

# Arithmetic Geometry of Orthogonal and Unitary Shimura Varieties BIRS, June 3-8, 2012

**Schedule** (PLEASE SEE "SURVIVAL INFO" AT THE BOTTOM)

## Sunday

16:00 Check-in begins (Front Desk – Professional Development Centre - open 24 hours)  
17:30 - 19:30 Buffet Dinner

## Monday

8:45 Welcome and information  
9:00 - 10:00 Steve Kudla: *Special cycles for unitary groups: the unramified case*  
10:00 - 10:30 Coffee  
10:30 - 11:30 Ben Howard: *A Gross-Zagier theorem for higher weight modular forms*  
11:30 - 13:30 Lunch  
13:45 Group Photo  
14:00 - 15:00 Sug-Woo Shin: *Nonemptiness of Newton strata for PEL type Shimura varieties*  
15:00 - 15:30 Coffee  
15:30 - 16:30 Keerthi Madapusi: *Regular integral models for orthogonal Shimura varieties and the Tate conjecture for K3 surfaces in finite characteristic.*

## Tuesday

9:00 - 10:00 Michael Rapoport: *On the geometry of unitary Shimura varieties in the ramified case*  
10:00 - 10:30 Coffee  
10:30 - 11:30 Torsten Wedhorn: *Reductions of PEL Shimura varieties and group theoretic invariants*  
11:30 - 13:30 Lunch  
13:30 - 14:30 Brian Smithling: *Moduli descriptions of some local models for Shimura varieties*  
14:30 - 15:00 Coffee  
15:00 - 16:00 Adrian Vasiu: *Arithmetic properties of good integral models of Shimura varieties of Hodge type*  
16:00 - 16:30 Break  
16:30 - 17:30 Ulrich Goertz: *Affine Deligne-Lusztig varieties in the Iwahori case*

## Wednesday

8:30 - 9:30 Jurg Kramer: *Arithmetic intersections on modular curves*  
9:30 - 9:40 Break  
9:40 - 10:40 Jose Burgos-Gil: *The singularities of the invariant metric of the sheaf of Jacobi forms on the universal elliptic curve*  
10:40 - 11:00 Coffee  
11:00 - 12:00 Ulf Kuhn: *Modularity of generating series for arithmetic Hecke correspondences*  
12:00 Lunch and free afternoon

## Thursday

9:00 - 10:00 Jayce Getz: *Twisted relative endoscopy*  
10:00 - 10:30 Coffee  
10:30 - 11:30 Ehud DeShalit: *Integral Structures in Locally Algebraic representations and Kirillov Models*  
11:30 - 13:30 Lunch  
13:30 - 14:30 Chung Pang Mok: *Endoscopic classification of automorphic representations on quasi-split unitary groups*  
14:30 - 15:00 Coffee  
15:00 - 16:00 Ulrich Terstiege: *On the arithmetic fundamental lemma in the minuscule case*  
16:00 - 16:30 Break  
16:30 - 17:30 David Helm: *The local Langlands correspondence for  $GL_n$  in families and completed cohomology of Shimura varieties.*

**Friday**

8:30 - 9:30 Matthew Greenberg: *Triple product p-adic L-functions for balanced weights*  
9:30 - 9:40 Break  
9:40 - 10:40 Ellen Eischen: *p-adic families of Eisenstein series for unitary groups*  
10:40 - 11:00 Coffee  
11:00 - 12:00 John Voight: *Semi-arithmetic points*  
12:00 Lunch and departure

## ===== ABSTRACTS =====

**BURGOS GIL**

A theorem by Mumford implies that every automorphic line bundle on a pure open Shimura variety, provided with an invariant smooth metric, can be uniquely extended as a line bundle on a toroidal compactification of the variety, in such a way that the metric acquires only logarithmic singularities. This result is the key to being able to compute arithmetic intersection numbers from these line bundles. Hence it is natural to ask whether Mumford's result remains valid for line bundles on mixed Shimura varieties. In this talk we will examine the simplest case, namely the sheaf of Jacobi forms on the universal elliptic curve. We will show that Mumford's result can not be extended to this case and that a new interesting kind of singularities appear. We will discuss some preliminary results. This is joint work with G. Freixas, J. Kramer and U. Kühn.

**DE SHALIT**

The Breuil-Schneider conjecture grew out of an attempt to formulate a general p-adic local Langlands correspondence. It predicts when a locally algebraic representation of  $GL_n$  over a p-adic field  $F$  should have an integral structure. Despite the elegant formulation, these integral structures are very elusive. Except for  $GL_2(\mathbb{Q}_p)$ , where the conjecture is established indirectly, there are only partial results. We shall describe local methods of Vigneras and of the speaker (with Kazhdan), which apply to certain smooth representations of  $GL_2(F)$ . We shall also mention a recent breakthrough by Sorensen which uses global methods (unitary Shimura varieties and eigenvarieties).

