PLANCK Component Separation using a Sparsity Prior

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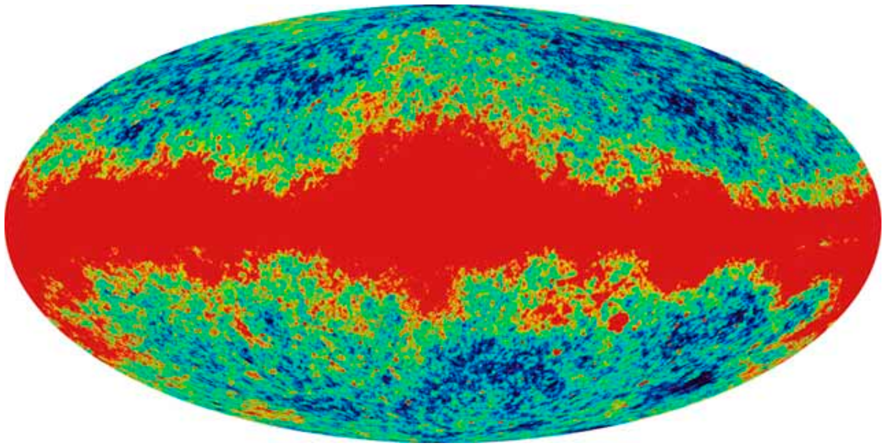
- Introduction
- Dictionaries on the Sphere
- Polarized Dictionaries on the Sphere
- Sparsity and Component Separation (temperature)
- Problems and new sparse approaches

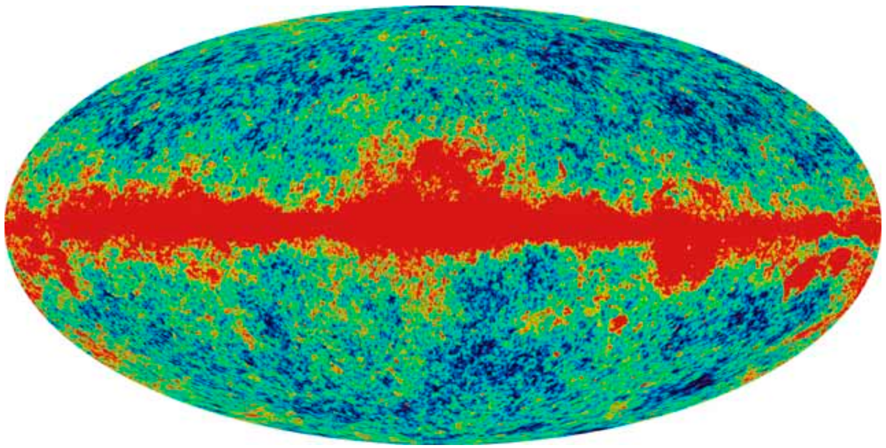
The Cosmic Microwave Background

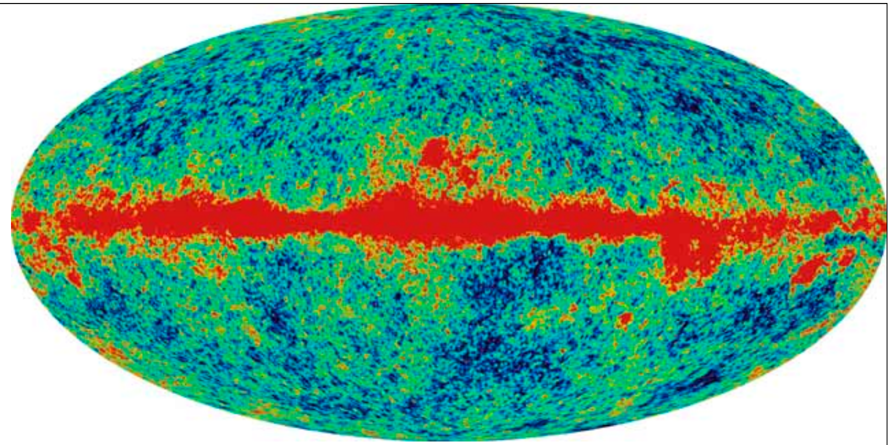
- The Universe is filled with a blackbody radiation field at a temperature of 3K.
- Predicted by G. Gamow in 1948
- Observed for the first time by Penzias and Wilson (1965)
- Confirmed by COBE (1990)
- Spectacular measurement of anisotropies by WMAP
- WMAP observed the CMB since 2002. Fifth and Last data release in August 2011.
- PLANCK first cosmological results in January 2013.

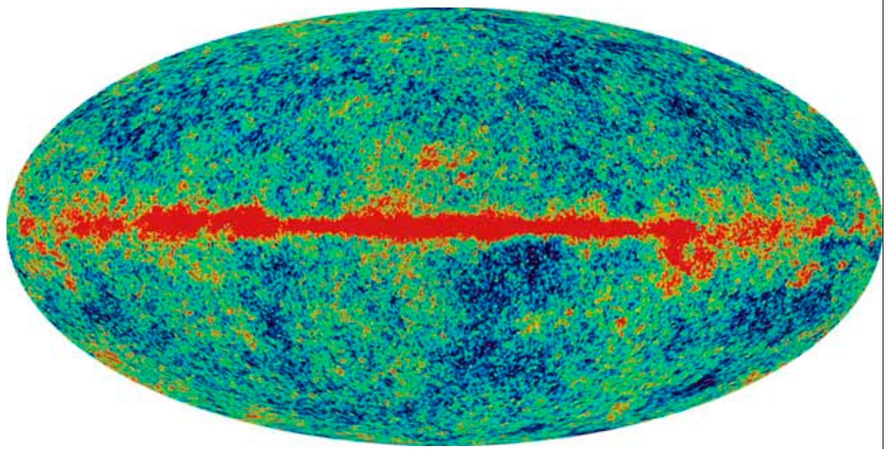
WMAP:five frequency maps

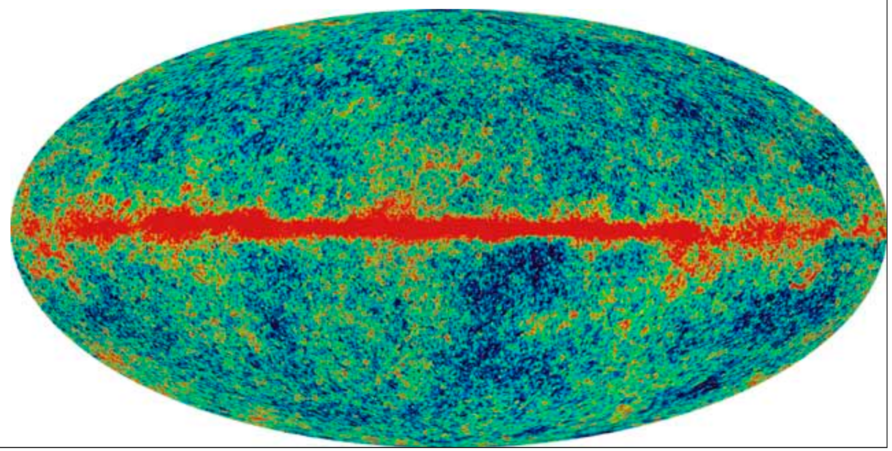


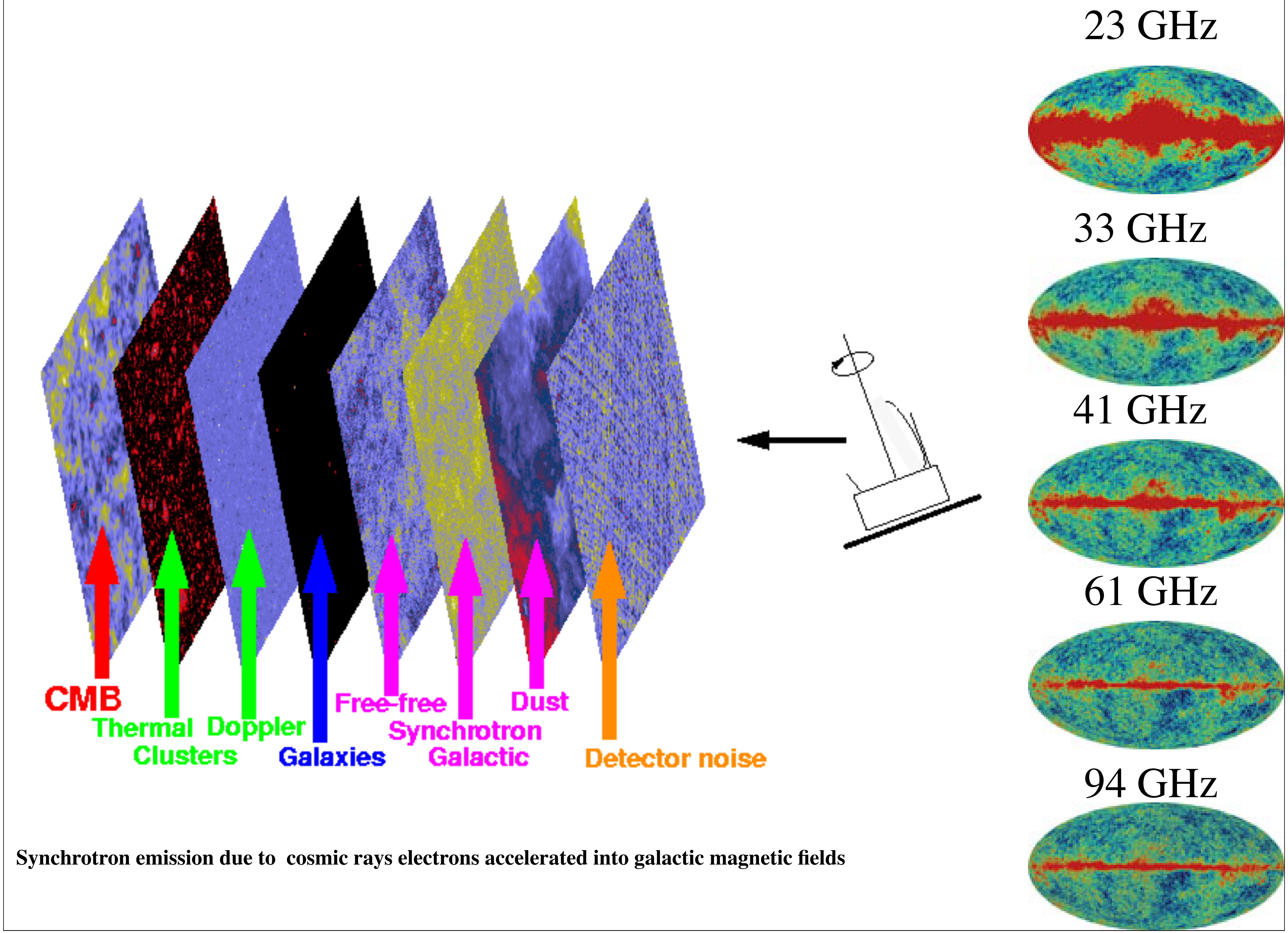
23, 33, 41, 61 and 93 GHz



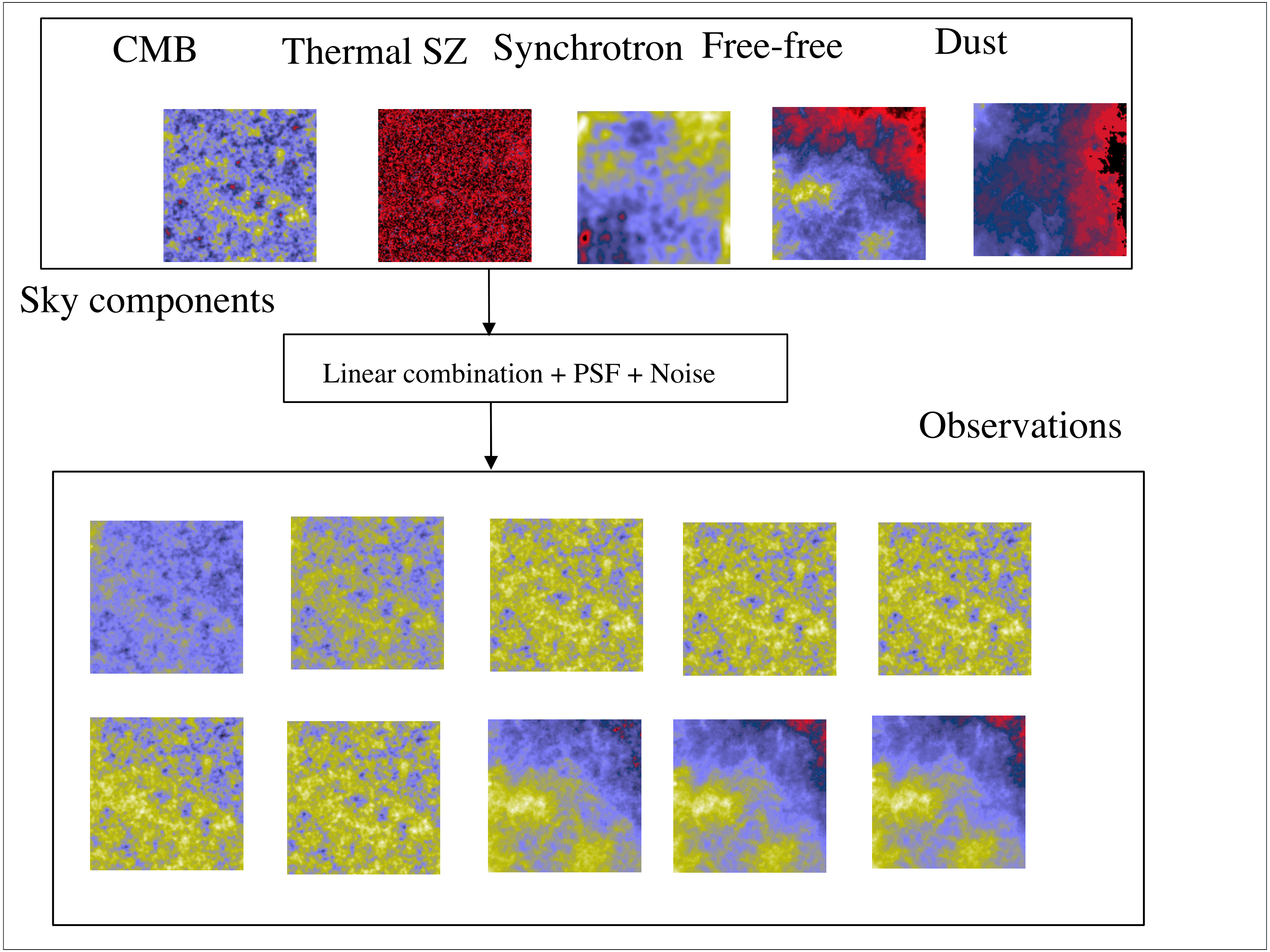




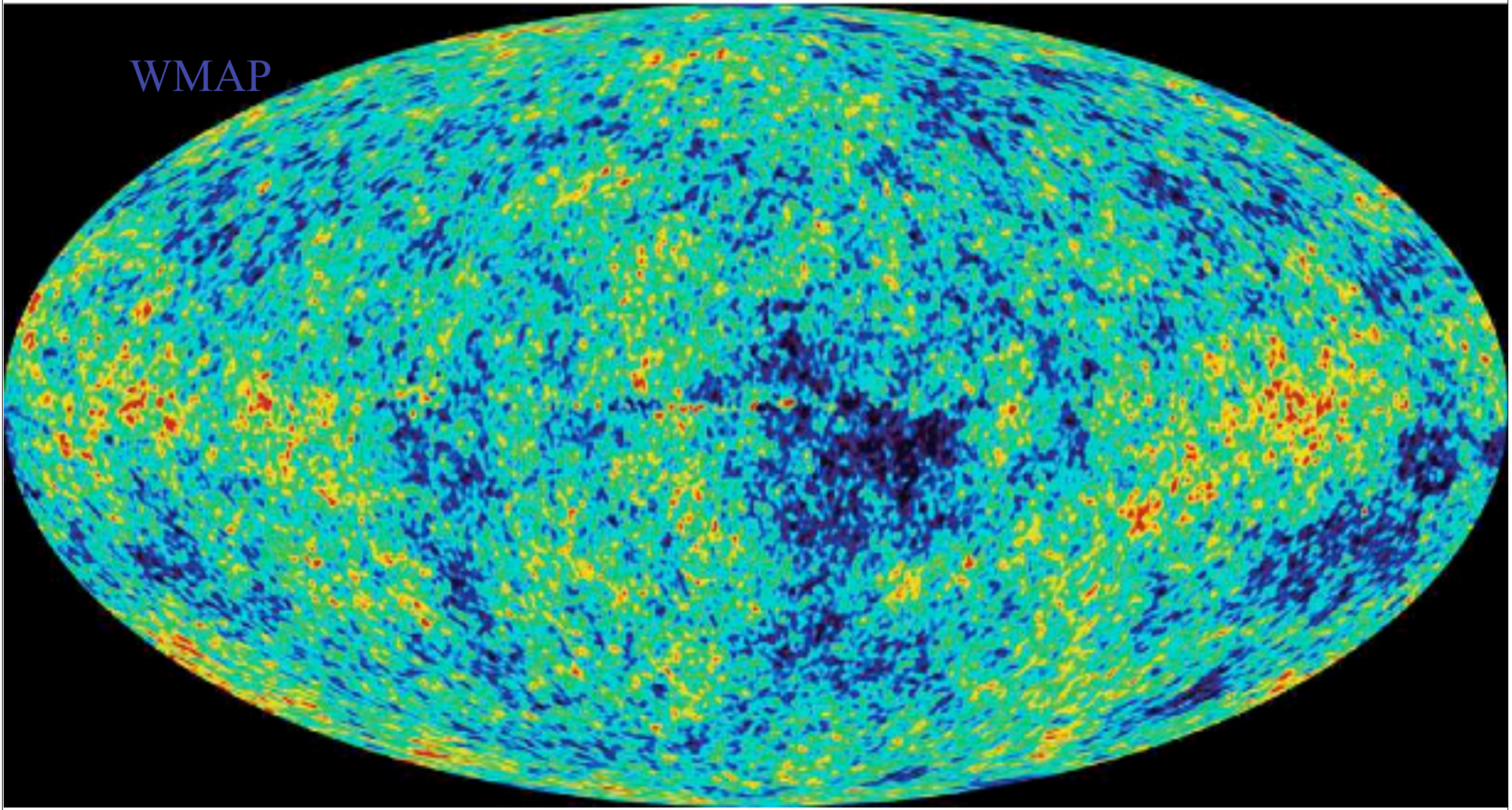




Synchrotron emission due to cosmic rays electrons accelerated into galactic magnetic fields

