Frontiers in Complex Dynamics
(Celebrating John Milnor's 80th birthday)

Araceli Bonifant (University of Rhode Island),
Mikhail Lyubich (Stony Brook University),
Scott Sutherland (Stony Brook University)

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1 Overview of the Field

The field of holomorphic dynamics, founded in the early 20^{th} century by Fatou and Julia, experienced explosive development in the 1980s and 1990s after the discovery of the Milnor-Thurston Kneading Theory, the appearance of pictures of the Mandelbrot set, and the proof of Sullivan's No Wandering Domain Theorem. In the 21^{st} century, new significant results emerged from the field, such as the construction of Julia sets of positive measure, major progress towards the long-standing MLC conjecture, a deeper understanding of various parameter slices, and applications of the pluri-potential theory to multi-dimensional dynamics. Interactions with other fields such as Kleinian groups, Teichmüller theory, hyperbolic and complex geometry, statistical physics and numerical analysis, have always played an insightful and stimulating role in the field. These developments are important for both mathematics and physics. The purpose of this conference was to examine and inspire further fruitful interactions in this field.

2 Recent Developments and Open Problems

The Fatou-Julia theory was one of the first applications of the concept of normal families as introduced by Montel. Indeed, Montel's theorem[1] plays a major role in Complex Dynamics. When one is interested in generalizing the Fatou-Julia theory to other spaces such as quasiregular maps, non-Archimedean maps or higher dimensional maps, one needs to have some version of Montel's theorem. For quasiregular maps[2] this

[1] A family of meromorphic functions omitting three fixed values is a normal family.

[2] A continuous mapping $f : D \to \mathbb{R}^n$ in the Sobolev space $W^{1,n}_{\text{loc}}(D, \mathbb{R}^n)$ is called K-quasiregular $K > 1$, if $|f'(x)|^n \leq K\,|\det f'(x)|$, a.e. $x \in D$. Here $n \geq 2$, $D \subset \mathbb{R}^n$ is a domain, $|f'(x)|$ is the operator norm of the differential of f. (In the plane, orientation preserving 1-quasiregular maps are precisely analytic functions of a single complex variable.)

was done by Miniowitz in [50]. An analog to Montel's theorem in higher complex dimensions was given by Fornæss and Sibony in [24] using the Kobayashi hyperbolicity of the complement of certain complex hypersurfaces in $\mathbb{CP}^k$. In his talk, Charles Favre presented a natural analog to Montel's theorem in a non-Archimedean context (see [23]).

As mentioned before, after remaining dormant for a long period of time, the study of holomorphic dynamical systems experienced major developments in the 1980's and 1990's. A major breakthrough took place in 2005, when Xavier Buff and Arnaud Chéritat (see [7] and [8]) proved the existence of Julia sets of positive measure.[3] A crucial tool in this development was the theory of renormalization for irrationally indifferent fixed points of holomorphic functions. Irrationally indifferent fixed points are always a source of interesting and delicate phenomena in complex dynamics. When the rotation number is of sufficiently high type (i.e. the coefficients in the continued fraction are large), the first return map to a certain fundamental region defines another irrationally indifferent function, which is called the *near-parabolic renormalization*. This theory was developed by Hiroyuki Inou and Mitsuhiro Shishikura in [28], and was presented at this conference by Shishikura.

The Classification theorem for Fatou components states that every periodic component U of the Fatou set of f is either an attracting basin, a parabolic basin, a Siegel disk (i.e., some iterate of f on U is analytically conjugate to an irrational rotation of the complex disc), or is a Herman ring (compare [63]). In 2004, Carsten Petersen and Saeed Zakeri proved by means of trans-quasiconformal surgery[4] on an associated Blaschke product model, that for almost every $0 < \theta < 1$ with θ irrational, the quadratic map $P_\theta = e^{2\pi i\theta}z + z^2$ had a Siegel disk whose boundary is a Jordan curve passing through the critical point of P_θ (see [55]). In fact they extended the class of rotation numbers for which such a surgery is possible to a much larger class PZ. In his talk, Arnaud Chéritat described how Gaofei Zhang has proved that trans-quasiconformal surgery is also possible for the matings studied by Michael Yampolsky and Saeed Zakeri in [68].

An interesting question related to parabolic cycles is whether or not the filled Julia set $\mathcal{K}(f)$ associated to a polynomial map of degree f and its corresponding Julia set $\mathcal{J}(f)$ could depend continuously on f. This question was addressed by Adrien Douady in 1994 (see [20]), who proved that: *If f_0 is a polynomial map without parabolic cycles, then $f \mapsto \mathcal{K}(f)$ is continuous at f_0*. Remaining open was the question of the possible limits of the filled Julia set $\mathcal{K}(f(z))$ as f approaches a polynomial map with a parabolic cycle. In his talk, John Hubbard discussed the limit of the filled Julia set $\mathcal{K}(f_c)$ in the Hausdorff metric when $f_c(z) = z^2 + c$ as c approaches a parabolic value. To do this he used parabolic blow-ups and a projective limit.

