# RESEARCH INTERESTS CONNECTED WITH CLUSTER ALGEBRAS – PIERRE-GUY PLAMONDON

Cluster algebras. Cluster algebras, introduced by S. Fomin and A. Zelevinsky [3], are commutative algebras with a distinguished set of generators called *cluster variables*. These generators are constructed using an iterated process called *mutation*. The combinatorial information needed to initiate this process is encoded in a finite quiver.

Additive categorification of cluster algebras. The combinatorial phenomena appearing in the theory of cluster algebras are known to be analogous to others appearing inside some triangulated categories. Namely, given a finite quiver without loops or 2-cycles and a non-degenerate potential on that quiver, C. Amiot [1] defined the cluster category associated to that quiver with potential. Using a formula similar to that of P. Caldero and F. Chapoton, one can then prove a series of nice correspondences between the cluster algebra and the cluster category associated to the same quiver. For instance, cluster variables in one correspond to some indecomposable objects in the other; mutation, to some distinguished triangles; products, to direct sums; **g**-vectors, to indices; and so on. The rich structure of cluster categories provides us with a good means to prove properties of cluster algebras (see, for instance, [5]).

Generic bases of cluster algebras. One of the main problems in the theory of cluster algebras is that of finding a basis which should, among other properties, contain a set of distinguished elements called *cluster monomials*. Despite several results for some classes of examples, the best ones being those of C. Geiss, B. Leclerc and J. Schröer [4], this problem is still largely open. One possible way of attacking it is by using the additive categorification described above. As in the work of G. Dupont [2], to each index in the cluster category (equivalently, to each $n$-tuple of integers) is associated an element of the *upper* cluster algebra. This element is defined to be the "generic" value taken by the Caldero–Chapoton formula under certain conditions given by the index. The elements thus constructed can be shown (see [6]) to be linearly independent, to be well-behaved with respect to mutation and to form the basis of [4] in the cases considered there. Hopefully, they form a basis of the upper cluster algebra in general.

## References


[1] Claire Amiot. *Cluster categories for algebras of global dimension 2 and quivers with potential*, Annales de l'Institut Fourier, Vol. 59 no. 6 (2009), p. 2525-2590.

[2] Grégoire Dupont. *Generic Cluster Characters*, International Mathematics Research Notices IMRN, rnr024, 2011.

[3] Sergey Fomin and Andrei Zelevinsky. *Cluster algebras I: Foundations*, J. Amer. Math. Soc. 15 (2002), no. 2, 497-529.

[4] Christof Geiss, Bernard Leclerc and Jan Schröer. *Generic bases for cluster algebras and the Chamber Ansatz*, to appear in J. Amer. Math. Soc., arXiv:1004.2781v2 [math.RT].

[5] Pierre-Guy Plamondon. *Cluster algebras via cluster categories with infinite-dimensional morphism spaces*, to appear in Comp. Math., arXiv:1004.0830v2 [math.RT].

[6] Pierre-Guy Plamondon. *Catégories amassées aux espaces de morphismes de dimension infinie, applications*, PhD thesis, available at the author's webpage.



*Date*: August 23, 2011.