

**Banff International Research Station  
Ordered Groups and Topology 5-day Workshop  
February 12 - 17, 2012**

**MEALS**

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

**MEETING ROOMS**

Morning lectures will be held in room 201 of the TransCanada Pipelines Pavilion (TCPL); in the afternoon lectures will be in room 202. LCD projector, overhead projectors and blackboards are available for presentations. All lectures are scheduled for 50 minutes duration.

**TENTATIVE SCHEDULE (revised Feb. 13)**

**Sunday,  
Feb. 12**

**16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)  
**17:30–19:30** Buffet Dinner, Sally Borden Building  
**20:00** Informal gathering in 2nd floor lounge, Corbett Hall  
Beverages and a small assortment of snacks are available on a cash honor system.

**Monday,  
Feb. 13**

**7:00–8:45** Breakfast  
**8:45–9:00** Introduction and Welcome by BIRS Station Manager, TCPL  
**9:00** **Dale Rolfsen** *Ordering Knot Groups*  
**10:00–10:30** Coffee Break, TCPL  
**10:30** **Dave Witte Morris** *On interactions of amenability with left-orderings*  
**11:30** Group Photo; meet on the front steps of Corbett Hall  
**11:40–13:00** Lunch  
**13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall  
**14:10** **Thomas Koberda** *Faithful actions of automorphisms on the space of orderings of a group*  
**15:10–15:30** Coffee Break, TCPL  
**15:30** **Cristóbal Rivas** *Left-ordering on free products of groups*  
**16:30** **Rachel Roberts** *Manifolds containing no  $R$ -covered foliations*  
**17:30–19:30** Dinner

**Tuesday,  
Feb. 14**

**7:00–9:00** Breakfast  
**9:00** **Cameron Gordon** *L-spaces and left-orderability*  
**10:00–10:30** Coffee Break, TCPL  
**10:30** **Liam Watson** *Dehn surgery and left-orderability*  
**11:30–13:30** Lunch  
**13:30** **Problem Session**  
**14:30–15:00** Coffee Break, TCPL  
**15:00** **Nathan Dunfield** *L-spaces and left-orderability: an experimental survey*  
**16:00** **Tye Lidman** *Left-Orderability and a Seiberg-Witten Smith Inequality*  
**17:30–19:30** Dinner

**Wednesday,  
Feb. 15**

**7:00–9:00** Breakfast  
**9:00** **Andrés Navas** *Random walks on left-orderable groups*  
**10:00–10:30** Coffee Break, TCPL  
**10:30** **Tetsuya Ito** *Ordering of mapping class groups and contact 3-manifolds*  
**11:30** **Adam Clay** *Left-orderability and foliations*  
**12:30–13:30** Lunch  
Free Afternoon  
**17:30–19:30** Dinner

**Thursday,  
Feb. 16**

**7:00–9:00** Breakfast  
**9:00** **Luis Paris** *A simple and fast method for determining short  $\sigma$ -expressions of braids*  
**10:00–10:30** Coffee Break, TCPL  
**10:30** **Patrick Dehornoy** *The ordered structure of the Klein bottle group and subword reversing*  
**11:30–13:30** Lunch  
**13:30** **Jozef Przytycki** *Orderings on Conway algebras, and Tutte algebras; is anything known?*  
**14:30–15:00** Coffee Break, TCPL  
**15:00** **William Menasco** *The Dehornoy floor and the Markov Theorem without Stabilization*  
**16:00** **Peter Linnell** *The spaces of left- and locally invariant orders*  
**17:30–19:30** Dinner

**Friday, Feb.  
17**

**7:00–9:00** Breakfast  
**9:00** **Steve Boyer** *Graph manifolds which are integral homology 3-spheres and taut foliations*  
**10:00** Coffee Break, TCPL  
**10:30–11:30** **Problem session – Informal discussion**  
**11:30–13:30** Lunch  
**Checkout by  
12 noon.**

\*\* 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

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**ABSTRACTS**  
**(in alphabetic order by speaker surname)**

Speaker: **Steve Boyer** (UQAM)

Title: *Graph manifolds which are integral homology 3-spheres and taut foliations*

Abstract: We show that a graph manifold, which is an integral homology 3-sphere and is neither the 3-sphere nor the Poincare homology sphere, admits a taut foliation which is transverse to the fibers in each Seifert piece. This result gives a new proof that such a manifold has a left-orderable fundamental group and is not an L-space.

This is joint work with Michel Boileau.

Speaker: **Adam Clay** (UQAM)

Title: *Left-orderability and foliations*

Abstract: Every left-ordering of  $\mathbb{Z} \times \mathbb{Z}$  corresponds to a line in the plane. As such, whenever  $M$  is a 3-manifold with torus boundary, we can say that every left-ordering of the fundamental group 'detects' a slope on the boundary. The idea of r-decay is to show via calculation that those slopes on the boundary of a 3-manifold which are not detected by a left-ordering correspond to those slopes for which the surgery manifold doesn't have a nice foliation, or is an L-space. In this talk, I will discuss the extent to which we can make precise the association between left-orderings and foliations, and outline how recent group-theoretic work of Bludov, Glass and Chiswell may allow for the development of 'gluing conditions' for foliations of 3-manifolds with torus boundary components. This is joint work with Liam Watson and Steve Boyer.

Speaker: **Patrick Dehornoy** (Caen)

Title: *The ordered structure of the Klein bottle group and subword reversing*

Abstract: The Klein bottle group has a simple but interesting ordered structure, which is connected with the fact that the group is a group of fractions of a Garside monoid in which divisibility is a linear ordering. On the other hand, subword reversing is a combinatorial method relevant for investigating presented groups, in particular those that are groups of fractions. We shall explain how to use this tool in the (easy) case of the Klein bottle group and its ordered structure, with the aim of subsequently applying it to more complex examples.

Speaker: **Nathan Dunfield** (Illinois, Champaign-Urbana)

Title: *L-spaces and left-orderability: an experimental survey*

Abstract: I will discuss the results of some computer experiments on small-volume hyperbolic 3-manifolds. Specifically, for the 11,031 such manifolds in the Hodgson-Weeks census, at least 27% are L-spaces and at least 2% have left-orderable fundamental groups. So far, these two subsets are disjoint, consistent with the conjecture of Boyer-Gordon-Watson that an irreducible rational homology 3-sphere is an L-space if and only if its fundamental group is not left-orderable.

Speaker: **Cameron Gordon** (University of Texas)

Title: *L-spaces and left-orderability*

Abstract: We will discuss evidence for the conjecture that a rational homology 3-sphere is an L-space if and only if its fundamental group is not left-orderable. This is joint work with Steve Boyer and Liam Watson.

Speaker: **Tetsuya Ito** (Tokyo)

Title: *Ordering of mapping class groups and contact 3-manifolds*

Abstract: The mapping class group of a surface with non-empty boundaries have a family of left orderings called Nielsen-Thurston type orderings. We will show that N-T orderings provide a new criterion for tightness of contact 3-manifolds. This relationship between ordering and contact geometry is based on the open book foliation theory, which was developed by the speaker and Keiko Kawamuro.

Speaker: **Thomas Koberda** (Harvard)

Title: *Faithful actions of automorphisms on the space of orderings of a group*

Abstract: I will sketch the ideas which show that the automorphism group of a residually torsion-free nilpotent group  $G$  acts faithfully on the space of left-invariant orderings of  $G$ . In the case where  $G$  is Gromov-hyperbolic, I will explain this theorem in the context of the geometry of  $G$ .

Speaker: **Tye Lidman** (UCLA)

Title: *Left-Orderability and a Seiberg-Witten Smith Inequality*

Abstract: If  $G$  is left-orderable, then any subgroup of  $G$  is automatically left-orderable as well. In terms of covering spaces, if the fundamental group of  $Y$  is left-orderable and  $Y'$  covers  $Y$ , then the fundamental group of  $Y'$  is also left-orderable. Boyer-Gordon-Watson have therefore asked the analogous question for  $L$ -spaces. However, the obvious methods fail for technical reasons. We study this question from the point of view of Seiberg-Witten theory and present some results in this direction. This is work in progress with Ciprian Manolescu.

Speaker: **Peter Linnell** (Virginia Tech)

Title: *The spaces of left- and locally invariant orders*

Abstract: I will report on separate work with two of my students Kelli Karcher (doctoral) and Li Hao (undergraduate). In the former we are studying the space of left orders of polycyclic groups. In the latter we are studying the space of locally invariant orders (LIO) of an arbitrary group  $G$ .

Recall that the space of left orders of a group  $G$  is the set of all left orders of  $G$  with the topology given by the subbase  $\{< \mid g < h\}$  (for  $g, h \in G$ ). Also an LIO of a group  $G$  is a strict partial order  $<$  such that for all  $r, g \in G$  with  $r \neq 1$ , either  $rg > g$  or  $r^{-1}g > g$ . Then the space of LIO's on  $G$  is defined in the same way as the space of left orders on  $G$ , so is the set of LIO's with the topology given by the subbase  $\{< \mid g < h\}$ .

Speaker: **William W. Menasco** (SUNY-University at Buffalo)

Title: *The Dehornoy floor and the Markov Theorem without Stabilization*

Abstract: The Markov Theorem without Stabilization (MTWS) [Birman-M] tells us that for a fixed braid index  $n$  there are a finite number of “modelled” isotopes (dependent only on  $n$ ) which take any closed  $n$ -braid immediately to a representative of minimal index. Once at minimal index there is a finite number of modelled isotopes (again, dependent only on the value of the braid index) that allows one to jump between conjugacy classes of minimal index. These isotopes which will grow in number as  $n$  grows make up the MTWS calculus for closed braids. Connections between the Dehornoy floor and isotopes of the MTWS calculus were first made by T. Ito. In this talk we will expand on these connections by showing a new characterisation of MTWS isotopes for braids. This talk will feature joint work with Doug Lafountain [Aarhus University] and Hiroshi Matsuda [Yamagata University].

Speaker: **Dave Witte Morris** (University of Lethbridge)

Title: *On interactions of amenability with left-orderings*

Abstract: Amenability is a fundamental notion in group theory, as evidenced by the fact that it can be defined in more than a dozen different ways. A few of these different definitions will be discussed, together with some commentary on the theorem that left-orderable amenable groups are locally indicable, and perhaps some speculation on other ways that amenability might be useful in the theory of left-orderings.

Speaker: **Andrés Navas** (USACH, Chile)

Title: *Random walks on left-orderable groups*

Abstract: Given a finitely-generated, left-orderable group endowed with a probability measure supported on a finite system of generators, we are interested on the behavior of typical random products. Among other results, I will sketch the ideas involved in a Polya's like recurrence theorem obtained in collaboration with Deroin, Kleptsyn and Parwani: there exists an interval in the group such that almost every path "crosses" this interval infinitely many times. Potential applications of these ideas will be discussed.

Speaker: **Luis Paris** (Bourgogne)

Title: *A simple and fast method for determining short  $\sigma$ -expressions of braids*

Abstract: Joint work with J. Fromentin. Let  $n \in \mathbb{N}$ ,  $n \geq 2$ , and  $i \in \{1, \dots, n-1\}$ . We say that a braid  $\beta \in B_n$  is  $\sigma_i$ -positive (resp.  $\sigma_i$ -negative) if it can be written

$$\beta = \beta_0 \sigma_i \beta_1 \cdots \sigma_i \beta_k \quad (\text{resp. } \beta_0 \sigma_i^{-1} \beta_1 \cdots \sigma_i^{-1} \beta_k),$$

with  $k \geq 1$  and  $\beta_0, \beta_1, \dots, \beta_k \in B_i$ . A celebrated Dehornoy's theorem says that, for any braid  $\beta \in B_n \setminus \{1\}$ , there exists a unique  $i \in \{1, \dots, n-1\}$  such that  $\beta$  is either  $\sigma_i$ -positive, or  $\sigma_i$ -negative, but not both. There are several proofs of this result. Most of them are effective, but the involved algorithms are slow (exponential complexity) and determine  $\sigma_i$ -positive expressions (or  $\sigma_i$ -negative expressions) whose lengths are exponential with respect to the word-length of the original braids. In this talk we present a simple algorithm of quadratic complexity which, given a braid  $\beta \in B_n \setminus \{1\}$ , determines a  $\sigma_i$ -positive expression (or a  $\sigma_i$ -negative expression) for  $\beta$ , whose length is bounded above by some constant times the word length of  $\beta$ .

Speaker: **Jozef Przytycki** (George Washington University)

Title: *Orderings on Conway algebras, and Tutte algebras; is anything known?*

Abstract: We consider various non-associative binary structures and ask whether they have orderings, and whether orderings lead to some interesting consequences. We concentrate on an entropic property,  $(a * b) * (c * d) = (a * c) * (b * d)$ , with Conway algebra (including Homflypt polynomial) and Tutte algebra as main examples. Another property of great interest is right self-distributivity,  $(a * b) * c = (a * c) * (b * c)$  with quandles, in particular Dehn quandles of surfaces, as premiere examples. We will stress that both structures satisfy "generative property" which we discuss in detail.

Speaker: **Cristóbal Rivas** (ENS-Lyon)

Title: *Left-ordering on free products of groups*

Abstract: Based on the concept of dynamical realization of a left-ordering, we exhibit a dynamical criterion for approximating the giving left-ordering. This criterion is used to show that no left-ordering on a free product of groups is isolated.

Speaker: **Rachel Roberts** (Washington University, St. Louis)

Title: *Manifolds containing no R-covered foliations*

Abstract: We show that there are 3-manifolds which have left orderable fundamental group but which do not contain R-covered foliations. This is joint work with Sergio Fenley.

Speaker: **Dale Rolfsen** (UBC-Vancouver)

Title: *Ordering Knot Groups*

Abstract: I will discuss orderability of knot groups, that is, fundamental groups of knot complements in 3-space. Howie and Short showed that all knot groups are locally indicable, and therefore left-orderable. In joint work with Bernard Perron and Adam Clay, I'll discuss criteria, involving roots of the Alexander polynomial, determining that certain knot groups are bi-orderable, while many others are not. Sketches of the proofs will be given. Among the applications: if a knot has bi-orderable group, then surgery on that knot cannot produce an L-space in the sense of Ozsváth and Szabó.

Speaker: **Liam Watson** (UCLA)

Title: *Dehn surgery and left-orderability*

Abstract: In light of the conjectured relationship between L-spaces and manifolds with non-left-orderable fundamental group, it is natural to study the behaviour of left-orderable groups in the context of Dehn surgery. This talk will describe some formal properties of Heegaard-Floer homology in this context, and establish analogous behaviour for left-orderable fundamental groups. This is joint work with Adam Clay.