Another important question related to parabolic points and to continuity is the one raised by John Milnor in [45] in which he conjecture that: *The parabolic Mandelbrot set $\mathcal{M}_1$ is homeomorphic to the Mandelbrot set $\mathcal{M} = \mathcal{M}_0$. Moreover, $\mathcal{M}_\lambda$ tends to $\mathcal{M}_1$ in the Hausdorff topology when $\lambda \to 1$.* This conjecture was recently proved by Carsten Petersen and Pascal Roesch (see [53]). To approach this conjecture they defined the notion of parabolic ray (the analog of external/internal rays in super-attracting basins) to construct parabolic puzzles which are similar to Yoccoz puzzles, but modified near the parabolic fixed point. The natural space to study these questions is the moduli space $\mathscr{M} = \mathcal{R}at_2/PSL(2,\mathbb{C})$, the set of all conjugacy classes of rational mappings of degree two, which John Milnor proved to be biholomorphic to $\mathbb{C}^2$ (see [45]).

Sarah Koch's talk made use of the moduli spaces for the 2-sphere with finitely many punctures. In her

[3]Adrien Douady was very interested in this problem even before 1998 when he assigned it to Chéritat as a PhD problem.

[4]Trans-quasiconformal surgery is a surgery where the invariant Beltrami form $\mu\ d\bar{z}/dz$ used in the surgery is a David-Beltrami form. A Beltrami form $\mu\ d\bar{z}/dz$ form in a domain U is called a David-Beltrami form if there are positive constants C, α, ϵ_0 such that $\text{area}\{z \in U : |\mu|(z) > 1 - \epsilon\} \leq Ce^{-\alpha/\epsilon}$ for all $\epsilon < \epsilon_0$.

talk, she studied the self-map g of the moduli space $\mathscr{M}_{\mathcal{P}}$ corresponding to a Thurston map[5] and constructed a "natural" compactification of the moduli space $\mathscr{M}_{\mathcal{P}}$ for which the map g is algebraically stable[6].

Also related to Thurston maps was the talk of Daniel Meyer, in which he considered Thurston maps that are expanding in a suitable sense. He showed that for these kind of maps, a suitable iterate $F = f^n$ is semiconjugate to $z^d : S^1 \to S^1$. That is, there is a Peano curve $h : S^1 \to S^2$ (onto) such that $F(h(z)) = h(z^d)$, where $d = \deg F$. This result is a generalization of a result of Milnor (see [46]), and in the group case corresponds to a result by Cannon-Thurston (see [11]).

Two postcritically finite maps $f,\, g : S^2 \to S^2$ with postcritical sets P_f and P_g respectively, are called *Thurston equivalent* if and only if there exists homeomorphisms $h_1\,, h_2 : S^2 \to S^2$ such that $g \circ h_1 = h_2 \circ f$, $h_1|_{P_f} = h_2|_{P_f}$, and h_1 and h_2 are homotopic relative to P_f. In [17], Adrien Douady and John Hubbard studied the behavior of Thurston's pullback map at infinity without specifying any structure at the boundary of the Teichmüller space. The question of finding the notion of the boundary for the Teichmüller space that would be appropriate for the problem is very natural. Thurston answered this question in the case where the Teichmüller space is given by surface diffeomorphisms. In his talk, Nikita Selinger explained that the characterization of rational maps is more complicated than the charaterization of diffeomorphisms by proving that there exist postcritically finite branched covers such that Thurston's pullback map does not extend to the Thurston boundary of the Teichmüller space. He defined the extension to the augmented Teichmüller space, and characterized the dynamics of Thurston's pullback map near invariant strata on the boundary of the augmented Teichmüller space (see [59]).

Considering a family of deformation spaces arising functorially from first principles in Teichmüller theory, Adam Epstein presented in his talk a methodology for proving the smoothness and transversality of dynamically natural subspaces of the moduli space of degree d rational maps, for example, those with specified critical orbit relations, cycles of specified period and multiplier, parabolic cycles of specified degeneracy and index, Herman ring cycles of specified rotation number, or some combination thereof (see [22]).

Curt McMullen's talk also touched on Teichmüller spaces. He divided his talk in two parts: The first part described how simple cycles of degree ≥ 2 play a central role in the combinatorics of the Mandelbrot set, the space of Blaschke products, Riemann surfaces, etc. (see [35]). The second part dealt with the Weil-Petersson metric[7] on Teichmüller spaces from the point of view of complex dynamics. By means of the thermodynamic formalism, he described very interesting results for the dimensions of the Julia sets, the moduli space of polynomials, and deformations of Blaschke products, among others (see [36]).