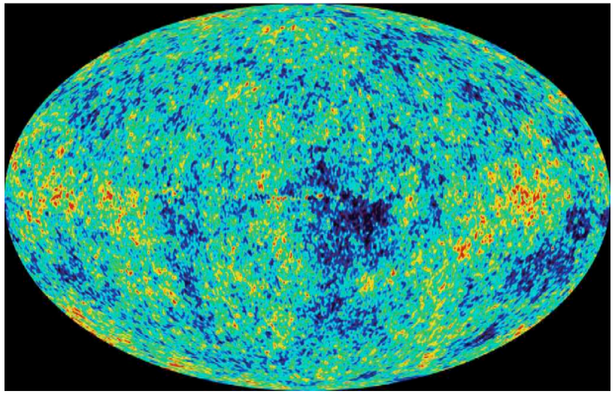

CMB
Thermal SZ
Synchrotron
Free-free
Dust
Sky components
Linear combination + PSF + Noise
Observations

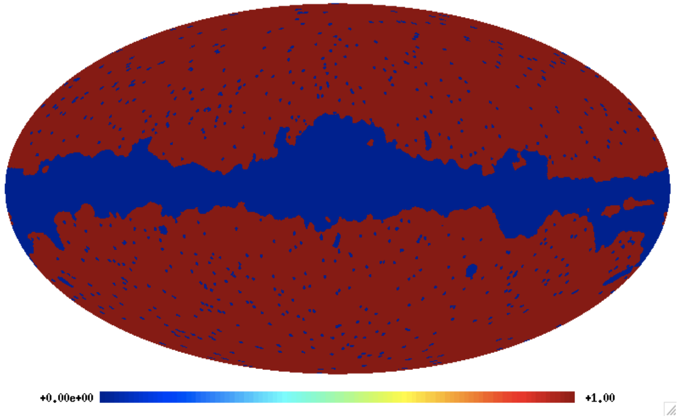
The CMB exhibits Fluctuations

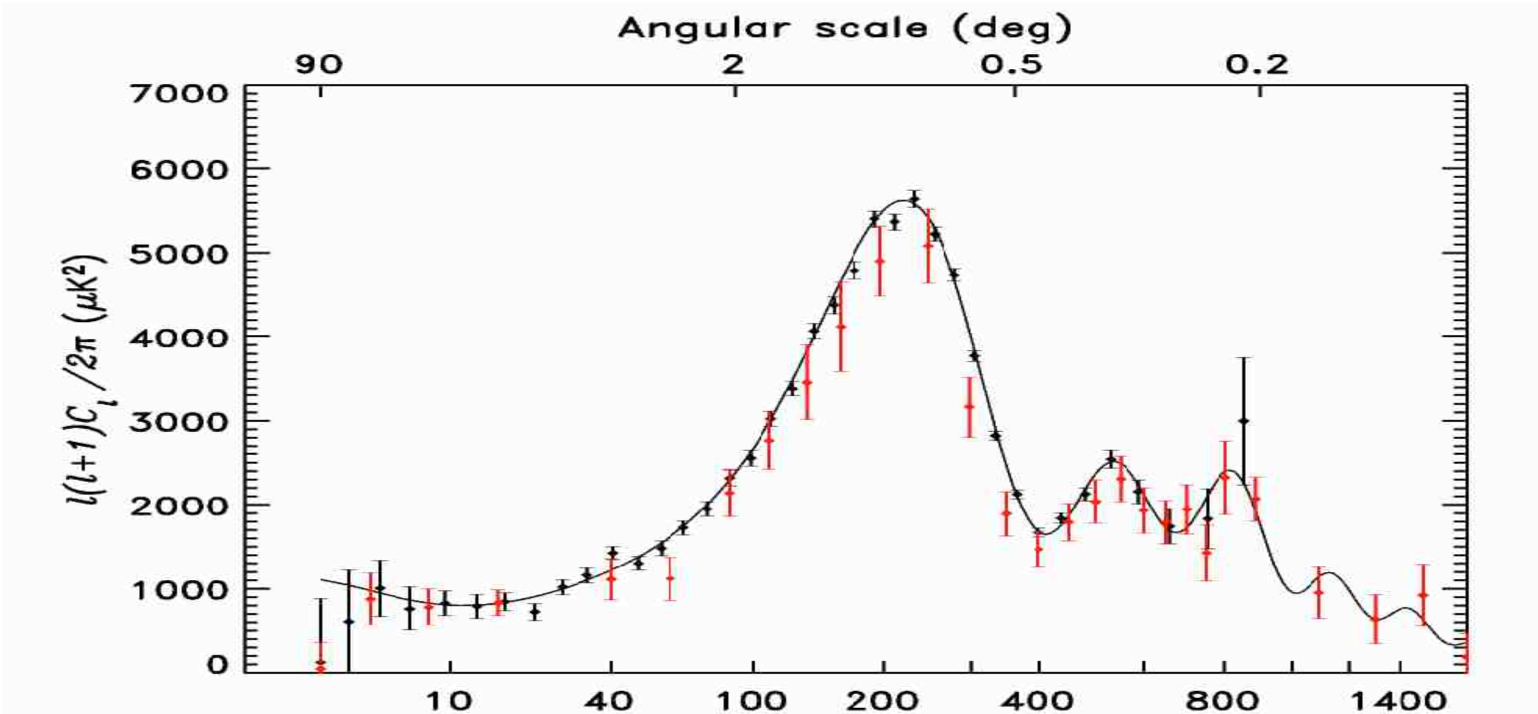


The Cosmic Microwave Background (CMB) is a relic radiation (with a temperature equals to 2.726 Kelvin) emitted 13 billion years ago when the Universe was about 370000 years old.

Power spectrum of WMAP







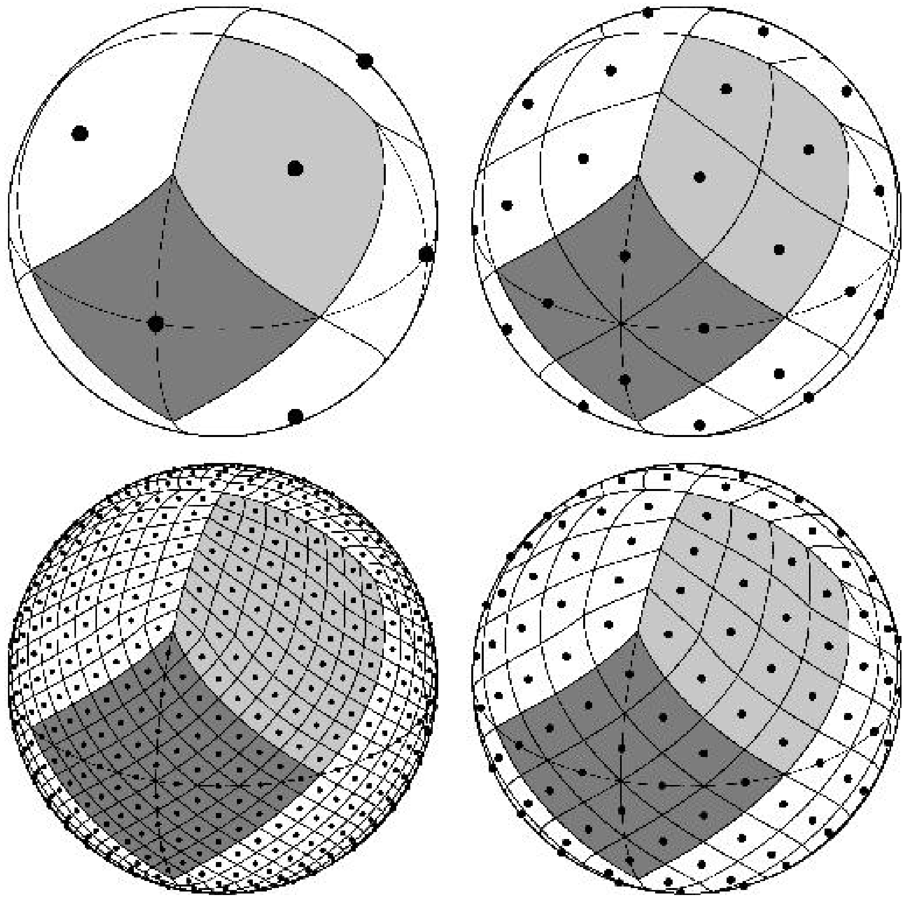
Healpix

K.M. Gorski et al., 1999, astro-ph/9812350,

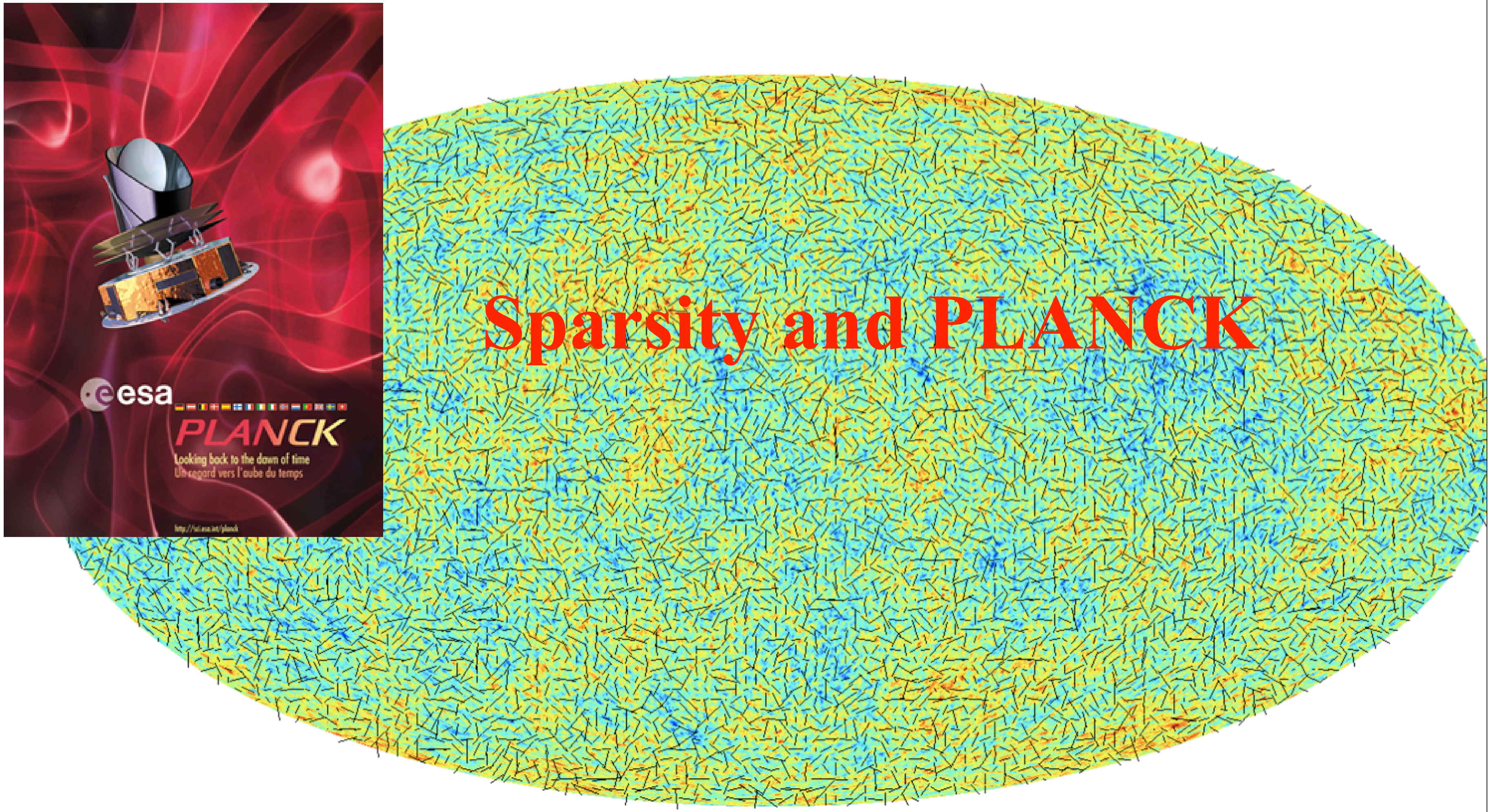
http://www.eso.org/science/healpix

- Pixel = Rhombus
- Same Surfaces
- For a given latitude : regularly spaced
- Number of pixels:

  $12 \times (N_{sides})^2$
- Included in the software:
  - Anafast
  - Synfast



Sparsity and PLANCK



- Successor of WMAP (better resolution, better sensitivity, more channels)
- Launched on May 14, 2009
- Two instruments LFI and HFI
- Nine maps at 30,44,70,100,143,217,353,545,857 GHz
- Angular resolutions: 33', 24', 14', 10', 7.1', 5', 5', 5', 5'

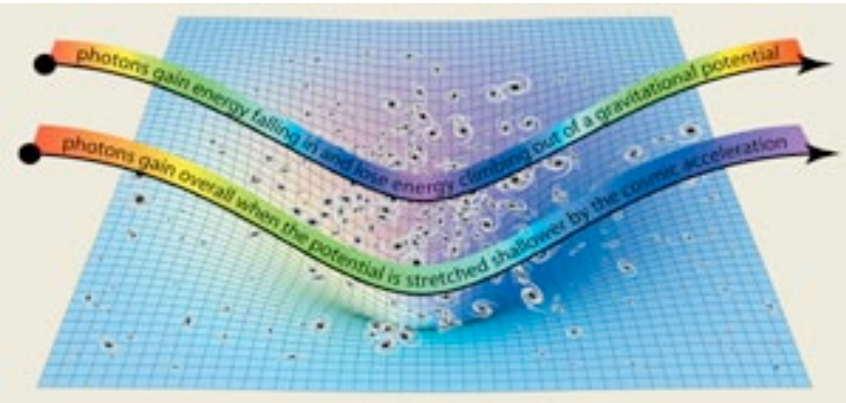
Many Cosmological Studies

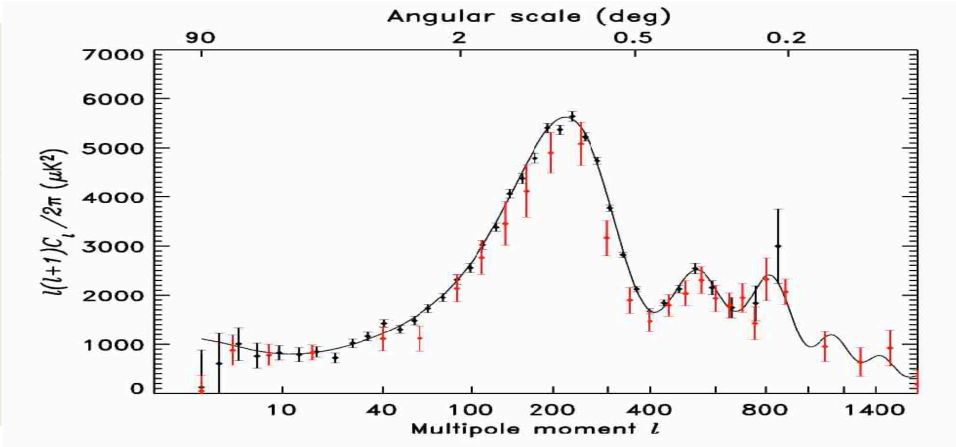
Power spectrum ==> Cosmological parameters

CMB map is contaminated by non-Gaussianities

- Lensing effect: (**L. Perotto, J. Bobin, S. Plaszczynski, J.-L. Starck and A. Lavabre "Reconstruction of the CMB lensing for Planck" , A&A, 5109 A4, 2010.).**
- Clusters of galaxies (SZ effect).
- Integrated Sachs Wolfe (ISW) effect

  F.-X. Dupe, A. Rassat, J.-L. Starck, M. J. Fadili , "An Optimal Approach for Measuring the Integrated Sachs-Wolfe Effect", arXiv:1010.2192