**EISCHEN**

Special values of certain L-functions can be expressed in terms of values of Eisenstein series at points on the Shimura variety for  $U(n,n)$ . One approach to p-adically interpolating values of these L-functions relies on construction of a p-adic family of Eisenstein series. In this talk, I will explain how to construct such a family of Eisenstein series, and I will explain how to p-adically interpolate certain values of both holomorphic and non-holomorphic Eisenstein series on  $U(n,n)$ .

**GETZ**

The theory of twisted endoscopy relates packets of automorphic representations on classical groups to automorphic representations on general linear groups, and is an essential ingredient in understanding the cohomology of Shimura varieties. We will introduce a (rudimentary) conjectural theory of twisted relative endoscopy. The goal of this theory is to relate periods of automorphic representations on classical groups to periods of automorphic representations on general linear groups (which are much better understood). These periods often define special algebraic cycles on the Shimura variety, and thus the theory of twisted relative endoscopy should give a means of studying what part of the cohomology (or arithmetic cohomology) of the Shimura variety can be explained by these cycles.

**GOERTZ**

In this talk, I will report on recent joint work with Xuhua He and Sian Nie on affine Deligne-Lusztig varieties in the Iwahori case, in particular about non-emptiness and their dimensions. Affine Deligne-Lusztig varieties are analogues of usual Deligne-Lusztig varieties in the context of an affine root system, and are a group-theoretic tool to study the reduction of Shimura varieties (of PEL type), especially the Newton stratification, the Kottwitz-Rapoport stratification, and how these to stratifications are related.

## GREENBERG

The conjecture of Gross and Prasad and its refinements constitute a framework for generalizing the famous formula of Gross and Zagier, relating the height of a Heegner point on an elliptic curve to the central derivative of an associated L-function, to the context of orthogonal and unitary Shimura varieties. In this talk, I would like to discuss work-in-progress with Marco Seveso dealing with p-adic analogues of these conjecture in a low-dimensional (though very rich!) test scenario. Specifically, I'll describe a construction of a triple product p-adic L-function in the case of "balanced weights." The key tools are existing classical special value formulae due to Gross-Kudla, Boecherer-Schulze-Pillot, and Ichino, and the Ash-Stevens theory of p-adic deformation of arithmetic cohomology.

## HELM

We discuss joint work with Matthew Emerton that describes a conjectural local Langlands correspondence for families of Galois representations. Emerton has shown that for two-dimensional Galois representations such a correspondence arises in his description of the completed cohomology of the modular tower. We will focus on some of the representation-theoretic questions that arise from our results, with an eye towards understanding the completed cohomology of other Shimura towers.

## HOWARD

I'll talk about an extension of the Gross-Zagier theorem to higher weight modular forms, expressing the height pairings of special cycles on unitary Shimura varieties to the central derivatives of Rankin-Selberg L-functions. This is joint work with Jan Bruinier and Tonghai Yang.

## KRAMER

In our talk we will report on asymptotic formulas for the arithmetic self-intersection of the relative dualizing sheaf equipped with the Arakelov metric on modular curves attached to congruence subgroups as the level tends to infinity. In case of the modular curve  $X_0(N)$  ( $N$  squarefree and not divisible by  $2,3$ ) such results are due to A. Abbes and E. Ullmo. We will present analogous results for the modular curve  $X_1(N)$  (for suitable squarefree  $N$ ), which then enable us to compute the Faltings height of the associated Jacobian  $J_1(N)$  asymptotically (as  $N$  tends to infinity).

## KUDLA

In the first part of this lecture I will review the definition of arithmetic special cycles on Shimura varieties for unitary groups of signature  $(n-1,1)$  and explain how the arithmetic 0-cycles arise in the computation of their height pairings. In the second part of talk, I will review the structure of the Rapoport-Zink space used in the p-adic uniformization of the supersingular locus of such a varieties in the case of an inert prime. Finally, I will give the definition of the analogous special cycles in this RZ space and explain their properties.