Renormalization in the Teichmüller disc was discussed in John Smillie's talk, who gave a geometric interpretation of the renormalization algorithm, and of the continued fraction map that he and Corinna Ulcigrai introduced in [60]. This algorithm provides us with a characterization of symbolic sequences for linear flows in the regular octagon. This algorithm is seen seen as renormalization on the Teichmüller disc of the octagon, and it is related to Teichmüller geodesic flow (see [61]). The connection made by Smillie and Ulcigrai is analogous to the classical relation between Sturmian sequences,[8] continued fractions and geodesic flow on the modular surface.

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[5]A Thurston map is a postcritically finite branched covering map $f : (S^2, \mathcal{P}) \to (S^2, \mathcal{P})$ with finite postcritical set $\mathcal{P}$.

[6]Geometrically, a meromorphic self-map f from a compact complex manifold to itself is algebraically stable if no hypersurface is contracted to something of lower dimension which is contained in the indeterminacy set of f.

[7]Let $\mathrm{Teich}(S)$ denote the Teichmüller space of S hyperbolic Riemann surface, which consists of all equivalence classes $[S, f, X]$ of marked Riemann surfaces (S, f, X), where $f : S \to X$ is a quasiconformal mapping. The Weil-Petersson norm $|\varphi|_{\mathrm{WP}}$ of $\varphi(z)\, dz^2$ is defined by $|\varphi|_{\mathrm{WP}} = \displaystyle\int_X \rho^{-2}(z) |\varphi(z)|^2 \, |dz|^2$ here $\varphi(z)\, |dz|$ is the hyperbolic metric on X.

[8]Sturmian sequences were introduced in the 19th. century, then studied by Marston Morse and Gustav Hedlund (see [51] and [52]) in the 1930's. They showed that Sturmian sequences provide a symbolic coding of the orbit of a point on a circle with respect to a rotation by an irrational number.

Carsten Petersen started his talk by revisiting the notion of conformal barycenter of a measure on the n-dimensional sphere S^n, as defined in 1986 by Adrian Douady and Clifford Earle (see [16]). Douady and Earle used the conformal barycenter to define conformally natural extensions of self-homeomorphisms of S^n. The aim of Petersen's talk was to extend rational maps from the Riemann sphere $\overline{\mathbb{C}} \cong S^2$ to the (hyperbolic) 3-ball B^3 and thus to S^3 by reflection (see [54]).

The study of the dynamics of cubic polynomials in $\mathbb{C}$, was initiated by Bodil Branner in [3] and it continued in a series of papers that she wrote jointly with John Hubbard (see [4] and [5]). Denote by $\mathcal{S}_p$ the parameter space for cubic polynomials with one marked (periodic) critical point a of period $p \geq 1$ and another critical point free. It was proved by Milnor in [47] that $\mathcal{S}_p$ is a smooth affine algebraic curve, who also conjectured in the same paper that $\mathcal{S}_p$ is always connected[9]. The study of $\mathcal{S}_p$ has proven to be difficult due to the prescence of branch points. Trying to get some insight into the geometry of this curve, Hiroyuki Inou described in his talk how he is using "Stereo-Viewer" a program developed by Ushiki [65], to visualize the bifurcation locus of $\mathcal{S}_p$ and its bifurcation measure $\mu_{\rm bif} = cT_+ \wedge T_-$, where $T_\pm = dd^c G_\pm$ and $G_\pm(a,v) = \lim_{n\to\infty} \frac{1}{3^n} \log^+ |F_{a,v}^{\circ n}(z)|$.

The existence of partially defined semiconjugacies[10] between rational functions acting on the Riemann sphere was established in Vladlen Timorin's talk. As an application of this result he considers the set $\mathcal{R}_2$, of Möbius conjugacy classes of quadratic rational functions with marked critical points. Following Mary Rees [58] and Milnor [44], he considers the slice $Per_k(0) \subset \mathcal{R}_2$ defined by the condition that the second critical point is periodic of period k, and says that a critically finite rational function $R \in Per_k(0)$ is of type C if the first critical point is eventually mapped to the second (periodic) critical point but does not belong to the cycle of the second critical point. In this way he obtains a partial semiconjugacy from a type C map in $Per_k(0) \subset \mathcal{R}_2$ to almost any rational function with a period k critical orbit (see [66]).

Fundamental examples in complex dynamics arise as algebraic self-maps of algebraic varieties. Arithmetic dynamics refers to the study of number theoretic phenomena arising in dynamical systems on algebraic varieties. The theory of p-adic (nonarchimedean) dynamics draws much of its inspiration from classical complex dynamics. As in complex dynamics, a fundamental question is to characterize orbits by their topological or metric properties. Recent progress in p-adic dynamics, especially in dimension one, has benefited from the introduction of Berkovich space into the subject. In his talk, Jan Kiwi established a correspondence between rescaling limits[11] and some repelling periodic orbits in the Berkovich projective line[12] over the field of Laurent series.