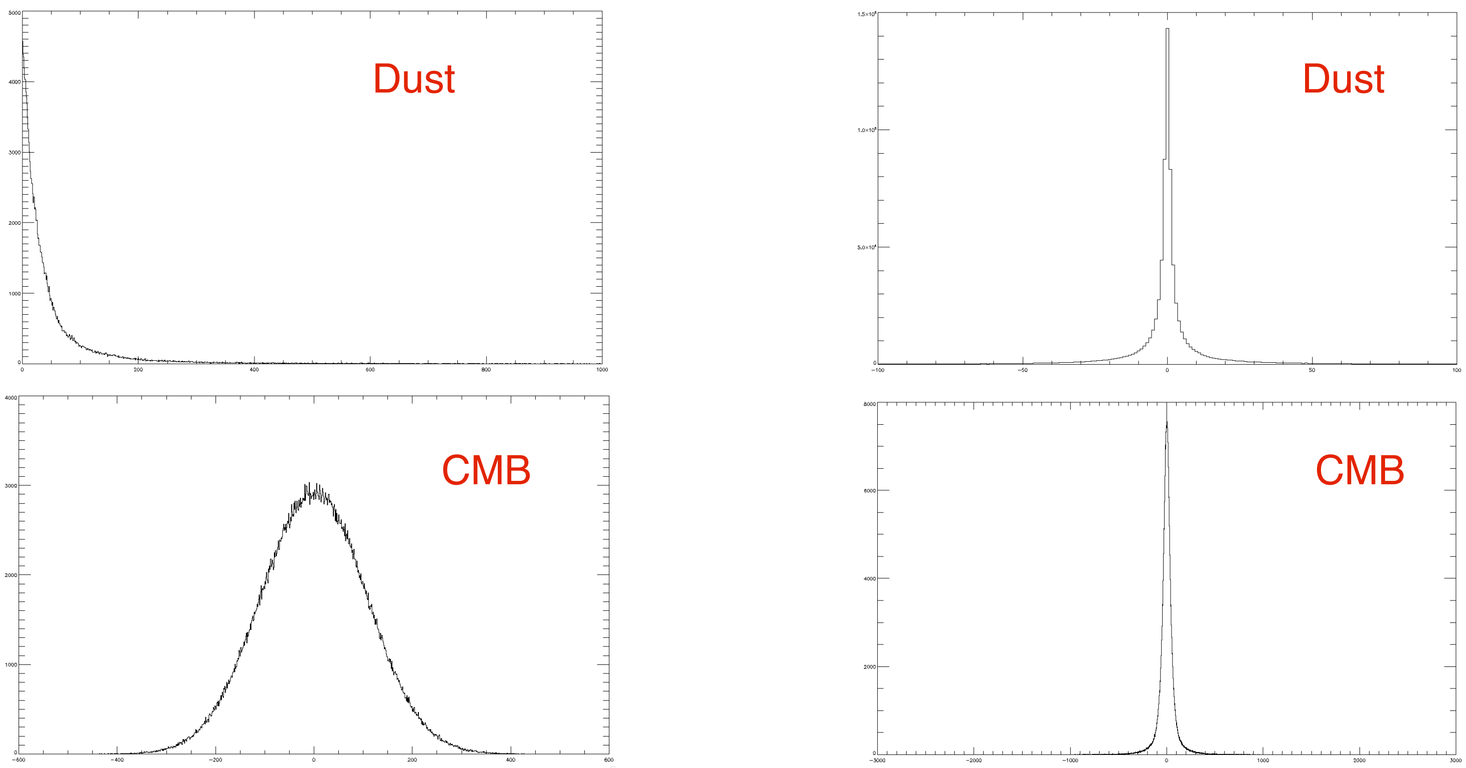
- Any other non-Gaussianity (Cosmic String, topology of the universe, etc)

- Is the CMB isotropic ?

Strong Constraint: Sparsity

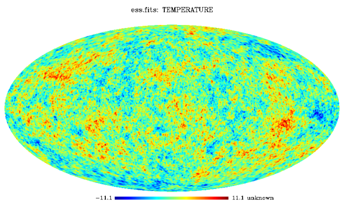
- Components are sparse in a wavelet dictionary

Spatial Domain | Wavelet Domain

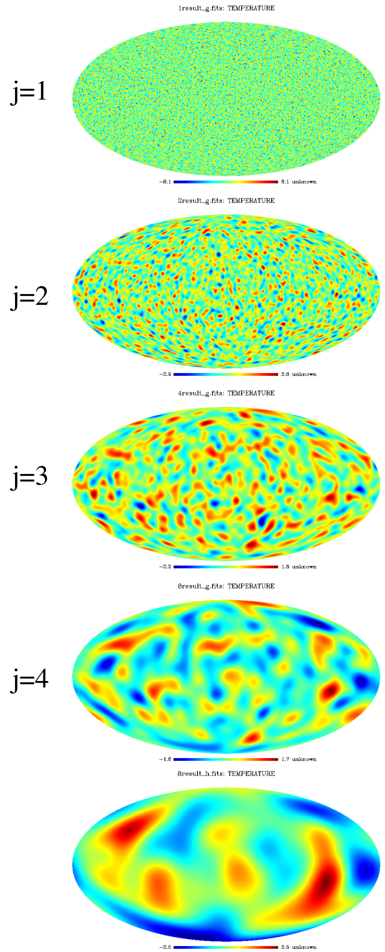


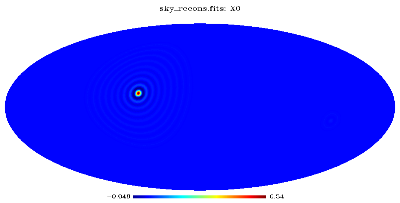
Isotropic Undecimated Wavelet on the Sphere

Wavelets, Ridgelets and Curvelets on the Sphere, Astronomy & Astrophysics, 446, 1191-1204, 2006.



Undecimated Wavelet Transform





$$\hat{\psi}_{\frac{l_c}{2^j}}(l,m) = \hat{\phi}_{\frac{l_c}{2^{j-1}}}(l,m) - \hat{\phi}_{\frac{l_c}{2^j}}(l,m)$$

$$\hat{H}_j(l,m) = \begin{cases} \dfrac{\hat{\phi}_{\frac{l_c}{2^{j+1}}}(l,m)}{\hat{\phi}_{\frac{l_c}{2^j}}(l,m)} & \text{if } l < \frac{l_c}{2^{j+1}} \quad \text{and} \quad m = 0 \\ 0 & \text{otherwise} \end{cases}$$

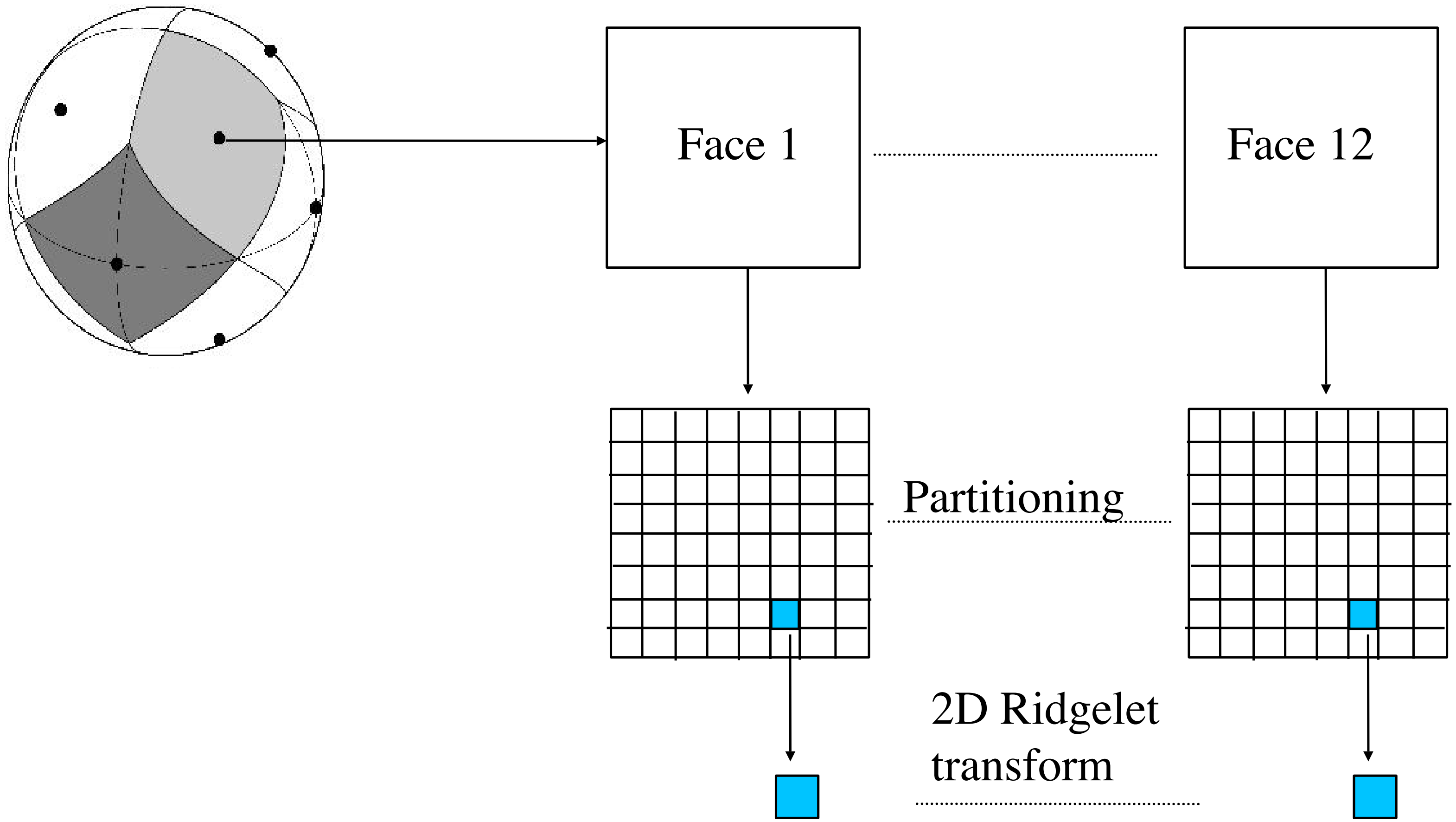
$$\hat{G}_j(l,m) = \begin{cases} \dfrac{\hat{\psi}_{\frac{l_c}{2^{j+1}}}(l,m)}{\hat{\phi}_{\frac{l_c}{2^j}}(l,m)} & \text{if } l < \frac{l_c}{2^{j+1}} \quad \text{and} \quad m = 0 \\ 1 & \text{if } l \geq \frac{l_c}{2^{j+1}} \quad \text{and} \quad m = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{c}_{j+1}(l,m) = \widehat{H}_j(l,m)\hat{c}_j(l,m)$$

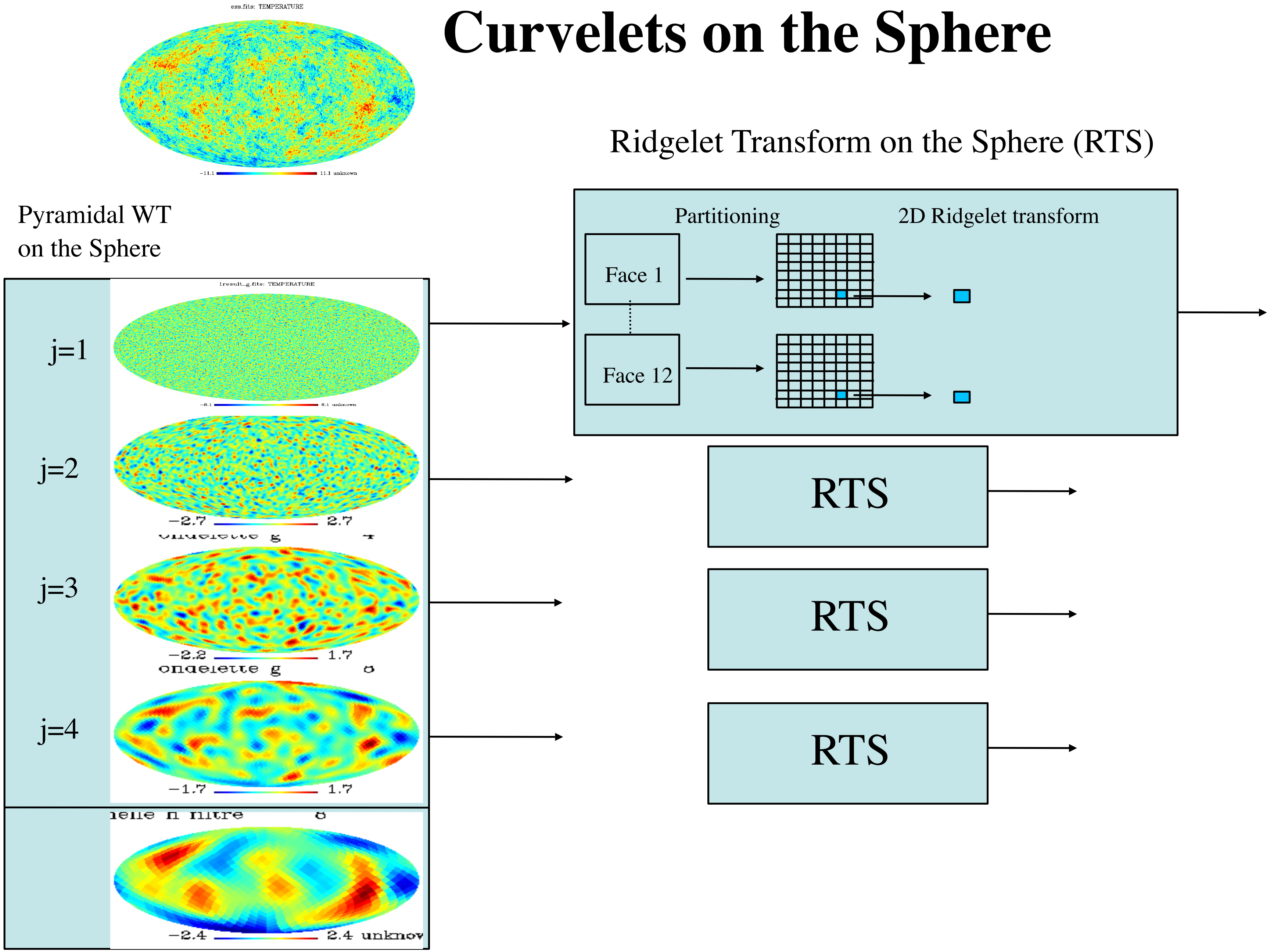
$$\hat{w}_{j+1}(l,m) = \widehat{G}_j(l,m)\hat{c}_j(l,m)$$

$$\boxed{c_0(\vartheta,\varphi) = c_J(\vartheta,\varphi) + \sum_{j=1}^{J} w_j(\vartheta,\varphi)}$$

