An exercise on the hypergeometric method: BIRS 2012

Exercise M1. (a) Find an explicit irrationality measure for $\theta = \sqrt[3]{3}$, i.e. an inequality of the form

$$\left| \sqrt[3]{3} - \frac{p}{q} \right| > c \cdot q^{-\lambda}$$

with $\lambda < 3$. (Hint : consider z = -1/8).

(b) Use this inequality to solve the Thue inequality

$$\left| x^3 - 3y^3 \right| \le 100.$$

- (c) Try something similar with $\theta = \sqrt[3]{5}$.
- (d) Do you think the method works for $\theta = \sqrt[3]{14}$?