Complex dynamical objects such as Julia sets, limit sets of quasi-Fuchsian groups, and so on, provide a rich source of examples of quasicircles with Hausdorff dimension greater than one. Astala conjectured in [2] that $1+k^2$ is the optimal bound on the dimension of K-quasicircles, where $k=(K-1)/(K+1)$. In 2009, Smirnov proved this conjecture (see [62]). The question of sharpness is an open problem with important connections to extremal behavior of harmonic measure (see [56]). In his talk Stanislav Smirnov, presented these themes explaining the intricate connections to the multifractal structure of harmonic measure which he is working on with Astala and Prause. In the case of conformal maps the multifractal analysis of harmonic measure provides a suitable framework for discussing the compression and expansion phenomena. He described how the multifractality of harmonic measure is reflected in the singularity of the welding.

[9] It is known that this curve is connected for periods $p \leq 5$.

[10] These semiconjugacies are defined on the complements to at most one-dimensional sets, and are holomorphic in a certain sense.

[11] In the literature rescaling limits appear as a relevant tool to study parameter spaces of rational maps near "infinity".

[12] The Berkovich affine line $\mathbb{A}^1_{\rm Berk}$ over K is a locally compact, Hausdorff, and path-connected topological space which contains K (with the topology induced by the given absolute value) as a dense subspace. The Berkovich projective line $\mathbb{P}^1_{\rm Berk}$ is obtained by adjoining to $\mathbb{A}^1_{\rm Berk}$ in a suitable manner a point at infinity; the resulting space $\mathbb{P}^1_{\rm Berk}$ is a compact, Hausdorff, and path-connected topological space which contains $\mathbb{P}^1(K)$ (with its natural topology) as a dense subspace.

Regarding topological entropy, John Milnor (see [43]) posed the Monotonicity Conjecture: *Within a family of real multimodal polynomial interval maps, the set of parameters for which the topological entropy*[13] *is constant is a connected set.* This conjecture was proved for quadratic maps by Milnor and William Thurston (see [48]) and for cubic maps by Milnor and Charles Tresser, (see [49] and also [18]).[14] In [6], Henk Bruin and Sebastian van Strien proved the general case of this conjecture. In his talk, van Strien described the main ideas of his work with Bruin and possible extensions of it.

William Thurston in his lecture characterized the values of entropy that can occur for postcritically finite maps.[15] In fact, a real number $a > 1$ is the topological entropy of a critically finite map of an interval to itself if and only if e^a is a weakly Perron number.[16] He proved the existence of a Mandelbrot-like containing all Galois conjugates of e^a for a critically finite quadratic map.

The study of dynamical systems is in many cases concerned with actions of $\mathbb{R}$ or $\mathbb{Z}$ on manifolds, i.e., with flows and diffeomorphisms. However it is possible to consider instead dynamical systems defined by actions of larger discrete or continuous groups. For infinite discrete groups, one possibility is *Zimmer's program* which seeks an understanding of actions of large groups on compact manifolds. Let G be an almost simple real Lie group. The *real rank* $rk_{\mathbb{R}}(G)$ of G is the dimension of a maximal abelian subgroup A of G that acts by $\mathbb{R}$-diagonalizable endomorphisms in the adjoint representation of G on its Lie algebra. Let Γ be a lattice in a simple Lie group G. Zimmer's conjecture states that *if* Γ *acts faithfully on a compact manifold* M *by diffeomorphisms, then* $\dim(M) \geq rk_{\mathbb{R}}(G)$. Serge Cantat initiated the study of Zimmer's program in the holomorphic and Kählerian setting in [12] and [13]. In his talk, Cantat explained how he and Abdelghani Zeghib continue pursuing this program (see [14]), providing a new line in Zimmer's program, and characterizing certain type of complex tori by a property of their automorphisms groups.

In 1991, Manfred Denker and Mariusz Urbański established in [19] the thermodynamic formalism for holomorphic endomorphisms of the Riemann sphere $\overline{\mathbb{C}}$ and Hölder continuous potentials with suffciently small oscillation (i.e., for the class of Hölder continuous functions f for which the pressure is larger than $\sup f$.) The existence and uniqueness of equilibrium states was also proved there. The corresponding Perron-Frobenius operator was shown to be almost periodic, and the system exact with respect to the equilibrium measure (compare [57]). A natural question that arises is regarding the existence and uniqueness of equilibria in higher dimensions, for example in complex projective spaces of an arbitrary dimension. In her talk, Anna Zdunik described her work with Mariusz Urbański (see [64]), in which they built the thermodynamic formalism for all holomorphic endomorphisms $f : \mathbb{P}^k \to \mathbb{P}^k$ of all complex projective spaces of an arbitrary dimension $k \geq 1$, and also for all Hölder continuous potentials[17] $\phi : \mathcal{J} \to \mathbb{R}$, with suffciently small value (depending only on the endomorphisms $f : \mathbb{P}^k \to \mathbb{P}^k$) of their oscillation $\sup(\phi) - \inf(\phi)$. She also described her work with Michal Szostakiewicz and Urbański involving statistical properties of non-maximal measures introduced in [64].