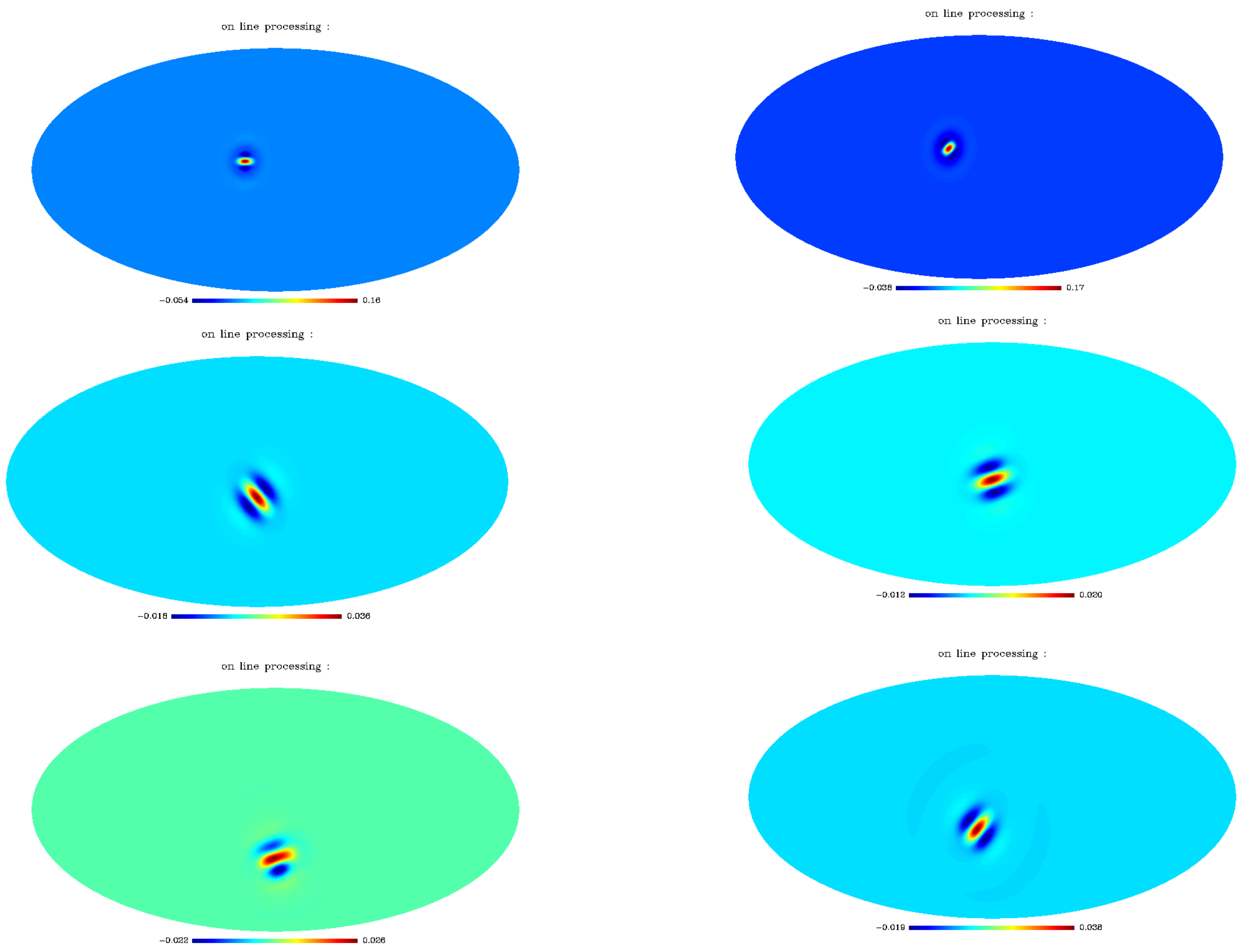
Ridgelets on the Sphere



Curvelets on the Sphere

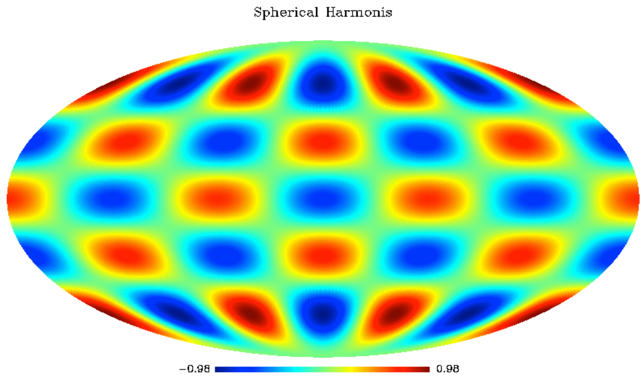


Example of curvelet functions on the sphere

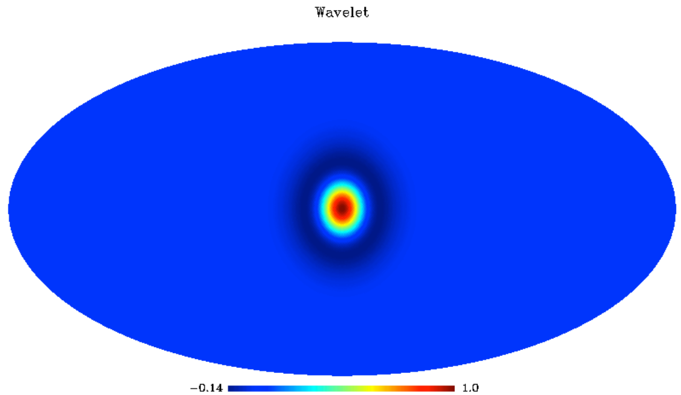


Dictionaries

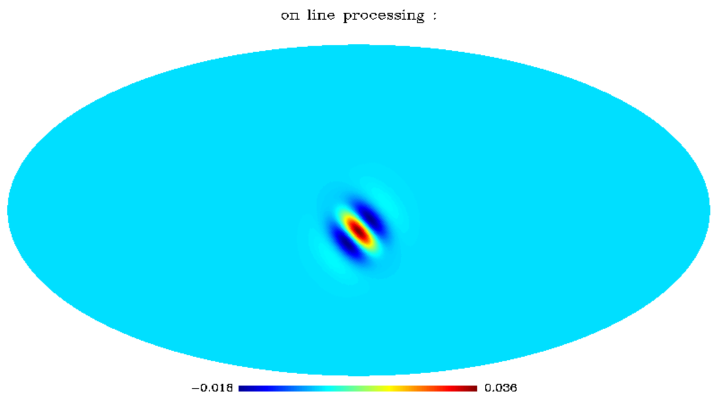
Spherical Harmonics



Wavelets



Curvelets

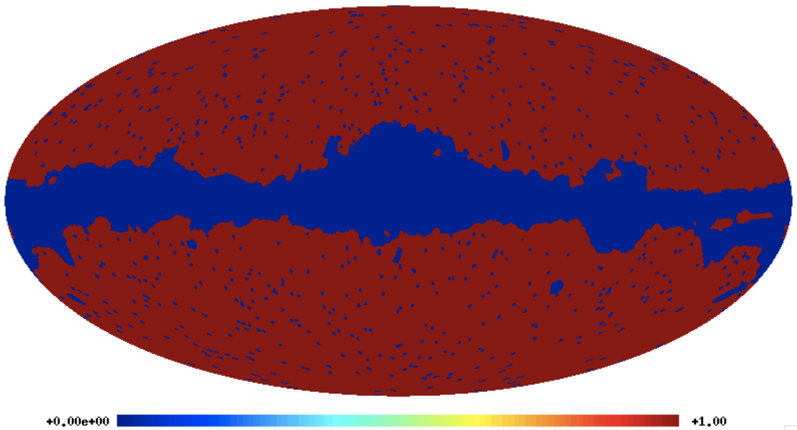


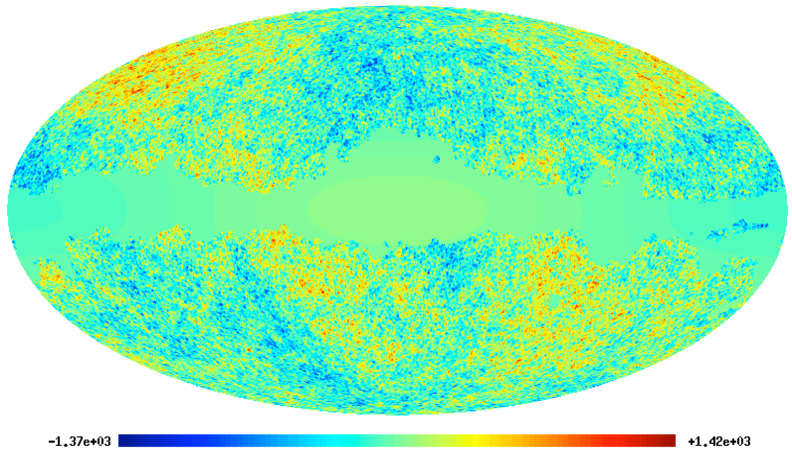
Interpolation of Missing Data: Sparse Inpainting

•*M. Elad, J.-L. Starck, D.L. Donoho, P. Querre, "Simultaneous Cartoon and Texture Image Inpainting using Morphological Component Analysis (MCA)", ACHA, Vol. 19, pp. 340-358, 2005.*
• P. Abrial, Y. Moudden, J.L. Starck, M.J. Fadili, J. Delabrouille, and M. Nguyen, "CMB Data Analysis and Sparsity" , **Statistical Methodology** , Vol 5, No 4, pp 289-298, 2008.

$$\min_{\alpha} ||\alpha||_p \quad \text{subject to} \quad Y = M\Phi\alpha$$

Where M is the mask: M(i,j) = 0 ==> missing data
M(i,j) = 1 ==> good data





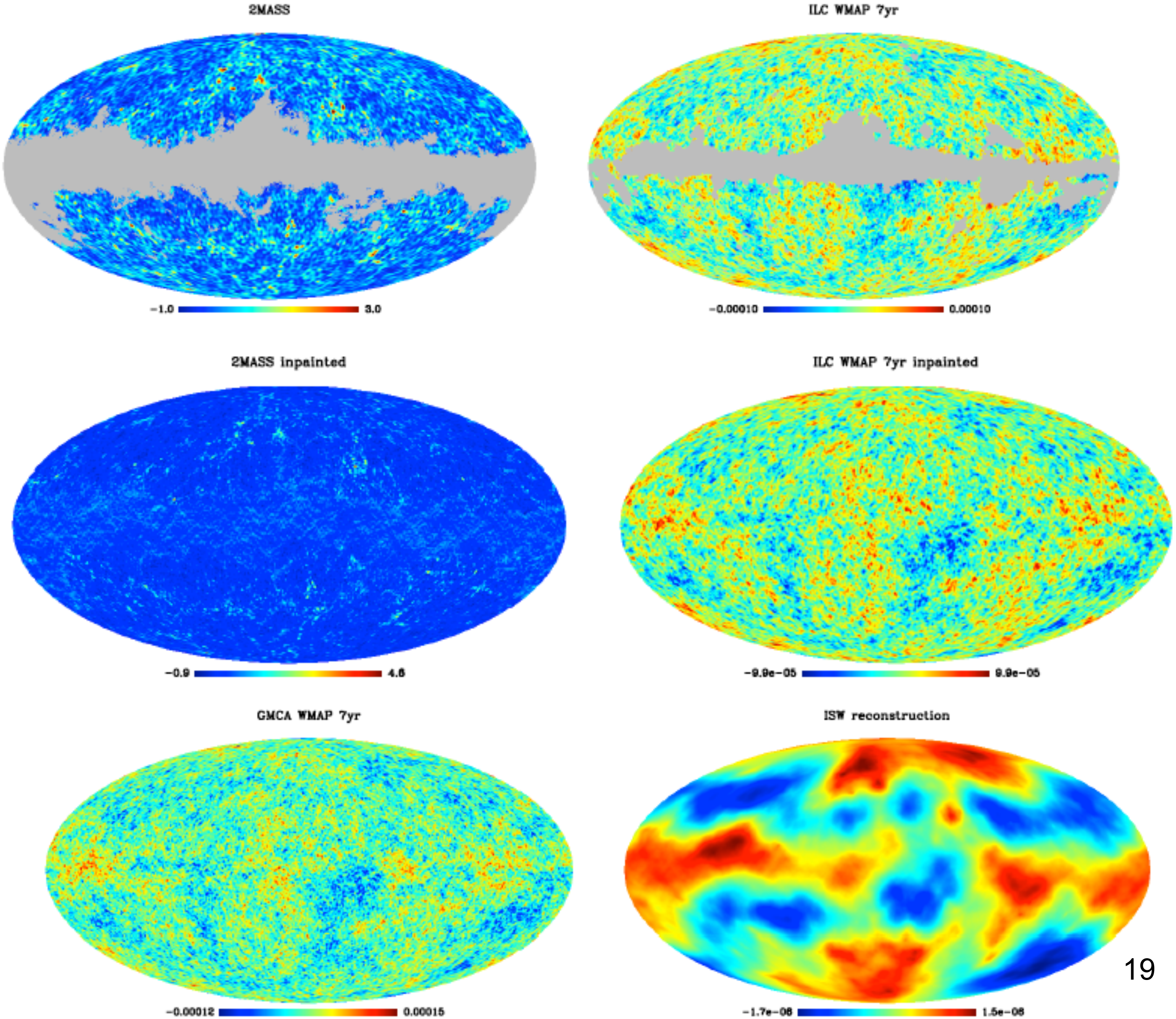
Sparse-Inpainting preserves the ISW and the weak lensing signal.

L. Perotto, J. Bobin, S. Plaszczynski, J.-L. Starck, and A. Lavabre, "Reconstruction of the CMB lensing for Planck", A&A, 2010.
F.-X. Dupe, A. Rassat, J.-L. Starck, M. J. Fadili , "An Optimal Approach for Measuring the Integrated Sachs-Wolfe Effect", arXiv:1010.2192

Theoretical justification through the sampling theory of Compressed Sensing ?
Rauhut and Ward, "Sparse Legendre expansion via l1 minimization", Constructive Approximation journal, submitted.

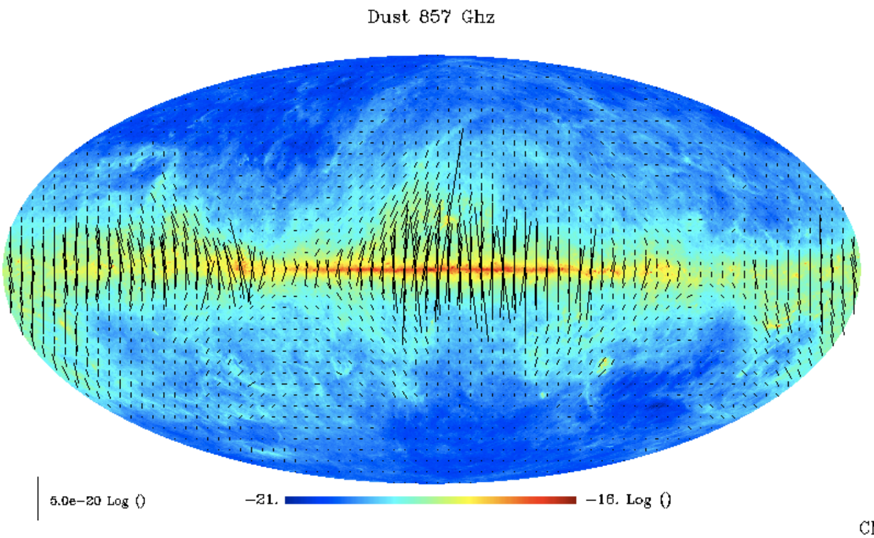
ISW Detection

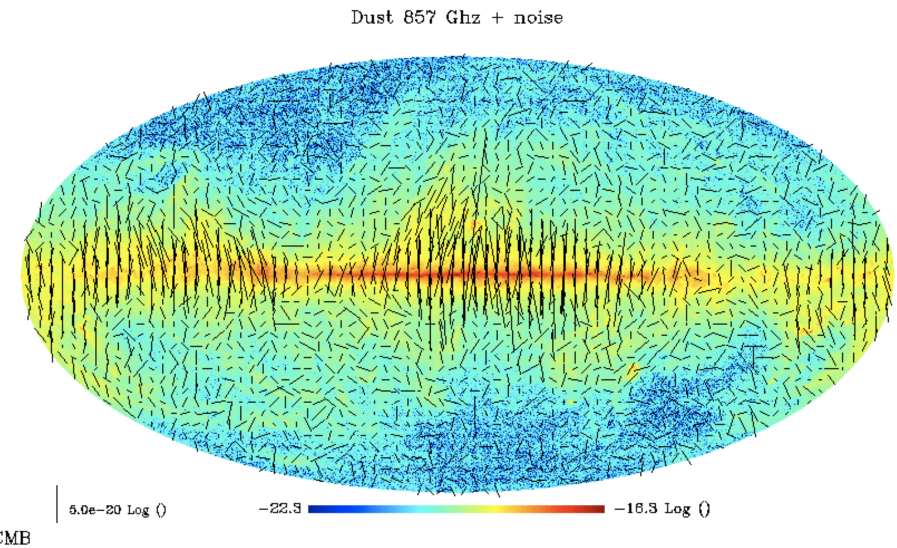
F.-X. Dupe, A. Rassat, J.-L. Starck, M. J. Fadili , "An Optimal Approach for Measuring the Integrated Sachs-Wolfe Effect", arXiv:1010.2192

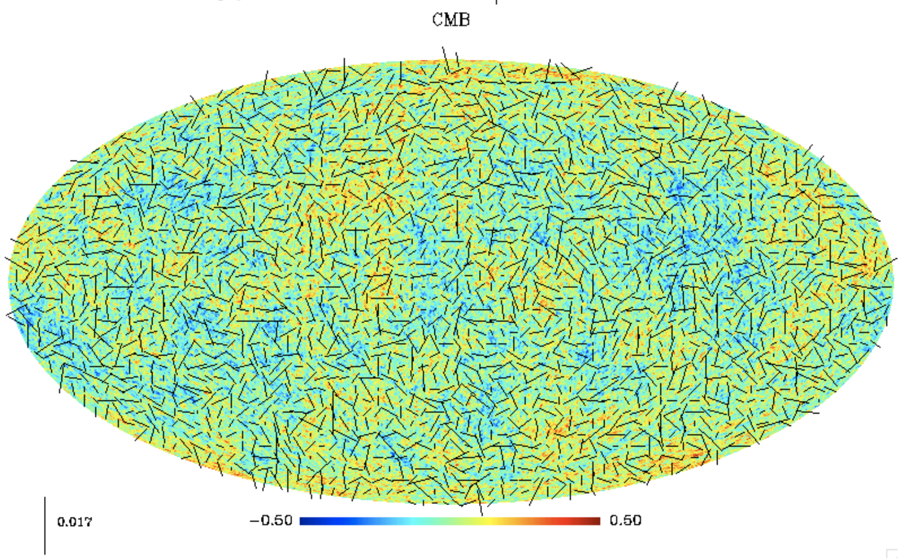


PLANCK POLARIZED DATA: T, Q, U

Magnitude $P = \sqrt{Q^2 + V^2}$ Orientation $\alpha = \arctan\left(U/Q\right)$







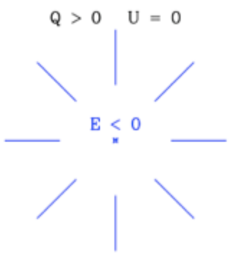
E/B Mode Decomposition

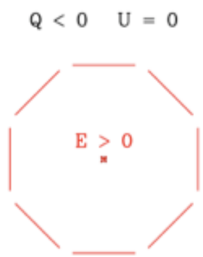
$$E = \sum_{\ell,m} a^E_{\ell m} Y_{\ell m} = \sum_{\ell,m} -\frac{{}_2a_{\ell m} + {}_{-2}a_{\ell m}}{2} Y_{\ell m}$$

$$B = \sum_{\ell,m} a^B_{\ell m} Y_{\ell m} = \sum_{\ell,m} i\frac{{}_2a_{\ell m} - {}_{-2}a_{\ell m}}{2} Y_{\ell m}$$

$$a^E_{lm} = -(a_{2,lm} + a_{-2,lm})/2$$

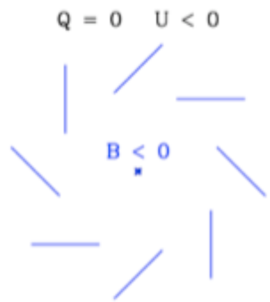
$$a^B_{lm} = i(a_{2,lm} - a_{-2,lm})/2$$

