and Min-Oo for submanifolds in S^n, with TS^n carrying a Calabi-Yau metric due to Stenzel. In this joint work with Marianty Ionel, we determine conditions under which the normal bundle of a *CR*-submanifold in $\mathbb{CP}^n$ is special Lagrangian with respect to the Stenzel metric on $T\mathbb{CP}^n$. We give examples in the case of hypersurfaces in $\mathbb{CP}^2$, and some nonexistence results in the totally real case.

Speaker: **Kovalev, Alexei (University of Cambridge)**
Title: **Asymptotically cylindrical** Spin(7) **manifolds**

Abstract: Riemannian manifolds with asymptotically cylindrical ends are essential ingredients in gluing theorems and also have a natural interpretation as having a 'boundary at infinity'. I will report on recent progress in constructing examples of asymptotically cylindrical 8-manifolds with special holonomy Spin(7). The method uses parts of Joyce's construction of compact Spin(7) manifolds modified in some important ways and can also be compared at some points with the known constructions of asymptotically cylindrical manifolds with holonomy G_2 and $\mathrm{SU}(n)$.

Speaker: **Lotay, Jason (University College London)**
Title: **Deforming** G_2 **conifolds**

Abstract: Two natural classes of G_2 manifolds are those which either have non-compact ends asymptotic to cones or have isolated conical singularities. Examples of the former are given by the first complete examples of G_2 manifolds due to Bryant and Salamon, and the latter play an important role in M-Theory. By the fundamental work of Joyce, a compact G_2 manifold M has a smooth moduli space of deformations of dimension $b^3(M)$. I will describe a natural extension of this result to the two aforementioned types of G_2 conifolds. In particular, I will show the stark contrast between the deformation theories in each case and give some applications. This is joint work with S. Karigiannis.

Speaker: **McKay, Benjamin (University College Cork)**
Title: **Some soliton solutions of a flow for** G_2**-structures**

Abstract: The Laplacian coflow of a G_2-structure is the flow in which the 4-form evolves by its Laplacian. Spiro Karigiannis, Mao-Pei Tsui, and I found a few examples of solitons for this flow.

Speaker: **Mettler, Thomas (Mathematical Sciences Research Institute)**
Title: **Holonomy reduction of 2-Segre structures**

Abstract: The Weyl metrisability problem on a projective surface M corresponds to finding holomorphic curves in a certain quasiholomorphic fibre bundle over M. In this talk I will show that there is a similar correspondence for reducing the holonomy group of a torsion-free 2-Segre structure on an even dimensional manifold.

Speaker: **Pacini, Tommaso (Scuola Normale Superiore)**
Title: **Gluing constructions for special Lagrangian conifolds in $\mathbb{C}^m$**

Abstract: I will present some recent gluing results as in my preprint "Special Lagrangian conifolds, II", available on the arXiv.

Speaker: **Parton, Maurizio (Universita di Chieti-Pescara)**
Title: **Spin(9), complex structures, and vector fields on spheres**

Abstract: Joint work with Paolo Piccinni and Victor Vuletescu. Although holonomy Spin(9) appears to be a very restrictive condition, weakened holonomy Spin(9) conditions have been proposed and studied in the last years. In this setting, a basic problem is to have a simple algebraic formula for the canonical 8-form Φ whose stabilizer is Spin(9), as happens for instance in the $\mathrm{Sp(n)}\cdot\mathrm{Sp(1)}$, G_2, and Spin(7) cases. I will show a nice relation between Φ and a family of almost complex structures J associated to the Spin(9) structure, leading to an algebraic formula for Φ. I will then show how the existence of more than 7 independent tangent vector fields on spheres is all the fault of Spin(9), more precisely, all the fault of the J's. Finally, if time permits, the case of metrics which are locally conformal to a parallel Spin(9) metric will be discussed.

Speaker: **Sá Earp, Henrique (Universidade Estadual de Campinas)**
Title: **Perspectives on G_2-instantons**

Abstract: Solutions to the Hermitian Yang–Mills problem over A. Kovalev's asymptotically cylindrical Calabi-Yau 3−folds induce instantons over compact 7−manifolds with holonomy group G_2, obtained by a twisted gluing procedure. Moreover, algebro-geometric monad constructions developed by M. Jardim can be used to generate numerous concrete examples of such G_2-instantons. I will present a survey of that study, punctuated by some open questions ranging from naïve to quite ambitious.

Speaker: **Smith, Aaron (University of Waterloo)**
Title: **A theory of multiholomorphic maps**

Abstract: In recent decades the phenomena associated to pseudoholomorphic curves in Kähler manifolds have led to the discovery of a number of interesting invariants of symplectic manifolds. I will introduce the generalizing framework of multiholomorphic mappings of which the theory of pseudoholomorphic curves forms one of a few families of examples. This is a theory pertaining to mappings (between Riemannian manifolds) which satisfy a particular PDE describing the intertwining of geometric data on domain and target. The higher-dimensional scenario is characterized by a significant amount of rigidity. This will be seen in particular on a family of examples of multiholomorphic maps which involve maps from a 3-manifold into a G_2 manifold. There are close relations to calibrated geometry and mathematical physics.

Speaker: **Wang, Mu-Tao (Columbia University)**
Title: **A separation of variables Ansatz for special Lagrangian submanifolds**

Abstract: I shall discuss new constructions of special Lagrangian submanifolds and self-similar solutions of Lagrangian mean curvature flows based on a separation of variables ansatz. A similar construction for Ricci solitons will also be discussed.

Speaker: **Warren, Micah (Princeton University)**
Title: **Calibrated geometry in the optimal transportation problem**

Abstract: It has been observed that Monge-Ampère equations are related to notions of special Lagrangian submanifolds in manifolds with signature (n, n). We discuss optimal transportation problems and find that there is a natural metric on the product manifold associated to a given problem. With respect to this metric, the graph of the solution to the optimal transportation problem is a calibrated current. We will say precisely what this means and discuss the many analogies between this setting and the Calabi-Yau setting.

Speaker: **Witt, Frederik (Universität Münster)**
Title: **A variational problem for spinors**

Abstract: We introduce a natural functional on the universal spinor bundle and discuss the Euler-Lagrange equation of the associated variational problem.

Speaker: **Ye, Rugang (University of California at Santa Barbara)**
Title: **The Laplacian flow**

Abstract: The Laplacian flow, introduced by R. Bryant, is a natural evolution equation and serves to deform closed G_2 structures to torsion-free G_2 structures which produce G_2 holonomy. We'll present short-time existence of the Laplacian flow, its stability around torsion-free G_2 structures, long time convergence under a small torsion condition, and the smoothness of the limit map of the Laplacian flow. Additional geometric properties of the Laplacian flow will also be discussed.