The nature of the leaves of a holomorphic foliation as abstract Riemann surfaces has received much attention. Alexey Glutsyuk [25] and Alcides Lins-Neto [31] have shown that on a generic foliation $\mathscr{F}$ of degree d the leaves are covered by the unit disc in $\mathbb{C}$. (In this case one says that the foliation is hyperbolic.) More precisely, Lins-Neto has shown that this is the case when all singular points have non-degenerate linear part. In [10], Alberto Candel and Xavier Gómez-Mont have shown that if all singularities are hyperbolic, the Poincaré metric on leaves is transversally continuous. In his talk Nessim Sibony described the sequel of

[13] The topological entropy $h(f)$ of a map from a compact topological space to itself, $f : X \to X$, is a numerical measure of the unpredictability of trajectories $x,\ f(x),\ f^2(x),\ \ldots$ of points under f.

[14] All these results use complex methods, and a purely real proof is still unknown.

[15] A differentiable map f of an interval to itself is postcritically finite or critically finite if the union of forward orbits of the critical points is finite.

[16] A *weakly Perron number* is the root of an integer monic polynomial with no other roots of larger modulus.

[17] As above $\mathcal{J}$ denotes the Julia set of the map $f : \mathbb{P}^k \to \mathbb{P}^k$, i.e., the topological support of the measure of maximal entropy.

his joint work with Tien-Cuong Dinh and Viet-Anh Nguyên (see [15]) on the statistical behavior of leaves of a possibly singular foliation by Riemann surfaces. He discussed the following: A geometric ergodic theorem, á la Birkhoff (averages of leaves converge towards the dd^c closed current T for T-almost every leaf). Heat equation with respect to a harmonic measure (the unbounded geometry of the leaves because of singularities forces the authors to develop a new approach for heat diffusion). Transverse regularity for the Poincaré metric.

We move now to areas closely related to complex dynamical systems.

In 1968, motivated by the relation between curvature and fundamental group, John Milnor introduced the notion of growth of a finitely generated group. In [40], he asked *"Are there groups of intermediate growth between polynomial and exponential?"* and *"What are the groups with polynomial growth?"* The first question was answered negatively by Rotislav Grigorchuk in [26]. Milnor and Wolf showed that the growth type of the fundamental group gives information about the curvature of a manifold, and that solvable groups which are not virtually nilpotent have exponential growth (see [41], [42] and [67]). The relation of the growth of a group with its amenability[18] was discovered by Adelson-Velskiĭ (see [1]).

Milnor also formulated the major conjecture about the coincidence of the class of groups of polynomial growth with the class of groups containing a nilpotent subgroup of finite index (i.e. with the class of virtually nilpotent groups). This conjecture was proved by Gromov in [27]. In his talk, Grigorchuk gave an overview of the results around Milnor's problem in this area, and explained how it lead to the arising of new directions in group theory such as self-similar groups, branch groups and iterated monodromy groups, and how it stimulated an intensive study of groups generated by finite automata.

The talk of William Goldman had the objective of describing recent developments following Milnor's contributions to the theory of manifolds with flat structures. Goldman described how Milnor's 1958 paper "On the existence of a connection with curvature zero" ([39]) began the development of the theory of characteristic classes of flat bundles, foliations and bounded cohomology. He also talked about Milnor's 1977 paper, "On fundamental groups of complete affinely flat manifolds," and how this paper clarified the theory of complete affine manifolds, and set the stage for startling examples of Margulis 3-manifold quotients of Euclidean 3-space by free groups of affine transformations (see [33] and [34]).

The Ehrenpreis conjecture, formulated by Leon Ehrenpreis in [21], has been open since 1970. It asserts that *if S and R are two closed Riemann surfaces of genus at least 2, then for any $K > 1$, one can find finite degree covers S_1 and R_1 of S and R respectively, such that there exists a K-quasiconformal map $f : S_1 \to R_1$.*

In his talk, Jeremy Kahn explained the proof of this conjecture in joint work with Vladimir Markovic (see [29] and [30]). In their proof, they developed the notion of the *good pants homology*[19] and showed that it agrees with the standard homology on closed surfaces. Combining these results with their previous work on the Surface Subgroup Theorem (see [29]), they obtained a proof of the Ehrenpreis conjecture.

