- 1. Cluster automorphisms: Ibrahim Assem, Vasilisa Shramchenko and I have introduced the notion of cluster automorphism of a cluster algebra \mathcal{A} as an automorphism of the **Z**-algebra \mathcal{A} which sends a cluster to a cluster and commutes with the mutations at that cluster. We study the cluster automorphism group for acyclic cluster algebras and for cluster algebras from surfaces. Using the combinatorial structure of the Auslander-Reiten quiver of the cluster category, we compute the cluster automorphism group in the finite types and in the euclidean (i.e. affine) types. Using the surface approach, we show that (a variation of) the mapping class group of the surface is isomorphic to a subgroup of the cluster automorphism group.
- 2. Cluster algebras and representation theory: Relation between tilting theory and cluster-tilting theory. Cluster-tilted algebras are endomorphism algebras (over the cluster category) of cluster-tilting objects, so to every cluster in a cluster algebra corresponds a cluster-tilted algebra. Tilted algebras are endomorphism algebras (over a hereditary algebra) of a tilting module. There is a surjective map ϕ from tilted algebras to cluster-tilted algebras given by taking a trivial extension of the tilted algebra. The map ϕ is defined on all algebras of global dimension at most two, and there are many examples of algebras that are not tilted but whose image under ϕ is cluster-tilted. In collaboration with Lucas David-Roesler, we are studying the algebras whose image under ϕ are cluster-tilted algebras of type corresponding to surfaces without punctures.
- 3. Expansion formulas for cluster variables in rank 2: Kyungyong Lee and I have found a combinatorial formula for the cluster variables of skew-symmetric cluster algebras of rank two in terms of subpaths of a specific lattice path in the plane. The formula is manifestly positive, providing a new proof of a result by Nakajima and Qin.
- 4. Cluster algebras from surfaces: Gregg Musiker, Lauren Williams and I have found combinatorial formulas for the cluster variables in cluster algebras from surfaces. Cluster variables correspond to arcs in the surface. The formula is in terms of perfect matchings of a so-called snake graph associated to the arc of the cluster variable. Adapting the combinatorial formulas in order to associate cluster algebra elements also to other curves in the surface, we are now working on constructing bases for the corresponding cluster algebra.

The order in this list is alphabetical on the names of the authors.