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My current research projects related to cluster algebras are as follows.

In joint work with Yuji Kodama, we demonstrate a link between cluster algebras and soliton solutions to the KP equation. More specifically, for each point $A \in (Gr_{k,n})_{\geq 0}$, there is a soliton solution $u_A(x, y, t)$ to the KP equation. If one fixes the time t, one can draw the *contour plot* of the solution, the locus in the plane where the solution is maximized. Such solutions to the KP equation model shallow water wave, and one may think of this contour plot as showing the locations of the peaks of the waves. What we've shown is a precise relation between these contour plots and the *reduced plabic graph* or *Postnikov diagrams* that correspond to clusters in the cluster algebra of the coordinate ring of the Grassmannian. More generally one can get a solution to the KP equation from any point A in the real Grassmannian (not necessary the non-negative part), and we are working to extend our results to this case. We are in the process of proving that any *regular* soliton solution that arises in this way must actually come from $(Gr_{k,n})_{\geq 0}$.

I also have two works in progress on cluster algebras from surface. In joint work with Musiker and Schiffler, we have constructed (conjectural) vector space bases for cluster algebras from surfaces, using principal coefficients. We have verified this conjecture for unpunctured surfaces. The main idea is to parameterize elements of the basis by collections of (tagged) arcs and also *closed loops* in the surface, and extend the notion of g-vector to closed loops. We define the elements associated to closed loops combinatorially, associating a certain *band graph* (on an annulus or Mobius strip) to each closed loop, and taking a weighted sum of the *good* matchings of that band graph. One can show that the set of good matchings has the structure of a distributive lattice, which in turn implies that the corresponding cluster algebra element has a well-defined g-vector. One then needs to show that our basis elements span (by proving a number of skein relations), and show that their g-vectors are all distinct. (Conceptually there is no major problem with extending our proofs to surfaces with punctures, but checking the 20 or so different kinds of skein relations involving notched arcs seems very painful.)

Musiker and I also have a paper in progress which provides some necessary tools for the paper on bases. This paper with Musiker provides expressions for our basis elements in terms of products of two by two matrices, and then proves skein relations, using principal coefficients. These results generalize results of Fock and Goncharov for the coefficient-free case. Note that we need such results for principal coefficients for our paper on vector space bases of these cluster algebras, in order to make sense of g-vectors.