In 1953, Calabi and Eckmann (see [9]) produced one of the first examples of non-Kähler compact complex manifolds and found new principal holomorphic bundles over the product of projective spaces. In his talk, Alberto Verjovsky presented his work with Laurent Meersseman (see [38]) describing a construction of principal bundles over any quasi-regular projective toric variety, which contains the Calabi-Eckmann fibration as a particular case. The fiber is a compact complex torus (not always of dimension one) and the total space belongs to a class of non-Kählerian (or even symplectic) compact complex manifolds introduced

[18]A group G is *amenable* if it admits almost *invariant subsets*; i.e., for every $\epsilon > 1$ and every finite $S \subset G$, a finite $F \subset G$ such that $\#(F \cup FS) \leq \epsilon \# F$.

[19]Good pants are pairs of pants whose cuffs have the length nearly equal to some large number $R > 0$.

in [32] and generalized in [37]. This bundle is a Seifert bundle. Their construction produces non-singular holomorphic principal bundles above any smooth projective toric variety.

3 Presentation Highlights

All the talks given in this conference were of high academic quality. It is important to point out that this was the first conference in which the proof of the Ehrenpreis conjecture was presented. Jeremy Kahn did a magnificent job in presenting it. This proof was the result of his recent collaboration with Vladimir Markovic.

All of the talks were videotaped and can be found at

http://www.math.sunysb.edu/jackfest/Videos/

4 Scientific Progress Made

As explained in the first section of this report, there has been a lot of progress in the area of complex dynamical systems and associated fields, among others: The field of arithmetic dynamics has now an analog to Montel's theorem. There is a theory of near-parabolic renormalization and it has been proved that there exist Julia sets of positive measure. The continuity of the map that assigns to each f its corresponding filled Julia set $\mathcal{K}(f)$ has been established for polynomial maps with parabolic cycles. It has been proved that the Mandelbrot set $\mathcal{M}_\lambda$ tends to $\mathcal{M}_1$ as λ converges to 1. For the space of rational maps an augmented Teichmüller space has been defined, and it has been possible to characterize the dynamics of the Thurston pullback map near the invariant strata on the boundary of this space. There is a characterization of symbolic sequences for linear flows in the regular octagon. The Monotonicity conjectured has been proved in the general case. There has been a lot of progress in the study of Zimmer's program and new lines of this program have been opened. The statistical behavior of leaves of singular foliations by Riemann surfaces has been studied. Non-singular holomorphic principal bundles have been constructed above any smooth projective toric variety. The Ehrenparis conjecture has been proved.

5 Outcome of the Meeting

This meeting was very successful. The main objective of the meeting which was favoring interaction between senior researchers, young researchers, and graduate students was without any doubt attained. All of the attendees benefited greatly from the talks, which were of the highest academic quality. We learned the current state of the art of complex dynamics and associated fields and new collaborations between participants the conference were begun. Senior researchers learned about examples they were looking for in this conference. This was indeed a very fruitful conference at all levels. We heard extremely positive comments from a great percentage of our participants after the conference was over. The best of all was the fact that John Milnor, who had some health problems at the beginning of the conference, was able to join us in the middle of the week.

References

[1] G.M. ADEL′SON-VEL′SKIĬ AND YU. A. ŠREĬDER. The Banach mean on groups, *Uspehi Mat. Nauk (N.S.)*, **12** (6(78)) (1957), 131–136.

[2] K. ASTALA, Area distortion of quasiconformal mappings, *Acta Math.*, **173** (1994), 37–60.

[3] B. BRANNER, The parameter space for complex cubic polynomials,*Chaotic dynamics and fractals* (Atlanta, Ga., 1985), 169–179, Notes Rep. Math. Sci. Engrg., 2, Academic Press, Orlando, FL, 1986.

[4] B. BRANNER AND J.H. HUBBARD, The iteration of cubic polynomials. I. The global topology of parameter space, *Acta Math.*, **160** (1988), 143–206.

[5] B. BRANNER AND J.H. HUBBARD, The iteration of cubic polynomials. II. Patterns and parapatterns, *Acta Math.*, **169** (1992), 229–325.

[6] H. BRUIN AND S. VAN STRIEN, Monotonicity of entropy for real multimodal maps, *arXiv:0905.3377*

[7] X. BUFF AND A. CHÉRITAT, Ensembles de Julia quadratiques de mesure de Lebesgue strictement positive, *C. R. Math. Acad. Sci. Paris* **341** (2005), 669–674.

[8] X. BUFF AND A. CHÉRITAT, Quadratic Julia Sets with Positive Area, *Proceedings of the International Congress of Mathematicians Hyderabad, India*, 2010.

[9] E. CALABI, B. ECKMANN, A class of compact, complex manifolds which are not algebraic, *Ann. Math.*, **58** (1953), 494–500.

[10] A. CANDEL AND X. GÓMEZ-MONT, Uniformization of the leaves of a rational vector field, *Ann. Inst. Fourier (Grenoble)*, **45** (1995), 1123–1133.

[11] J. CANNON AND W. THURSTON Group invariant Peano curves, *Geometry & Topology* **11** (2007) 1315–1355.

[12] S. CANTAT, Version Kählérienne d'une conjecture de Robert J. Zimmer, *Ann. Sci. École Norm. Sup.*, **37** (2004), 759–768.

[13] S. CANTAT, Caractérisation des exemples de Lattès et de Kummer, *Compos. Math.*, **144** (2008), 1235–1270.

[14] S. CANTAT AND A. ZEGHIB, Holomorphic actions, Kummer example and Zimmer program, *arXiv:1011.4768* (2010).

[15] T-C. DINH, V-A. NGUYÊN AND N. SIBONY, Entropy for hyperbolic Riemann surface laminations Part II, *arXiv:1105.2307*.

[16] A. DOUADY AND C.J. EARLE, Conformally natural extension of homeomorphisms of the circle, *Acta Math.*, **157** (1986), 23–48.

[17] A. DOUADY AND J.H. HUBBARD A proof of Thurston's topological characterization of rational functions, *Acta Math.* **171** (1993), 263–297.

[18] S.P. DAWSON, R. GALEEVA, J. MILNOR AND C. TRESSER, A monotonicity conjecture for real cubic maps, *Real and Complex dynamical systems (B. Branner and P. Hjorth Eds)*, (Kluwer, Dordrecht, 1995) 165–183.

[19] M. DENKER AND M. URBAŃSKI, Ergodic theory of equilibrium states for rational maps, *Nonlinearity* **4** (1991), 103–134.

[20] A. DOUADY, Does a Julia set depend continuously on the polynomial? *Proceedings of Symposia in Applied Mathematics* **49** (1994), 91–.

[21] L. EHRENPREIS, Cohomology with bounds, In *1970 Symposia Mathematica, Vol. IV (INDAM, Rome)* (1968/69), 389–395, Academic Press, London.

[22] A. EPSTEIN, Transversality Principles in Holomorphic Dynamics, *Manuscript.*

[23] C. FAVRE, J. KIWI AND E. TRUCCO, A non-Archimedean Montel's theorem, *arXiv:1105.0746*.

[24] J.E. FORNÆSS AND N. SIBONY, Complex Dynamics in Higher Dimension I, *Complex analytic methods in dynamical systems (Rio de Janeiro,* 1992). *Astŕisque* **222** (1994), 201–231.

[25] A.A. GLUTSYUK, Hyperbolicity of the leaves of a generic one-dimensional holomorphic foliation on a nonsingular projective algebraic variety, *Proc. Steklov Inst. Math.*, **213** (1996), 83–103.

[26] R.I. GRIGORCHUK, Degrees of growth of finitely generated groups and the theory of invariant means, *Izv. Akad. Nauk SSSR Ser. Mat.*, **48** (1984), 939–985.

[27] M. GROMOV, Groups of polynomial growth and expanding maps, *Inst. Hautes Études Sci. Publ. Math.*, **53** (1981), 53–73.

[28] H. INOU AND M. SHISHIKURA, The renormalization of parabolic fixed points and their perturbation, *Preprint*, 2008.

[29] J. KAHN, V. MARKOVIC, Immersing almost geodesic surfaces in a closed hyperbolic three manifold, (2009) *arXiv:0910.5501*

[30] J. KAHN AND V. MARKOVIC, The good pants homology and a proof of the Ehrenpreis conjecture, (2011) *arXiv:1101.1330*.

[31] A. LINS NETO, Uniformization and the Poincaré metric on the leaves of a foliation by curves, *Bol. Soc. Brasil. Mat. (N.S.)*, **31** (2000), 351–366.

[32] S. LÓPEZ DE MEDRANO AND A. VERJOVSKY, A new family of complex, compact, non symplectic manifolds, *Bol. Soc. Mat. Bra.* **28** (1997), 253–269.

[33] G.A. MARGULIS, Free completely discontinuous groups of affine transformations, *Dokl. Akad. Nauk SSSR*, **272** (1983), 785–788.

[34] G.A. MARGULIS, Complete affine locally flat manifolds with a free fundamental group, *Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI)*, **134** (1984), 190–205.

[35] C. MCMULLEN, Dynamics on the unit disk: short geodesics and simple cycles, *Comment. Math. Helv.* **85** (2010), 723–749.

[36] C. McMullen, Thermodynamics, dimension and the Weil-Petersson metric, *Invent. Math.*, **173** (2008), 365–425.

[37] L. Meersseman, A new geometric construction of compact complex manifolds in any dimension, *Math. Ann.*, **317** (2000), 79–115.