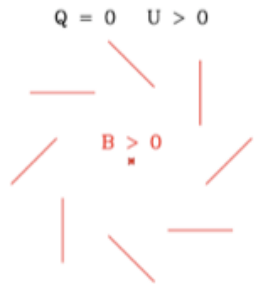




$$Q = -\sum_{l,m}\left(a^E_{l,m} Z^+_{l,m} + i a^B_{l,m} Z^-_{l,m}\right)$$

$$U = -\sum_{l,m}\left(a^B_{l,m} Z^+_{l,m} - i a^E_{l,m} Z^-_{l,m}\right)$$



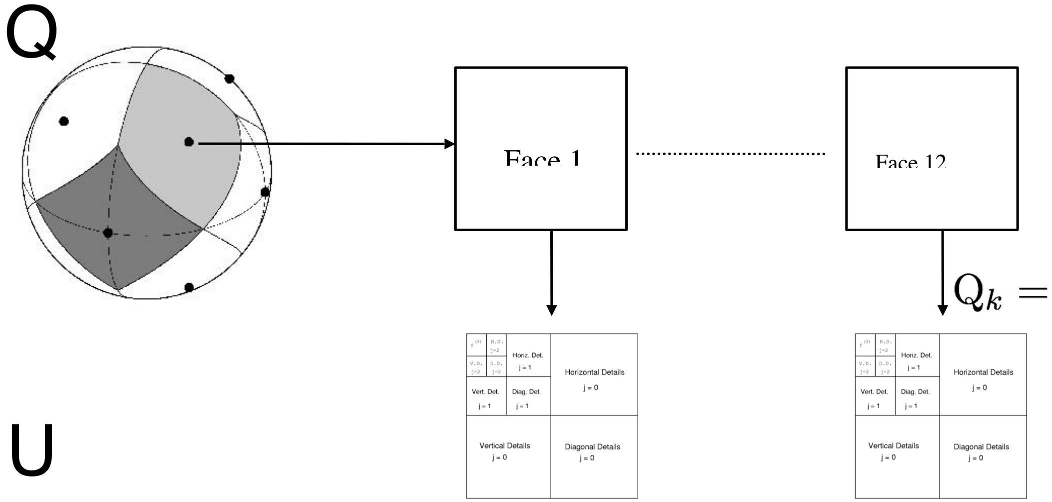


$$Z^+_{l,m} = ({}_2Y_{l,m} + {}_{-2}Y_{l,m})/2$$

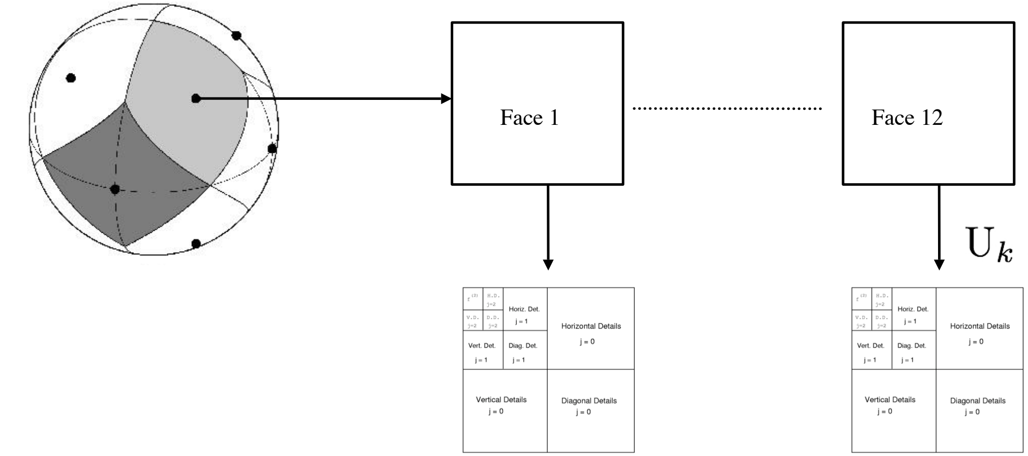
$$Z^-_{l,m} = ({}_2Y_{l,m} - {}_{-2}Y_{l,m})/2$$

E and B mode are closely related to the curl-free and div-free components of the vector field

Orthogonal Q-U Polarized Wavelet on the Sphere



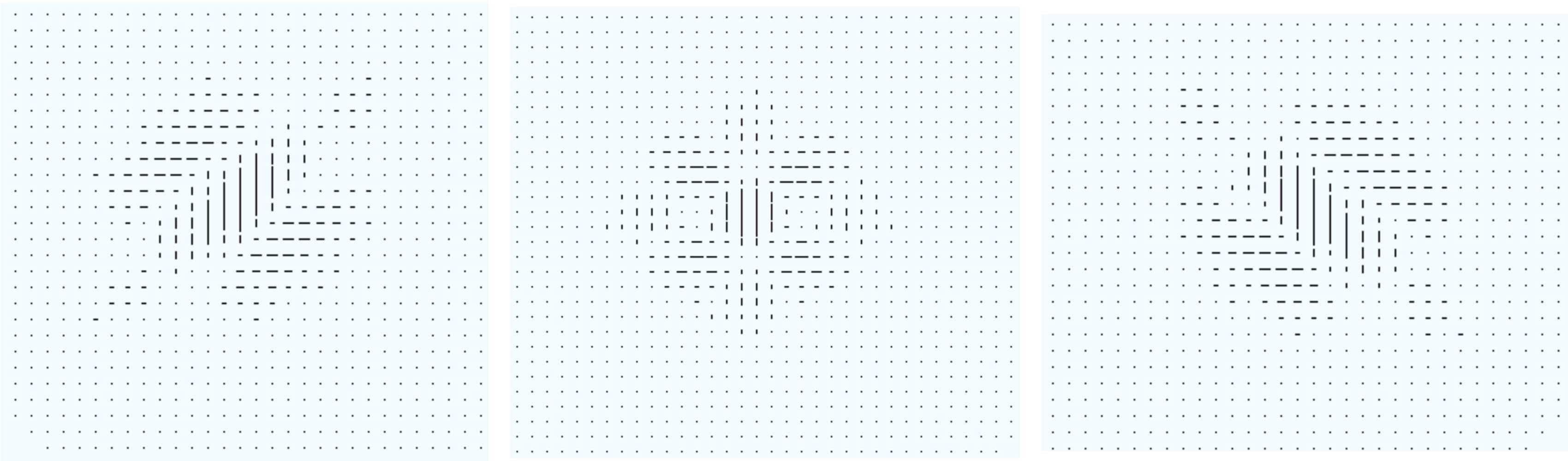
$$Q_k = \sum_p c^Q_{J,p}\phi_{j,k}(p) + \sum_p \sum_{j=1}^{J} \psi_{j,k}(p) w^Q_{j,p}$$



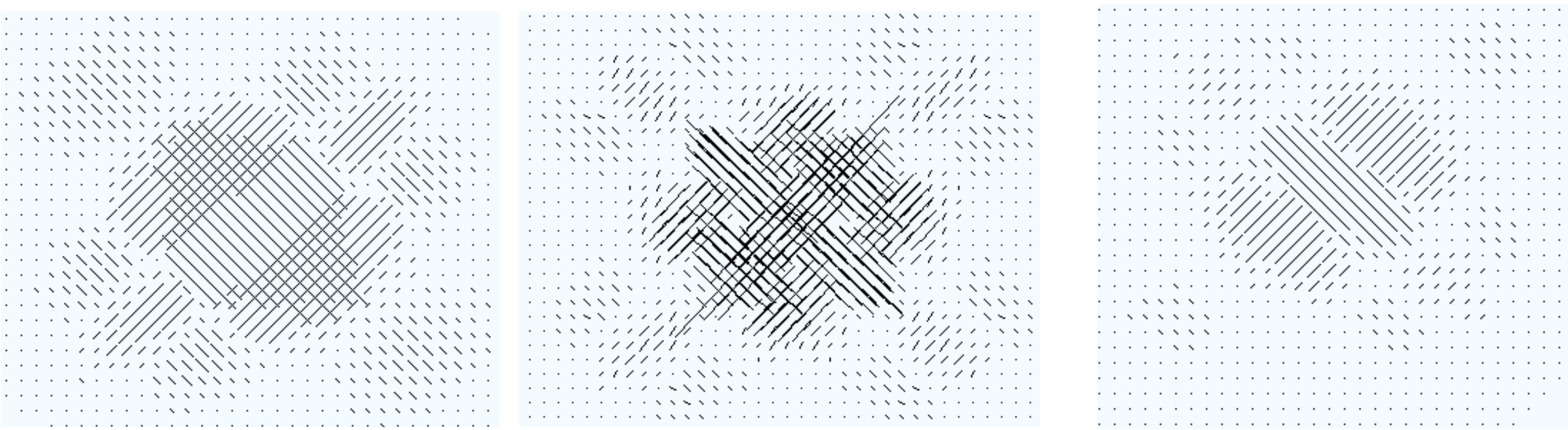
$$U_k = \sum_p c^U_{J,p}\phi_{j,k}(p) + \sum_p \sum_{j=1}^{J} \psi_{j,k}(p) w^U_{j,}$$

$$(Q \pm iU)_k = \sum_p (c^Q_{J,p} \pm c^U_{J,p})\phi_{j,k}(p) + \sum_p \sum_{j=1}^{J} \psi_{j,k}(p)(w^Q_{j,p} \pm w^U_{j,p})$$

Q,U Orthogonal Wavelet Decomposition



Q-wavelet coefficient backprojection



U-wavelet coefficient backprojection

E/B Undecimated Wavelet Transform for Polarized Data

J.-L. Starck, Y. Moudden and J. Bobin, "Polarized Wavelets and Curvelets on the Sphere", Astronomy and Astrophysics, 497, 3, pp 931--943, 2009.

$$
\begin{aligned}
E = & \quad \sum_{\ell,m} a^E_{\ell m} Y_{\ell m} = \sum_{\ell,m} -\frac{{}_2a_{\ell m} + {}_{-2}a_{\ell m}}{2} Y_{\ell m} \\
B = & \quad \sum_{\ell,m} a^B_{\ell m} Y_{\ell m} = \sum_{\ell,m} i \frac{{}_2a_{\ell m} - {}_{-2}a_{\ell m}}{2} Y_{\ell m}
\end{aligned}
$$

Wavelet Transform of E and B are obtained by:

$$w_j^E =< E, \psi_j >$$

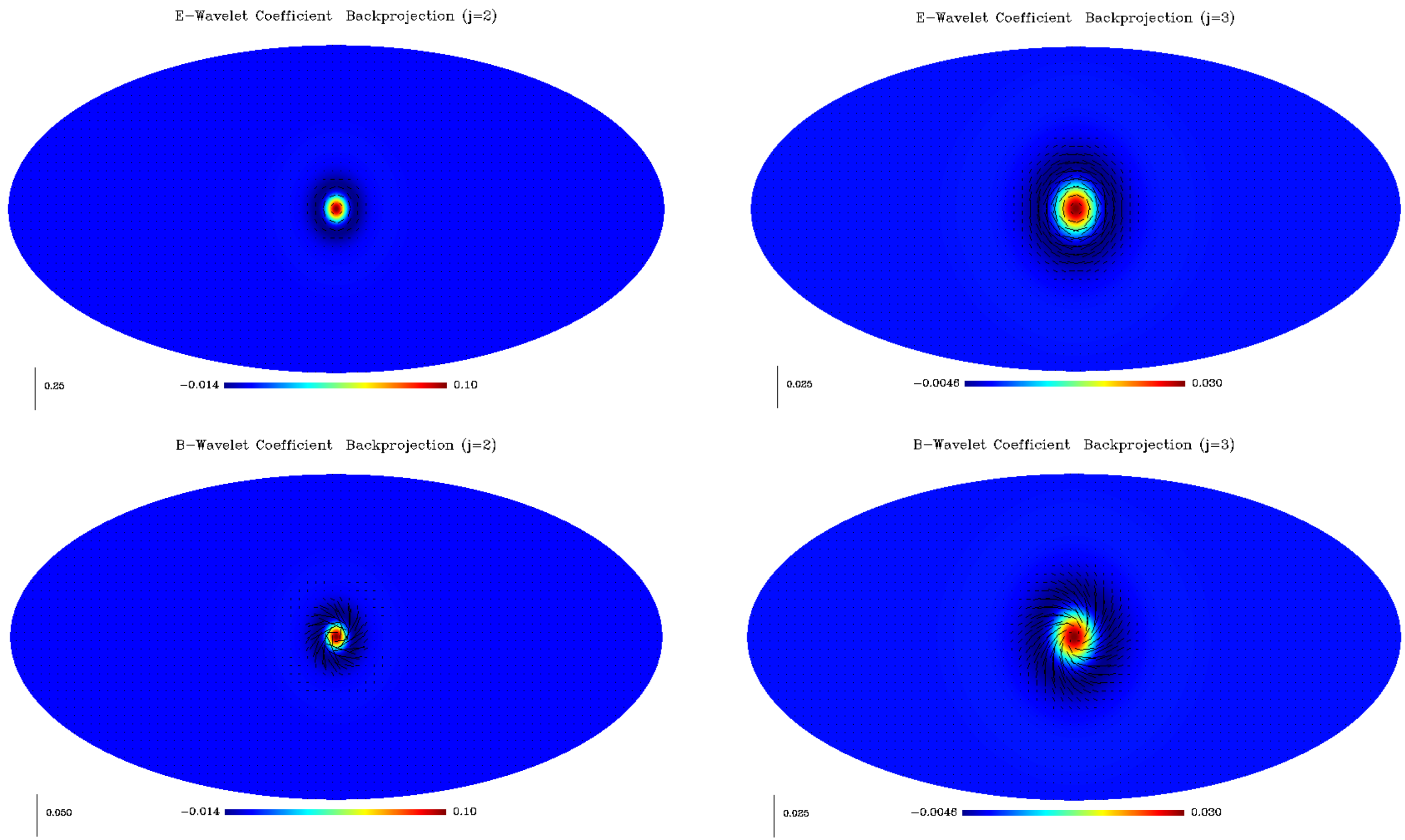
$$w_j^B =< B, \psi_j >$$

Furthermore, if we use the spherical isotropic wavelet construction of (starck et al, 2006), we have

$$E(\theta, \phi) = c_J^E(\theta, \phi) + \sum_{j=1}^{J} w_j^E(\theta, \phi)$$

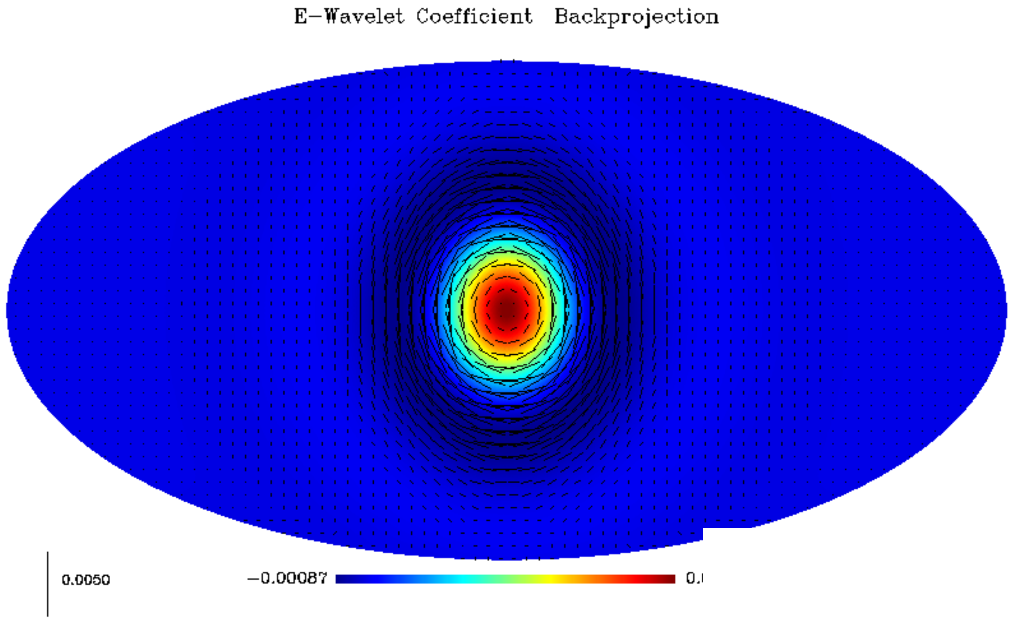
$$B(\theta, \phi) = c_J^B(\theta, \phi) + \sum_{j=1}^{J} w_j^B(\theta, \phi)$$

Wavelet and E/B Mode Decomposition

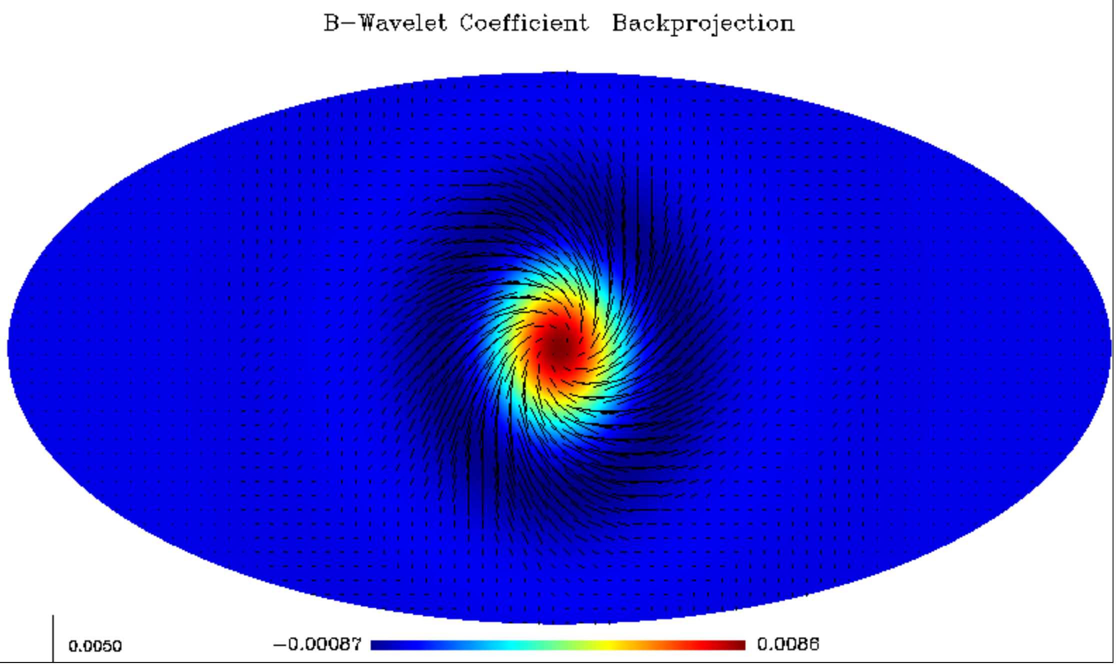


Polarized Dictionary: E/B Polarized Wavelet

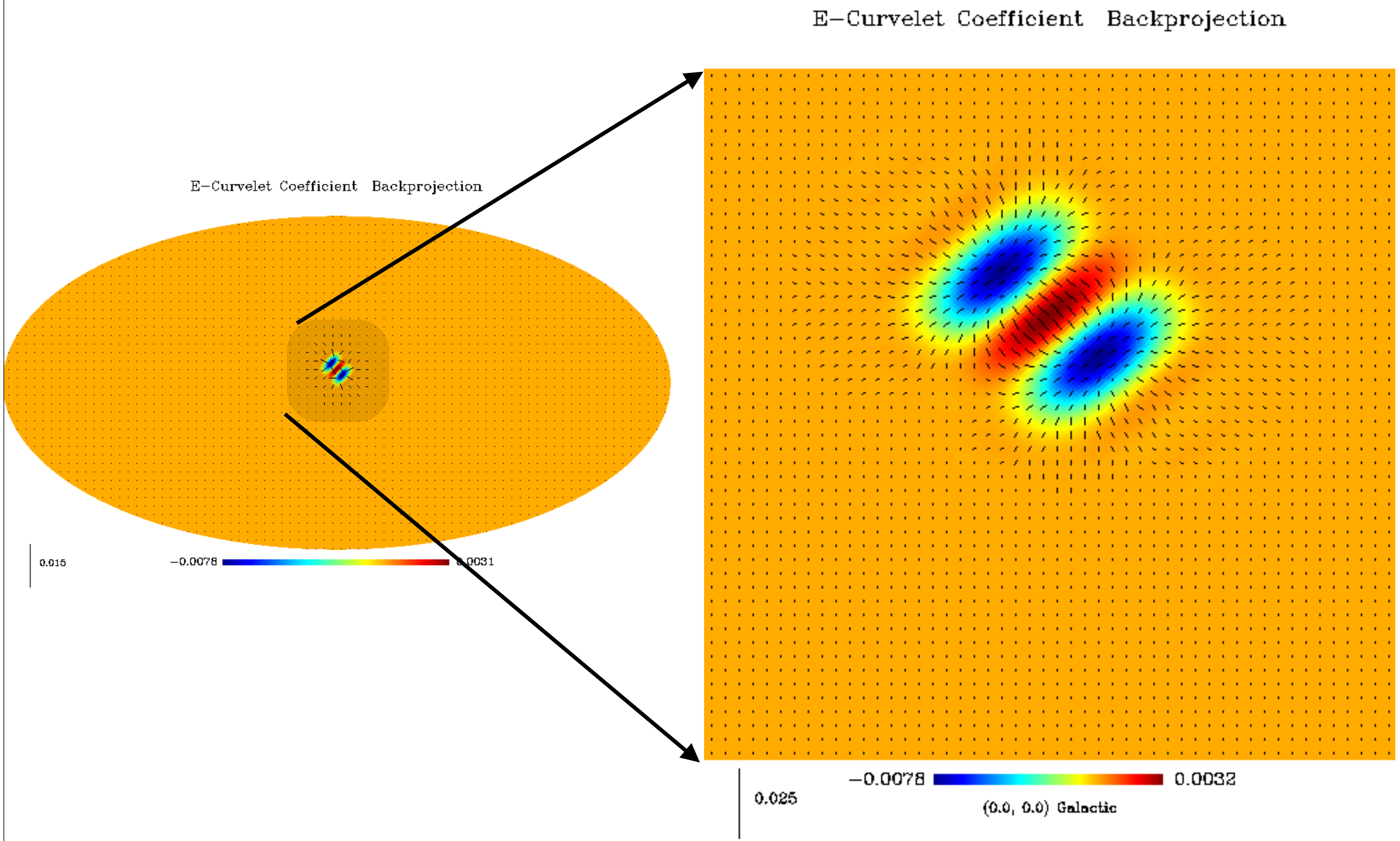
J.-L. Starck, Y. Moudden and J. Bobin, "Polarized Wavelets and Curvelets on the Sphere", Astronomy and Astrophysics, 497, 3, pp 931--943, 2009.



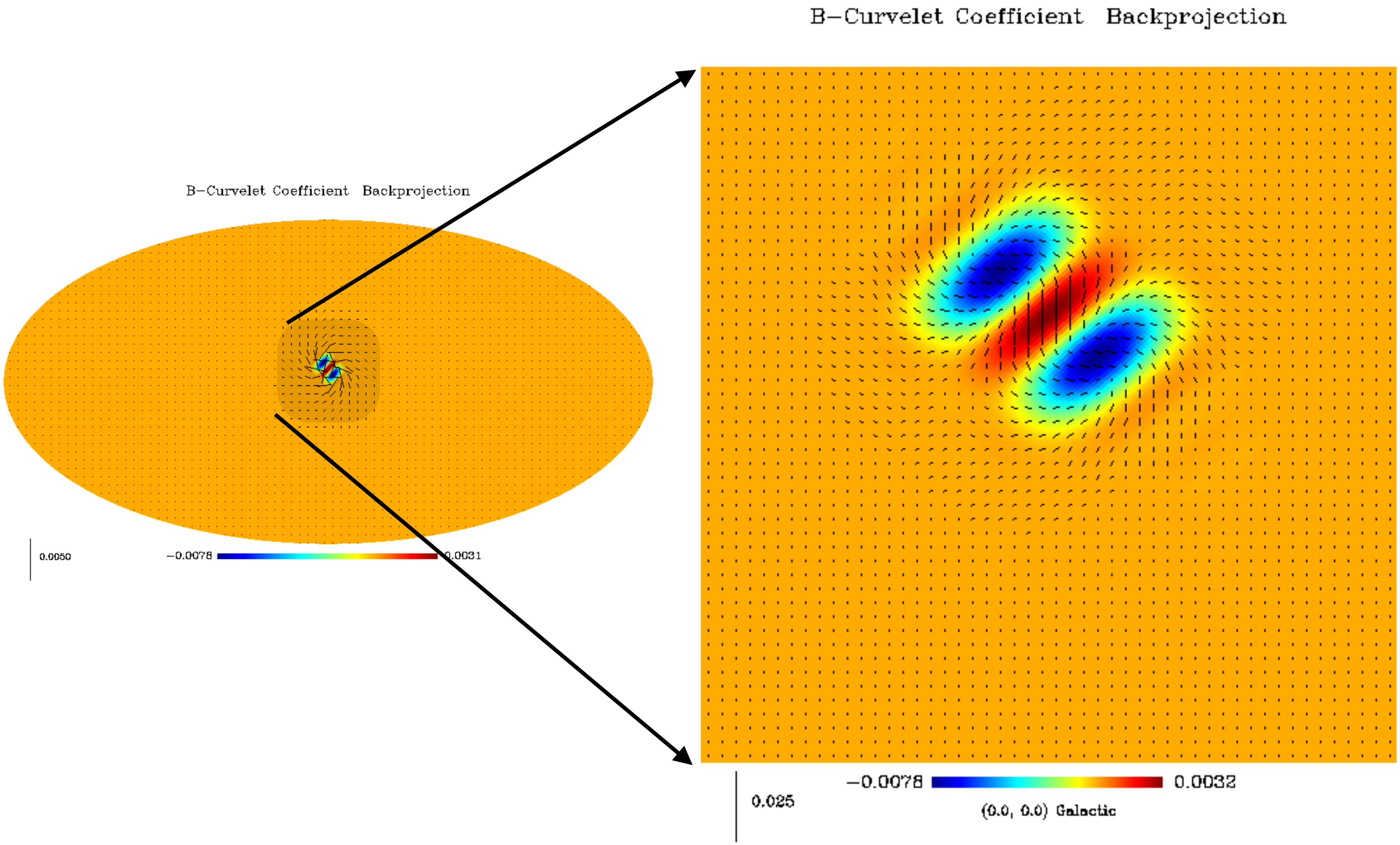
j=4



Curvelet and E/B Mode Decomposition



Curvelet and E/B Mode Decomposition



E/B Undecimated Wavelet Reconstuction

$$Q + iU = \sum_{lm} a_{2,lm} \; {}_2Y_{lm} \qquad\qquad Q - iU = \sum_{lm} a_{-2,lm} \; {}_{-2}Y_{lm}$$

$$Q = -\frac{1}{2}\sum_{\ell,m} a^E_{\ell m}({}_2Y_{\ell m} + {}_{-2}Y_{\ell m}) + i a^B_{\ell m}({}_2Y_{\ell m} - {}_{-2}Y_{\ell m}) = \sum_{\ell,m} a^E_{\ell m} Z^+_{\ell m} + i a^B_{\ell m} Z^-_{\ell m}$$

$$U = -\frac{1}{2}\sum a^B_{\ell m}({}_2Y_{\ell m} + {}_{-2}Y_{\ell m}) - i a^E_{\ell m}({}_2Y_{\ell m} - {}_{-2}Y_{\ell m}) = \sum_{\ell,m} a^B_{\ell m} Z^+_{\ell m} - i a^E_{\ell m} Z^-_{\ell m}$$

As we have: $E = c^E_J + \sum_{j=1}^{J} w^E_j$ and $B = c^B_J + \sum_{j=1}^{J} w^B_j$

Then

$$Q(\theta,\phi) = \sum_{l,m} c^E_{J,l,m} Z^+_{l,m} + i c^B_{J,l,m} Z^-_{l,m} + \sum_j \sum_{l,m} w^E_{j,l,m} Z^+_{l,m} + i w^B_{j,l,m} Z^-_{l,m}$$

$$U(\theta,\phi) = \sum_{l,m} c^B_{J,l,m} Z^+_{l,m} - i c^E_{J,l,m} Z^-_{l,m} + \sum_j \sum_{l,m} w^B_{j,l,m} Z^+_{l,m} - i w^E_{j,l,m} Z^-_{l,m}$$

$$c^{X,+}_J = c^X_J \sum_{\ell,m} Y^\dagger_{\ell m} Z^+_{\ell m} \quad \text{and} \quad c^{X,-}_J = c^X_J \sum_{\ell,m} Y^\dagger_{\ell m} Z^-_{\ell m}$$

Polarized Data Denoising

$$Q(\theta,\phi) = \sum_{l,m} c^E_{J,l,m} Z^+_{l,m} + i c^B_{J,l,m} Z^-_{l,m} + \sum_j \sum_{l,m} \tilde{w}^E_{j,l,m} Z^+_{l,m} + i \tilde{w}^B_{j,l,m} Z^-_{l,m}$$

$$U(\theta,\phi) = \sum_{l,m} c^B_{J,l,m} Z^+_{l,m} - i c^E_{J,l,m} Z^-_{l,m} + \sum_j \sum_{l,m} \tilde{w}^B_{j,l,m} Z^+_{l,m} - i \tilde{w}^E_{j,l,m} Z^-_{l,m}$$

Where

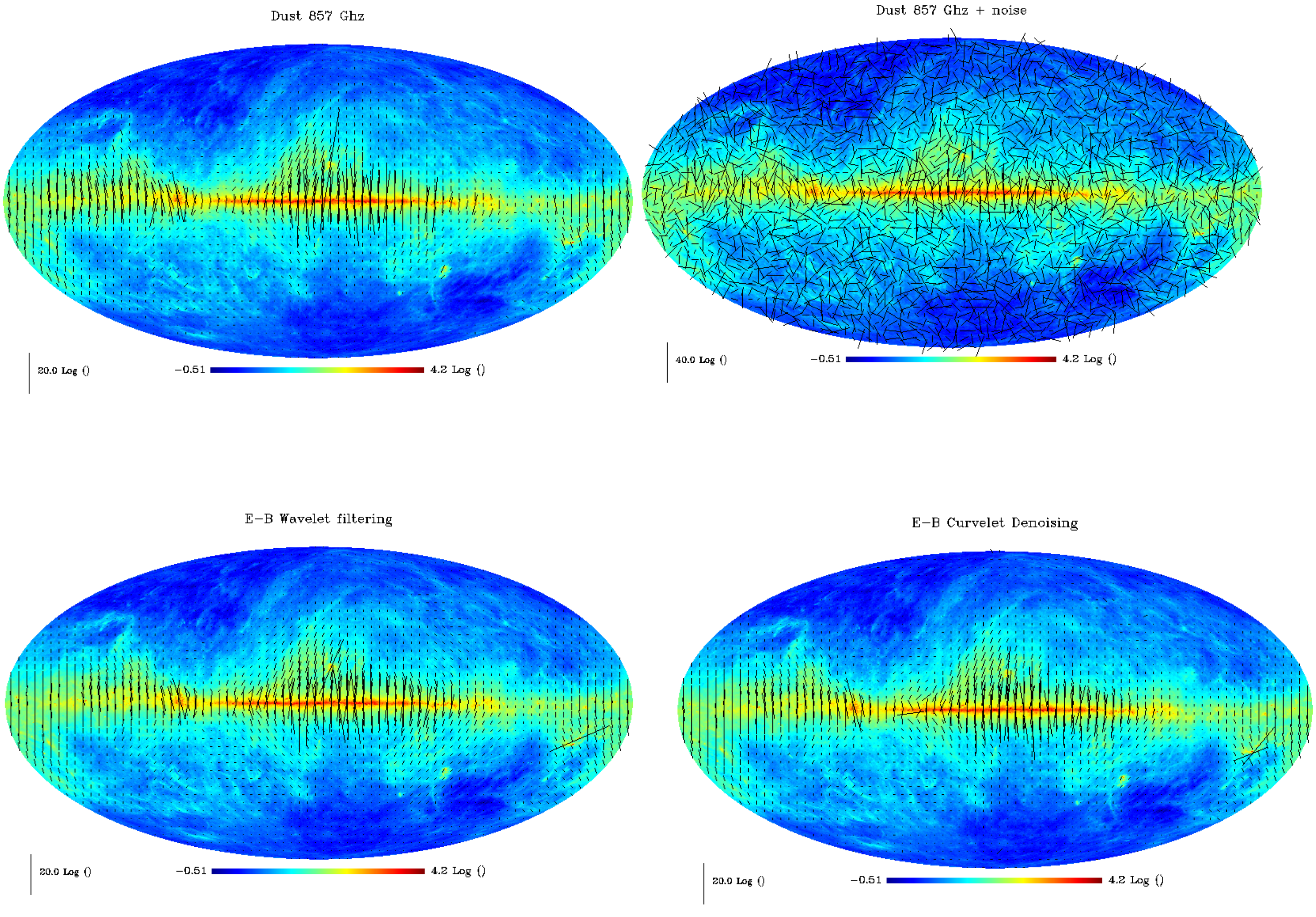
$$\tilde{w}^E_{j,k} = \delta(w^E_{j,k})$$

$$\tilde{w}^B_{j,k} = \delta(w^B_{j,k})$$

Hard thresholding corresponds to the following non linear operation:

$$\tilde{w}_{j,k} = \begin{cases} w_{j,k} & \text{if } \mid w_{j,k} \mid \geq T_j \\ 0 & \text{otherwise} \end{cases}$$

Polarized Data Denoising



MRS Version V2.0 available since June 2010

Wavelet, Ridgelet and Curvelet on the Sphere :

Software available at: http://jstarck.free.fr/mrs.html

J.-L. Starck, P. Abrial, Y. Moudden and M. Nguyen, ***Wavelets, Ridgelets and Curvelets on the Sphere***, *Astronomy & Astrophysics*, 446, 1191-1204, 2006.

1. Wavelet transforms
   - Continuous Wavelet Transform (Mexican Hat)
   - Orthogonal Wavelets
   - Undecimated isotropic wavelet transform (Spline, Meyer and Needlet filters).
   - Pyramidal wavelet transform
2. Ridgelet and Curvelet Transforms
3. Denoising using Wavelets and Curvelets
4. Gaussianity tests: Skewness, Kurtosis, Moment of order 5 and 6, Max, Higher Criticism
5. Astrophysical Component Separation (ICA on the Sphere): JADE, Fast ICA, GMCA.
6. Sparse Inpainting.

Polarized Spherical Wavelets and Curvelets: SparsePol/Version 1.0

Software available at: http://jstarck.free.fr/mrsp.html

J.-L. Starck, Y. Moudden and J. Bobin, "Polarized Wavelets and Curvelets on the Sphere", Astronomy and Astrophysics, 497, 3, pp 931--943, 2009.

Morpho-Spectral Diversity

Data: $X = [x_1, \ldots, x_m]$

Source: $S = [s_1, \ldots, s_n]$

$$X = [x_1, \ldots, x_m] = AS$$

$$x_l = \sum_{i=1}^{n} a_{i,l} s_i$$

$$\min_\alpha \|\alpha\|_p \text{ s.t } \mathbf{X} = \sum_{\gamma \in \Gamma} \alpha_\gamma \psi_\gamma$$

$\mathbf{\Phi_A} = [\mathbf{\Phi_{A,1}}, \mathbf{\Phi_{A,2}}]$ Spatial Dictionary with

$\mathbf{\Phi_S}$ Spectral Dictionary

$$\mathbf{\Psi} = [\mathbf{\Phi_{A,1}} \otimes \mathbf{\Phi_S}, \mathbf{\Phi_{A,2}} \otimes \mathbf{\Phi_S}]$$

Sparse Component Separation Method:

Generalized Morphological Analysis Methodv(GMCA)

- J. Bobin, J.-L. Starck, M.J. Fadili, and Y. Moudden, "Sparsity, Morphological Diversity and Blind Source Separation", IEEE Trans. on Image Processing, Vol 16, No 11, pp 2662 - 2674, 2007.
- .J. Bobin, J.-L. Starck, M.J. Fadili, and Y. Moudden, "Blind Source Separation: The Sparsity Revolution", Advances in Imaging and Electron Physics , Vol 152, pp 221 -- 306, 2008.

Source: $X = [x_1, x_n]$ Data: $Y = [y_1, ..., y_n] = AX + N$

We define a dictionary ϕ

=> GMCA searches a sparse solution X in the dictionary ϕ subject to the constraint that the norm $\| Y - AX \|^2$ is minimal.

GMCA + sky model : we can easily introduce in the component separation a priori knowledge in order to improve the separation.

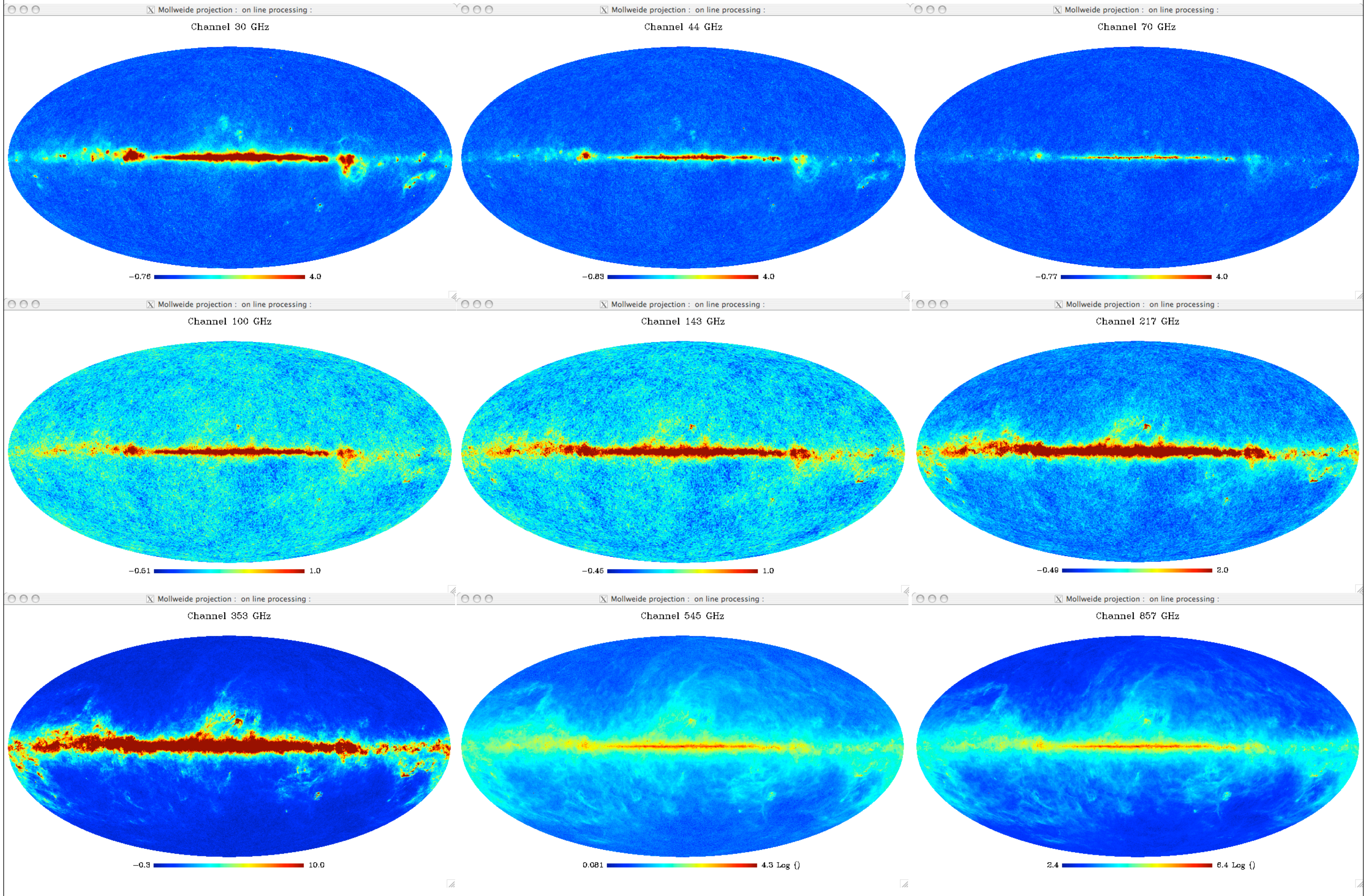
- J. Bobin, Y. Moudden,J.-L. Starck, M.J. Fadili, and N. Aghanim, ***"SZ and CMB reconstruction using GMCA"***, Statistical Methodology, astro-ph/0712.0588, 2008.

- CMB, SZ: The spectrum is known
- Free-Free: The spectrum is know up to a scale factor.

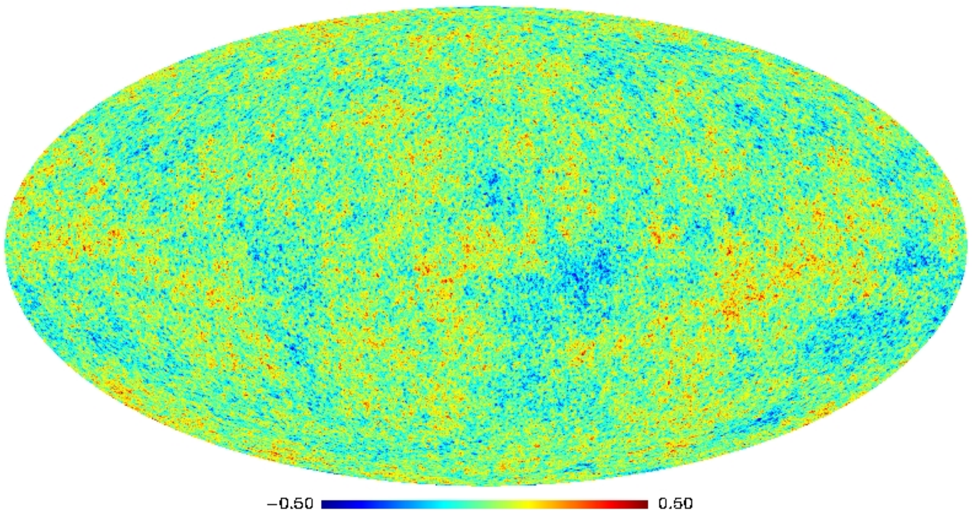
We have nine channels and we search for nine sources:

3 sources are modeled (CMB,SZ, Free-Free) and 6 are not modeled.

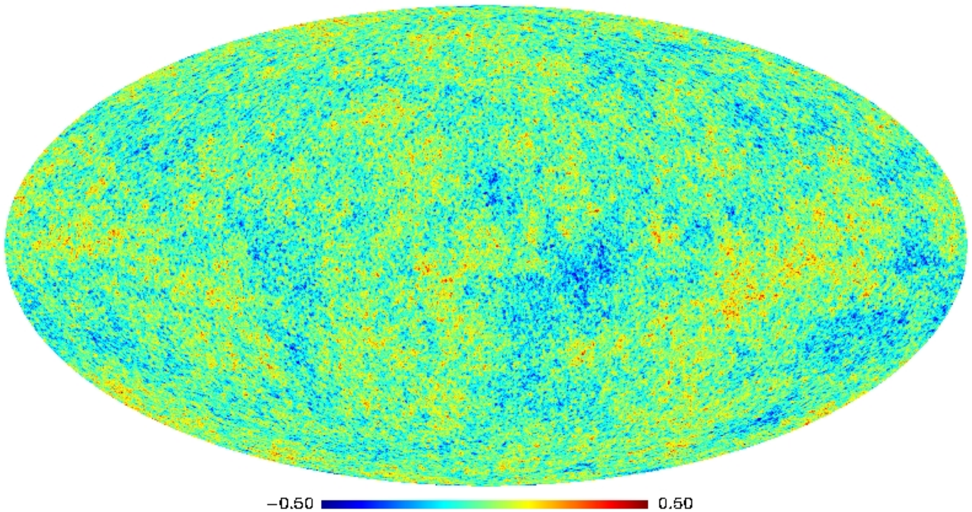
PLANCK Simulated Data



Planck - WG2 - Challenge 2



Input simulated CMB map (mK)



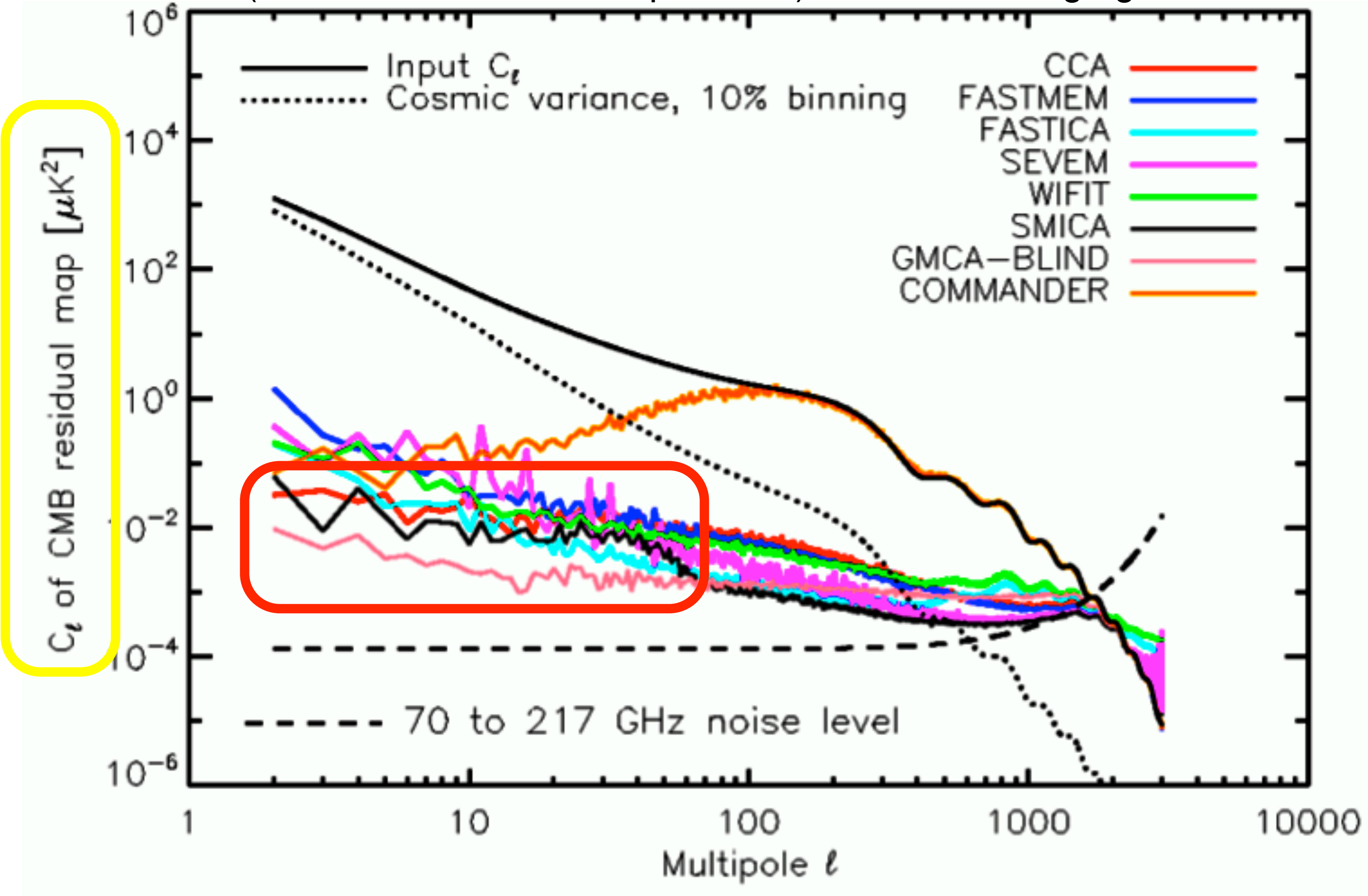
Estimated map with GMCA

Planck - WG2 - Challenger 2

comparaisons - power spectrum

GMCA has the lowest spectral residuals at low l.

Plot shows C_l of (reconstructed CMB – input CMB) evaluated at high galactic latitudes.