## KUHN

The aim of the talk is to explain our approach to Kudla's conjectures for the case of the product of two modular curves. The mayor difficulties in this situation are of analytical nature. We present a mild modification of this Green function that satisfies the requirements of being a Green function in the sense of Arakelov theory on the natural compactification in addition. Only this allows us to define arithmetic special cycles and to show that the generating series of those modified arithmetic special cycles is as predicted by Kudla's conjectures a modular form with values in the first arithmetic Chow group. Moreover its intersection with the arithmetic canonical class yields essentially the derivative of an Eisenstein series. This is joint work with Rolf Berndt: <http://xxx.uni-augsburg.de/abs/1205.6417>  
<http://www.math.uni-hamburg.de/home/kuehn/berndt-kuehn-part-II.pdf>

## MADAPUSI

We construct regular integral canonical models for Shimura varieties of orthogonal type with maximal parahoric level, and we show that certain moduli spaces of polarized K3 surfaces can be viewed as open sub-schemes of such integral models. Using a result of Kisin, this then implies the Tate conjecture for K3 surfaces in odd characteristic  $p$ , as long as they admit a polarization of degree indivisible by  $p^2$ . The same methods also work to prove the Tate conjecture for cubic fourfolds in odd characteristic.

## MOK

Recently Arthur has established the endoscopic classification of automorphic representations on orthogonal and symplectic groups (modulo stabilization of the twisted trace formula). In this talk we report on the current work on extending Arthur's results to unitary groups.

## RAPOPORT

I will explain structure theorems for the formal moduli space of  $p$ -divisible groups of Picard type of signature  $(1, n-1)$  for a ramified quadratic extension of  $\mathbb{B}Q_p$ . The underlying reduced scheme possesses a stratification by Deligne-Lusztig varieties for symplectic groups over  $\mathbb{B}F_p$ ; the strata are parametrized by simplices in the Bruhat-Tits building of a  $p$ -adic unitary group. This is joint work with U. Terstiege and S. Wilson; the results are analogous to the results of I. Vollaard and T. Wedhorn in the case of an unramified quadratic extension of  $\mathbb{B}Q_p$ .

## SHIN

The Newton strata for Shimura varieties arising from unramified PEL data of type A and C are known to be nonempty whenever expected by Vasiu and Viehmann-Wedhorn based on earlier work by many. I will explain a different approach to prove the result in the unramified case via Honda-Tate theory and Galois cohomology. Assuming a result on the existence of F-crystals more can be shown.

## SMITHLING

Local models are certain schemes introduced to model the étale-local structure of  $p$ -adic integral models of Shimura varieties. A general definition of them in the setting of PEL Shimura varieties was given by Rapoport and Zink; recently Pappas and Zhu have formulated a general group-theoretic definition of them for tamely ramified groups and established many good properties for them. Unfortunately the local models do not in general admit ready moduli-theoretic interpretations. To facilitate applications, it is therefore of interest to describe them in moduli-theoretic terms when possible. In the case of orthogonal and ramified unitary groups, such a description has been proposed by Pappas and Rapoport. I will report on some progress towards proving their conjecture.

## TERSTIEGE

The arithmetic fundamental lemma conjecture of Wei Zhang connects the derivative of a certain orbital integral with an intersection number on a Rapoport-Zink space parameterizing  $p$ -divisible groups of unitary type of signature  $(1, n-1)$ . It arises in the relative trace formula approach to the arithmetic Gan-Gross-Prasad conjecture. This talk is about joint work with M. Rapoport and W. Zhang proving the AFL-conjecture in the minuscule case.

## VASIU

We present an accessible survey of known results on arithmetic properties of good integral models of Shimura varieties of Hodge type in arbitrary unramified mixed characteristic  $(0, p)$ . We will also include some of those proofs that are the most useful for the further studies of such arithmetic properties.

## VOIGHT

We present a method for constructing algebraic points on elliptic curves defined over totally real fields, combining the theory of Belyi maps and quaternionic Shimura varieties; our method generalizes the construction of Heegner points arising from classical modular curves. In particular, we report on some computational investigations of these points.

## WEDHORN

It is explained how to attach to a point in the parahoric reduction of Shimura varieties of PEL type (i.e. those reductions that are studied in the book of Rapoport and Zink) a group theoretic invariant which encodes the isomorphism class of the (multi)chain of  $p$ -divisible groups with additional structures associated to that point. This invariant spawns several other invariants which lead to the Newton stratification, the Ekedahl-Oort stratification and the Kottwitz-Rapoport stratification. In the case of good reduction several properties of these stratifications

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## MEALS

\*Breakfast (Buffet): 7:00 – 9:30 am, Sally Borden Building, Monday – Friday

\*Lunch (Buffet): 11:30 am – 1:30 pm, Sally Borden Building, Monday – Friday

\*Dinner (Buffet): 5:30 – 7:30 pm, Sally Borden Building, Sunday – Thursday

\*Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

## MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

## GROUP PHOTO

The meeting place for the group photograph is the foyer of TCPL. However, the photo is taken outside so it may be necessary for people to bring a jacket.

## CHECKOUT

**Friday by 12 noon.** Participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon.