

Lie algebras, torsors and cohomological invariants

Arriving Sunday, September 30 and departing Friday October 5, 2012

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday

16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

17:30–19:30 Buffet Dinner, Sally Borden Building

20:00 Informal gathering in 2nd floor lounge, Corbett Hall

Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–9:15 Breakfast

9:15–9:30 Introduction and Welcome by BIRS Station Manager, TCPL

9:30–10:20 E. Bayer-Fluckiger, Embeddings of maximal tori of type CM in orthogonal groups

10:20–10:40 Coffee Break, TCPL

10:40–11:30 A. Vishik, Stable and Unstable operations in Algebraic Cobordism

11:30–13:00 Lunch

13:00–14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

14:00–14:10 Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).

14:10–14:30 Coffee Break, TCPL

14:30–15:20 A. Merkurjev, Generic values of quadratic forms and essential dimension

15:30–16:00 S. Baek, On the torsion of Chow groups of Severi-Brauer varieties

17:30–19:30 Dinner

Tuesday

7:00–9:30 Breakfast

9:30–10:20 V. Popov, Rational functions on semisimple Lie algebras and the Gelfand-Kirillov Conjecture

10:20–10:40 Coffee Break, TCPL

10:40–11:30 E. Neher, Derivations of algebras obtained by étale descent

11:30–14:00 Lunch

14:00–14:30 Z. Chang, Twisted Loop Algebras Based on Conformal Superalgebras

14:30–14:50 Coffee Break, TCPL

14:50–15:20 I. Dimitrov, Constructing subrepresentations via the Borel-Weil-Bott theorem

15:30–16:00 N. Lemire, Stably Cayley Groups over Fields of Characteristic 0

17:30–19:30 Dinner

Wednesday

- 7:00–9:30** Breakfast
9:30–10:20 R. Parimala, Bounding symbol lengths in Galois cohomology
10:20–10:40 Coffee Break, TCPL
10:40–11:30 D. Saltman, Finite u Invariant and Bounds on Cohomology Symbol Lengths
11:30–13:30 Lunch
Free Afternoon
17:30–19:30 Dinner

Thursday

- 7:00–9:30** Breakfast
9:30–10:20 V. Chernousov, On the genus of a division algebra
10:20–10:40 Coffee Break, TCPL
10:40–11:30 S. Garibaldi, Algebraic groups and weak commensurability
11:30–14:00 Lunch
14:00–14:30 M. Florence, Central simple algebras of index p^n in characteristic p
14:30–14:50 Coffee Break, TCPL
14:50–15:20 O. Haution, Invariants of upper motives
15:30–16:00 V. Kiritchenko, Schubert calculus for equivariant algebraic cobordism
17:30–19:30 Dinner

Friday

- 7:00–9:00** Breakfast
9:00–9:30 S. Cernele, Essential dimension and error-correcting codes
09:40–10:10 R. Loetscher, Essential p -dimension of algebraic groups, whose connected component is a torus
10:10–10:40 Coffee Break, TCPL
10:40–11:30 P. Gille, Topological properties of torsors and homogeneous spaces over valued fields
11:30–13:30 Lunch
Checkout by 12 noon.

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Sanghoon Baek**

Title: *On the torsion of Chow groups of Severi-Brauer varieties*

Abstract: Kapenko determined the torsion in Chow groups of codimension 2 cycles on Severi-Brauer varieties. In this talk, we extend his results: for a large class of central simple algebras we provide upper bounds for the annihilators of the torsion subgroups of Chow groups of the corresponding Severi-Brauer varieties.

Speaker: **Eva Bayer-Fluckiger**

Title: *Embeddings of maximal tori of type CM in orthogonal groups*

Abstract: Let k be an algebraic number fields. Embeddings of maximal tori in orthogonal groups have been studied in several papers, and occur in various arithmetic questions. The case of tori of type CM (that is, tori associated to CM étale algebras) is of special interest of some of the applications. The aim of this talk is to give necessary and sufficient criteria for such an embedding to exist under some conditions, which are fulfilled in the CM case.

Speaker: **Shane Cernele**

Title: *Essential dimension and error-correcting codes*

Abstract: Let p be a prime, $r \geq 3$, and $n_i = p^{a_i}$ for positive integers a_1, \dots, a_r . Define $G = GL_{n_1} \times \dots \times GL_{n_r}$ and let μ be a central subgroup of G , over a field of characteristic zero. The Galois cohomology set $H^1(K, G/\mu)$ classifies r -tuples of central simple algebras satisfying linear equations in the Brauer group $Br(K)$. We study the essential dimension and essential p -dimension of G/μ . To any central subgroup μ , we associate a finite module C called the code associated to μ , and define a weight function w from C to the positive integers. We show that the essential dimension of G/μ depends only on C . Using general cohomological methods as well as results of Karpenko and Merkurjev, and Popov, we can give lower and upper bounds on the essential dimension of G/μ in terms of w and C . For some subgroups μ we find matching lower and upper bounds.

Speaker: **Zhihua Chang**

Title: *Twisted Loop Algebras Based on Conformal Superalgebras*

Abstract: Superconformal algebras are infinite dimensional Lie superalgebras of interest in theoretical physics. They are closely related to twisted loop algebras based on conformal superalgebras. To classify twisted loop algebras based on a given conformal superalgebra, differential conformal superalgebras were introduced by V. Kac, M. Lau, and A. Pianzola in 2009. In this talk, I will briefly introduce the general theory of twisted forms of differential conformal superalgebras, after which I will talk about the classification of twisted loop algebras based on the $N = 1, 2, 3$, small $N = 4$ and large $N = 4$ conformal superalgebras.

Speaker: **Vladimir Chernousov**

Title: *On the genus of a division algebra*

Abstract: Joint work with A. Rapinchuk and I. Rapinchuk. We define the genus $gen(D)$ of a finite-dimensional central division algebra D over a field K as the collection of all classes $[D]$ in the Brauer group $Br(K)$ that are represented by central division K -algebras D' having the same maximal subfields as D . In the talk we describe some situations where $gen(D)$ reduces to a single element and where it can be guaranteed to be finite.

Speaker: **Ivan Dimitrov**

Title: *Constructing subrepresentations via the Borel-Weil-Bott theorem*

Abstract: An embedding $G \subset G'$ of reductive algebraic groups gives rise to an embedding $G/B \subset G'/B'$ of the corresponding homogeneous varieties. For any line bundle L' on G'/B' one has the natural map of cohomologies $\pi : H^q(G'/B'; L) \rightarrow H^q(G/B, L)$, where L is the restriction of L' to G/B . The Borel-Weil-Bott theorem implies that the dual map π , when nonzero, is a G -module homomorphism $\pi : V \rightarrow V'$, where V and V' are irreducible modules respectively over G and G' . Varying L' (and respectively q) so that $H^q(G'/B'; L) = (V')^*$ we obtain a purely geometric construction of certain irreducible G -submodules of V' which we call cohomological components. In this talk I will discuss several types of embeddings $G \subset G'$ and the corresponding cohomological components their properties, relationship to other interesting problems as well as necessary and sufficient conditions for nonvanishing of π . In the case when G is embedded diagonally into $G' = G \times G$, the cohomological components lie on faces of the Littlewood-Richardson cone of codimension equal to the rank of G . With an appropriate choice of an embedding of G/B , one can also obtain generators of the algebra of invariant polynomials on the Lie algebra of G as cohomological components. I will mention results from a joint work with Mike Roth as well as results of Valdemar.

Speaker: **Mathieu Florence**

Title: *Central simple algebras of index p^n in characteristic p*

Abstract: Let k be a field of characteristic $p > 0$, and let A/k be a central simple algebra of index $d = p^n$ and exponent p^e . Using a result of Hochschild, of which we provide a new proof, we show that A is Brauer equivalent to the tensor product of at most $d - 1$ cyclic algebras of degree p^e . This improves drastically the previously known upper bounds, mainly due to Teichmüller, Mammone and Merkurjev.

Speaker: **Skip Garibaldi**

Title: *Algebraic groups and weak commensurability*

Abstract: The notion of weak commensurability introduced by Gopal Prasad and Andrei Rapinchuk gives deep connections between algebraic groups and differential geometry, as well as new tools for studying algebraic groups over number fields. We discuss recent applications to the questions:

- If two simple linear algebraic groups over a number field have the same isogeny classes of maximal tori, must the groups be isogenous?
- If two locally symmetric spaces M_1 and M_2 are weakly commensurable – i.e., if $\mathbb{Q} \cdot L(M_1) = \mathbb{Q} \cdot L(M_2)$ where L denotes the set of lengths of closed geodesics—must M_1 and M_2 have a common finite-sheeted cover?

Although there are well-known examples where the answer to each of these questions is “no”, the answer is nonetheless frequently “yes”.

Speaker: **Philippe Gille**

Title: *Topological properties of torsors and homogeneous spaces over valued fields*

Abstract: This is a report on work in progress with Laurent Moret-Bailly. Let K be the fraction field of a henselian valuation ring R of positive characteristic p . Let Y be a K -variety, H an algebraic group over K , and $f : X \rightarrow Y$ an H -torsor over Y . We consider the induced map $X(K) \rightarrow Y(K)$, which is continuous for the topologies deduced from the valuation. If Z denotes the image of this map, we investigate the following questions:

- (a) Is Z locally closed (resp. closed) in $Y(K)$?
- (b) Is the continuous bijection $X(K)/H(K) \rightarrow Z$ a homeomorphism ?

Speaker: **Olivier Haution**

Title: *Invariants of upper motives*

Abstract: Let H be a homology theory for algebraic varieties over a field k . To a complete k -variety X , one naturally associates an ideal of the coefficient ring $H(k)$. We show that, when X is regular, this ideal depends only on the upper Chow motive of X . This generalises the classical results asserting that this ideal is a birational invariant of smooth varieties for particular choices of H , such as the Chow group. When H is the Grothendieck group of coherent sheaves, we obtain a lower bound on the canonical dimension of varieties. When H is the algebraic cobordism, we give a new proof of a theorem of Levine and Morel. Finally we discuss some splitting properties of geometrically unirational field extensions of small transcendence degree.

Speaker: **Valentina Kiritchenko**

Title: *Schubert calculus for equivariant algebraic cobordism*

Abstract: I will talk about Schubert calculus in equivariant algebraic cobordism of complete flag varieties. This is joint work with Amalendu Krishna (arXiv:1104.1089 [math.AG]). For a connected reductive group G over a field k of zero characteristic and a k -split maximal torus T in G , the T -equivariant algebraic cobordism ring of the flag variety G/B (where B is a Borel subgroup in G) admits a Borel type presentation. There are also analogs of divided difference operators acting on this ring. These ingredients allow one to study Schubert calculus in equivariant cobordism, that is, generalized Schubert cycles (given by the classes of Bott-Samelson resolutions) and their multiplication. I will discuss results and open problems in this direction.

Speaker: **Nicole Lemire**

Title: *Stably Cayley Groups over Fields of Characteristic 0*

Abstract: A linear algebraic group is called a Cayley group if it is equivariantly birationally isomorphic to its Lie algebra. It is stably Cayley if the product of the group and some torus is Cayley. Cayley gave the first examples of Cayley groups with his Cayley map back in 1846. Over an algebraically closed field of characteristic 0, Cayley and stably Cayley simple groups were classified by Lemire, Popov and Reichstein in 2006. In recent joint work with Blunk, Borovoi, Kunyavskii and Reichstein, we classify the simple stably Cayley groups over an arbitrary field of characteristic 0.

Speaker: **Roland Löttscher**

Title: *Essential p -dimension of algebraic groups, whose connected component is a torus*

Abstract: Algebraic groups whose connected component is a torus arise naturally as normalizers of maximal tori in reductive groups. In this talk I will present joint work with Mark MacDonald, Aurel Meyer and Zinovy Reichstein, where we attempt to compute the essential p -dimension of such groups over fields of characteristic not p .

Speaker: **Alexander Merkurjev**

Title: *Generic values of quadratic forms and essential dimension*

Speaker: **Erhard Neher**

Title: *Derivations of algebras obtained by étale descent*

Abstract: We will describe derivations of Lie algebras obtained by étale descent and discuss applications to multiloop algebras and extended affine Lie algebras. The talk is based on joint work with Arturo Pianzola.

Speaker: **Raman Parimala**

Title: *Bounding symbol lengths in Galois cohomology*

Abstract: Bounding symbol lengths in Galois cohomology has had important implications to bounding the u -invariant of fields. This approach leads to finiteness of the u -invariant of function fields in one variable over a totally imaginary number field, provided a conjecture of Colliot Thelene on the Brauer-

Manin obstruction and the existence of zero cycles of degree one on smooth projective varieties over number fields holds.

Speaker: **Vladimir Popov**

Title: *Rational functions on semisimple Lie algebras and the Gelfand-Kirillov Conjecture*

Abstract: The talk is aimed at describing recent solution of the rationality problem for fields of rational functions on semisimple Lie algebras and the intimately related construction of counterexamples to the Gelfand–Kirillov conjecture on the fields of fractions of universal enveloping algebras of simple Lie algebras. This solution exploits a notion generalizing that of the usual torsor.

Speaker: **David Saltman**

Title: *Finite u Invariant and Bounds on Cohomology Symbol Lengths*

Abstract: At a AIM workshop in January 2011, Parimala asked whether in a field with finite u invariant there was a bound on the “symbol length” of any element of μ_2 cohomology in any degree. We answer this question in the affirmative for fields of characteristic 0, and at the same time get bounds on the Galois groups that realize all the properties of these cohomology elements and show that our results extend to finite field extensions.

Speaker: **Alexander Vishik**

Title: *Stable and Unstable operations in Algebraic Cobordism*

Abstract: We will show how to describe and effectively construct (unstable) additive operations $A \rightarrow B$, where A is a theory obtained from Algebraic Cobordism of M.Levine-F.Morel by change of coefficients, and B is any Generalized Oriented Cohomology Theory. Among the applications of this technique are the following major results:

1. Description of unstable operations in Algebraic Cobordism theory. The description of stable ones comes as well (much easier).
2. The Theorem claiming that multiplicative operations $A \rightarrow B$ (where A, B are as above) are in 1-to-1 correspondence with the homomorphisms of the respective formal group laws.
3. The construction of Integral (!) Adams operations in Algebraic Cobordism and all the theories obtained from it by change of coefficients (giving classical Adams operations in case of K_0).
4. The construction of Symmetric Operations for all primes p (previously known only for $p = 2$), and the construction of Tom Dieck - style Steenrod operations in Algebraic Cobordism.