[38] L. Meersseman and A. Verjovsky, Holomorphic principal bundles over projective toric varieties, *J. Reine Angew. Math.*, **572** (2004), 57–96.

[39] J. Milnor, On the existence of a connection with curvature zero, *Comm. Math. Helv.*, **32** (1958), 215–223.

[40] J. Milnor, Problem 5603, *Amer. Math. Monthly* **75** (1968), 685–686.

[41] J. Milnor, A note on curvature and fundamental group, *J. Differential Geometry*, **2** (1968), 1–7.

[42] J. Milnor, Growth of finitely generated solvable groups *J. Differential Geometry*, **2** (1968), 447–449. *Collected Papers of John Milnor V*, Editors: H. Bass and T.Y. Lam (2011), 155–157.

[43] J. Milnor, Remarks on iterated cubic maps, *Experimental Math.* **1** (1992) 5–24.

[44] J. Milnor, Geometry and Dynamics of Quadratic Rational Maps, *Experimental Math.*, **2** (1993), 37–83.

[45] J. Milnor, On rational maps with two critical points, *Experiment. Math.* **9** (2000) 481–522.

[46] J. Milnor, Pasting Together Julia Sets: A Worked Out Example of Mating, *Experiment. Math.* **13**, (2004), 55–92.

[47] J. Milnor, Cubic Polynomial Maps with Periodic Critical Orbit: Part I, in Complex Dynamics Families and Friends, ed. D. Schleicher, A. K. Peters 2009, pp. 333–411.

[48] J. Milnor and W. Thurston, On iterated maps of the interval, *Dynamical systems (College Park, MD, 198687)*, 465–563, *Lecture Notes in Math.*, **1342**, Springer, Berlin, 1988.

[49] J. Milnor and C. Tresser, On entropy and monotonicity for real cubic maps, *Commun. Math. Phys.*, **209** (2000), 123–178.

[50] R. Miniowitz, Normal families of quasimeromorphic mappings, *Proc. Amer. Math. Soc.* **84** (1982), 35–43.

[51] M. Morse and G.A. Hedlund, Symbolic Dynamics, *American Journal of Mathematics*, **60** (1938), 815–866.

[52] M. Morse and G.A. Hedlund, Symbolic Dynamics II: Sturmian trajectories, *American Journal of Mathematics*, **62** (1940), 1–42.

[53] C.L. Petersen and P. Roesch, Parabolic tools *Journal of Difference Equations and Applications*, **16** (2010), 715–738.

[54] C.L. Petersen, Conformally Natural extensions revisited, (2011) *arXiv:1102.1470*.

[55] C.L. PETERSEN AND S. ZAKERI, On the Julia set of a typical quadratic polynomial with a Siegel disk, *Annals of Mathematics*, **159** (2004), 1–52.

[56] I. PRAUSE AND S. SMIRNOV, Quasisymmetric distortion spectrum, *Bull. London Math. Soc.*, **43** (2011), 267–277.

[57] F. PRZYTYCKI, On the Perron-Frobenius-Ruelle operator for rational maps on the Riemann sphere and for Hölder continuous functions. *Bol. Soc. Bras. Mat.*, **20** (1990), 95–125.

[58] MARY REES, A partial description of the parameter space of rational maps of degree two: Part 1, *Acta Math.*, **168** (1992), 11–87.

[59] N. SELINGER, Thurston's pullback map on the augmented Teichmüller space and applications, *arXiv:1010.1690*.

[60] J. SMILLIE AND C. ULCIGRAI, Beyond Sturmian sequences: coding linear trajectories in the regular octagon, *Proc. London Math. Soc.* **102** (2011) 291–340.

[61] J. SMILLIE AND C. ULCIGRAI, Geodesic flow on the Teichm´uller disk of the regular octagon, cutting sequences and octagon continued fractions maps, *arXiv:1004.2265*.

[62] S. SMIRNOV, Dimension of quasicircles, *Acta Math.*, **205** (2010), 189–197.

[63] D. SULLIVAN, Quasiconformal homeomorphisms and dynamics I. Solution of the Fatou-Julia problem on wandering domains, *Ann. of Math.*, **122** (1985), 401–418.

[64] M. URBAŃSKI AND A. ZDUNIK, Ergodic Theory for holomorphic endomorphisms of complex projective spaces, preprint.

[65] S. USHIKI, Stereo Viewer, http://www.math.h.kyoto-u.ac.jp/ ushiki/

[66] V. TIMORIN, Partial holomorphic semiconjugacies between rational functions, *arXiv:1008.4670*.

[67] J.A. WOLF, Growth of finitely generated solvable groups and curvature of Riemanniann manifolds, *J. Differential Geometry*, **2** (1968) 421–446.

[68] M. YAMPOLSKY, AND S. ZAKERI, Mating Siegel quadratic polynomials, *J. Amer. Math. Soc.* **14** (2001), 25–78.