- **Sam Leach** (SISSA), June 19, 2008, WG2 meeting, Munich

Limitations

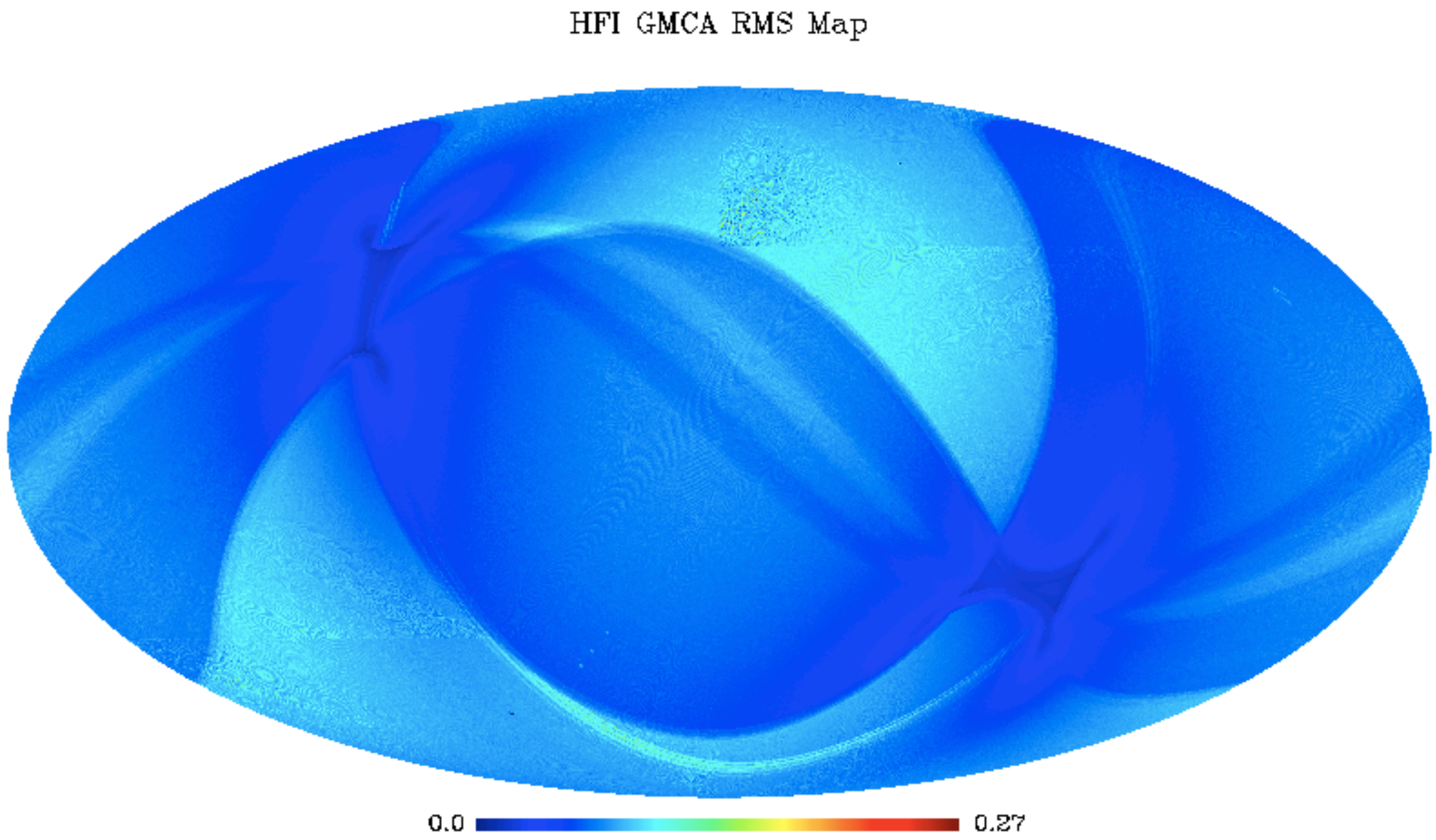
GMCA Model: $Y = A X + N$

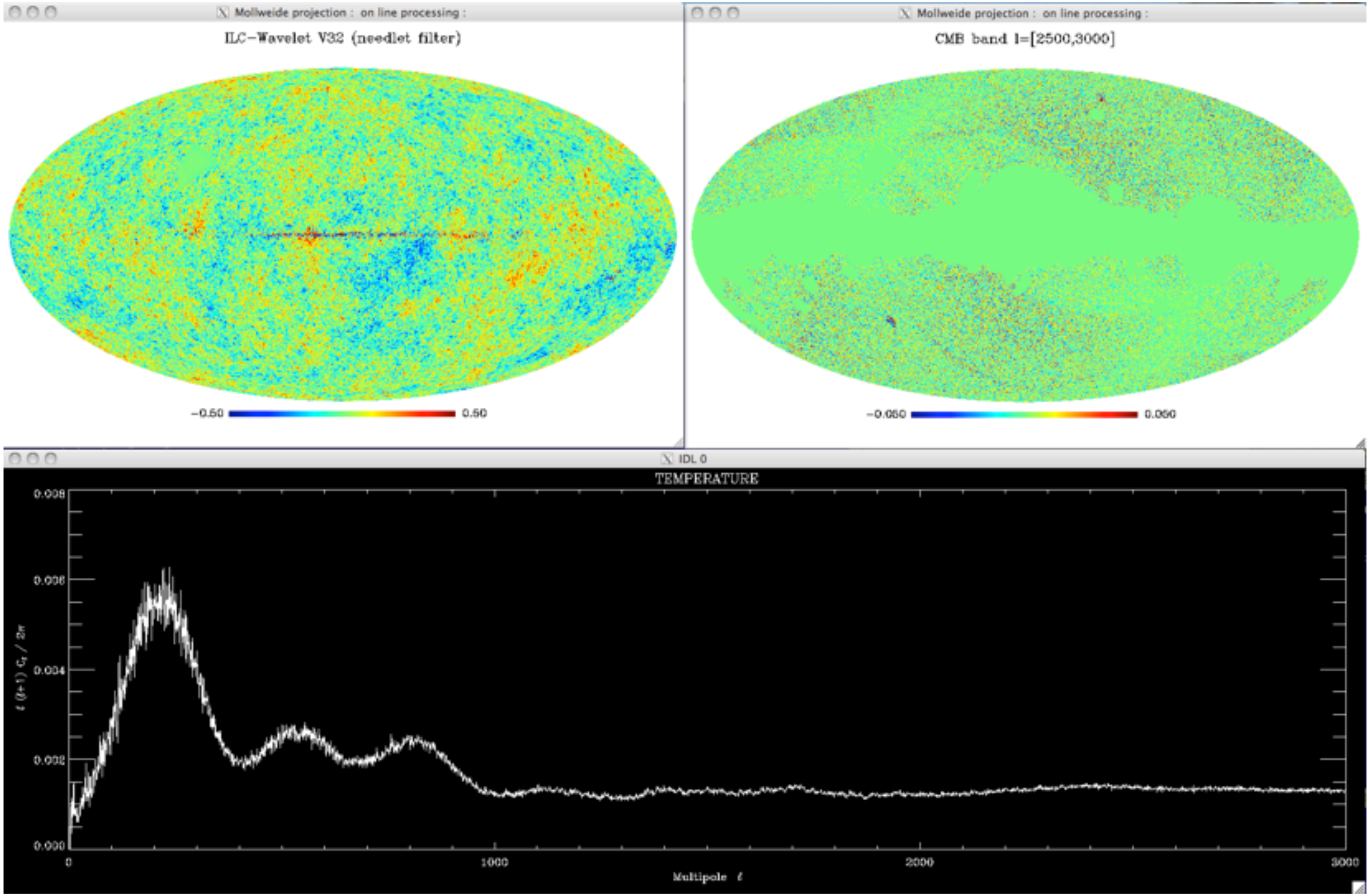
But three main problems:
i) A is spatially variant.
ii) This model does not take into account the beam.
iii) Noise is not homogeneous.

- Limitation of GMCA:

- One matrix to describe the whole sky (i.e. the simplest model !)
- PSF were not taken into account properly
- Non stationary noise.

Non stationary noise problem





Mollweide projection : on line processing :
ILC-Wavelet V32 (needlet filter)
−0.50
0.50
Mollweide projection : on line processing :
CMB band l=[2500,3000]
−0.050
0.050
IDL 0
TEMPERATURE
0.008
0.006
0.004
0.002
0.000
ℓ (ℓ+1) Cℓ / 2π
0
1000
2000
3000
Multipole ℓ

Non stationary noise problem

The sparse inpainted solution was obtained by minimizing:

$$\min_{\alpha} \frac{1}{2\sigma^2} \|Y - A\Phi\alpha\|^2 \quad \text{s.t.} \quad \alpha \;\; \text{is sparse}$$

A sparse inpainted and denoised solution can be obtained by minimizing:

$$\min_{\alpha} \frac{1}{2} \|Y - A\Phi\alpha\|_{\mathbf{\Sigma}}^2 + \lambda \|\alpha\|^2 \quad \text{s.t.} \quad \alpha \;\; \text{is sparse}$$

The notation $\|.\|_{\mathbf{\Sigma}}^2$ stands for the Frobenius norm of $\mathbf{Y}$ in the noise covariance metric : $\|Y\|_{\mathbf{\Sigma}}^2 = \text{ Trace}\left(\mathbf{Y}^T \mathbf{\Sigma}^{-1} \mathbf{Y}\right)$.

Forward-Backward Splitting Algorithm

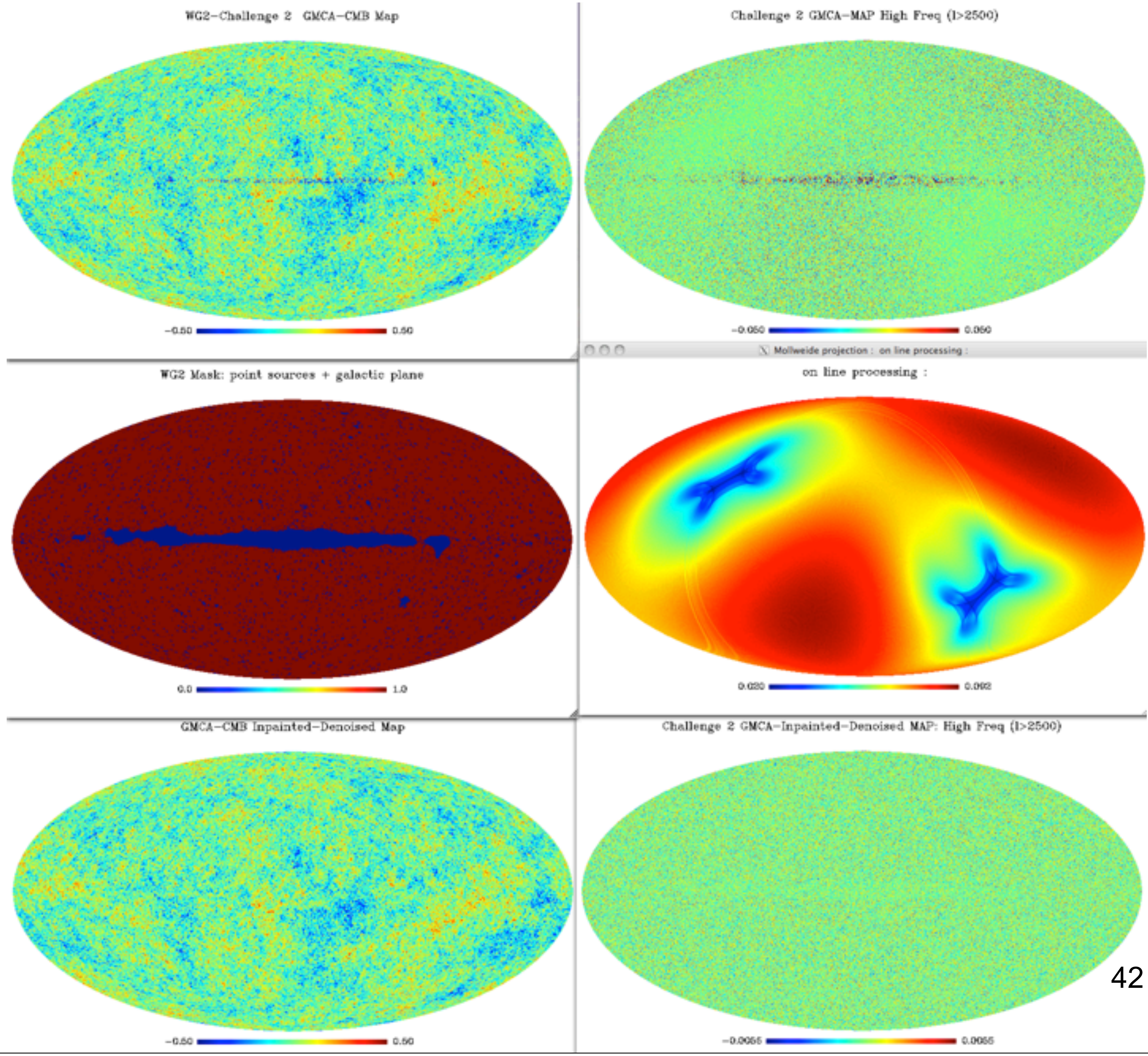
$\min_\alpha \; F_1(\alpha) + F_2(\alpha)$

Forward-backward splitting is a generalization of the classical gradient projection method for constrained convex optimization:

$$\alpha^{(n+1)} = \text{prox}_{\mu_n F_1} \left(\alpha^{(n)} - \mu_n \nabla F_2(\alpha^{(n)})\right)$$

P. L. Combettes and V. R. Wajs, "Signal recovery by proximal forward-backward splitting", Multiscale Modeling and Simulation, vol. 4, no. 4, pp. 1168-1200, November 2005

WG2 - Challenge 2



The BEAM problem

1- Work in the spherical harmonic domain (SMICA)
2- Perform the component separation several times:
  . one with all channels up to l=300,
  . Repeat with less channels up to 500, 800, 1200, 3000.
  . Merge all results

The second approach could be done in much more elegant way using the Wavelet-Vaguelette Decomposition (Donoho, 1995, Abramovich, 1998).

Wavelet-Vaguelette GMCA Decomposition

Inverse problem

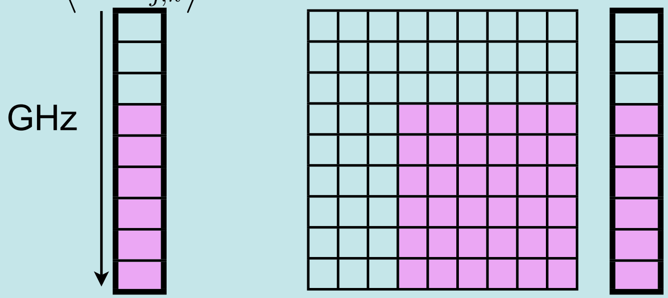
$$y = Kf + n \xrightarrow{\text{WVD}} f = \sum_j \sum_k \langle Kf, \Psi_{j,k} \rangle \psi_j, k \qquad \text{with } K^* \Psi_{j,k} = \psi_{j,k}$$

$$\tilde{f} = \sum_j \sum_k \Delta(\langle y, \Psi_{j,k} \rangle) \psi_j, k$$

Multi-channel WVD

$$Y_i = \sum_j K_i (A_j X)_i + N_i \longrightarrow \tilde{X}_s = \left[\sum_j \sum_k \tilde{A}_j^+ \left\langle Y_i, \Psi_{j,k}^{(i)} \right\rangle \psi_{j,k} \right]_s$$

$$\beta_{j,k} = \left\langle Y_i, \Psi_{j,k}^{(i)} \right\rangle \quad = \quad A_j \quad \alpha_{j,k} = \langle X_s, \psi_{j,k} \rangle \qquad \text{with } K_i^* \Psi_{j,k}^{(i)} = \psi_{j,k}$$

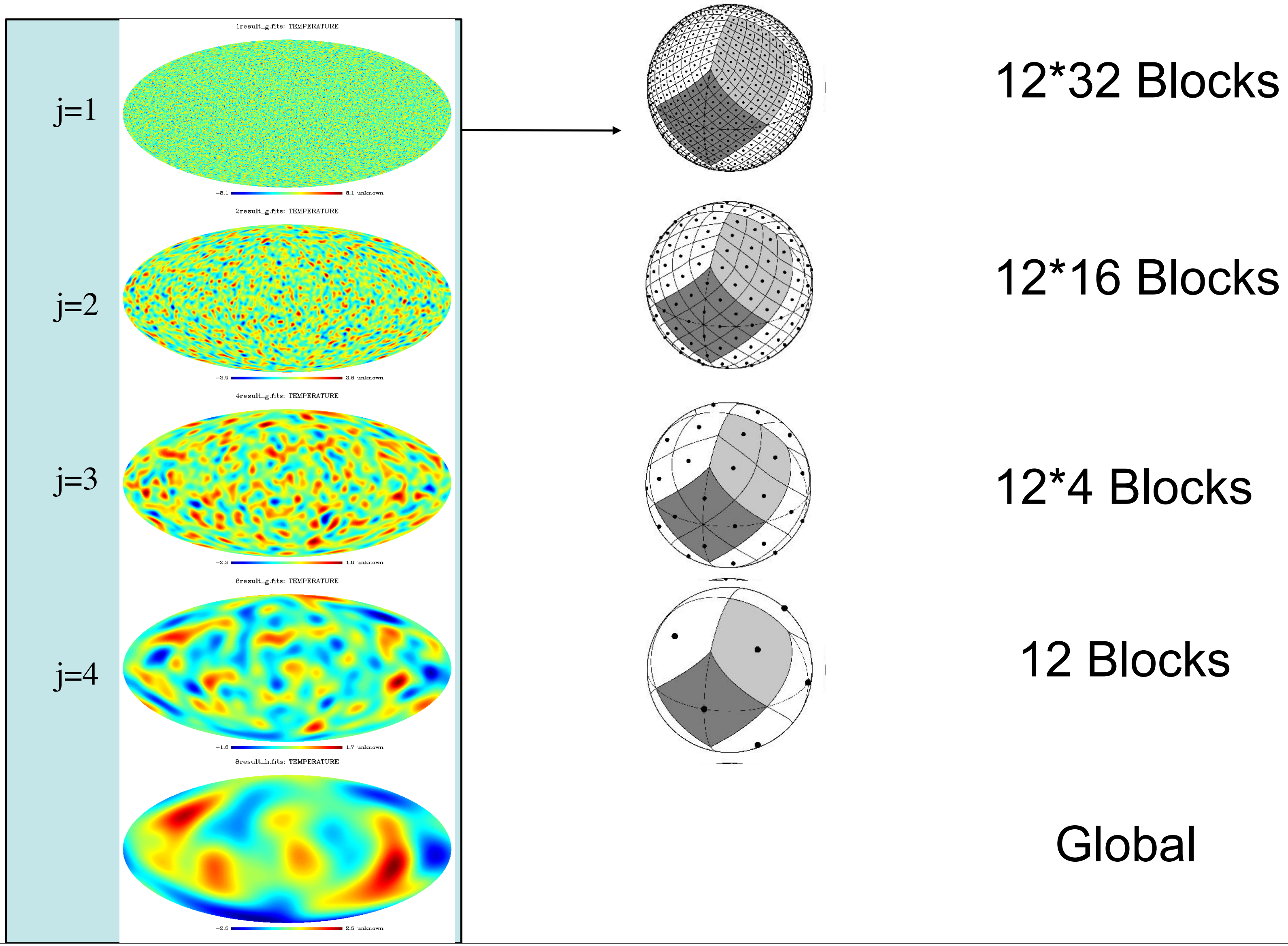


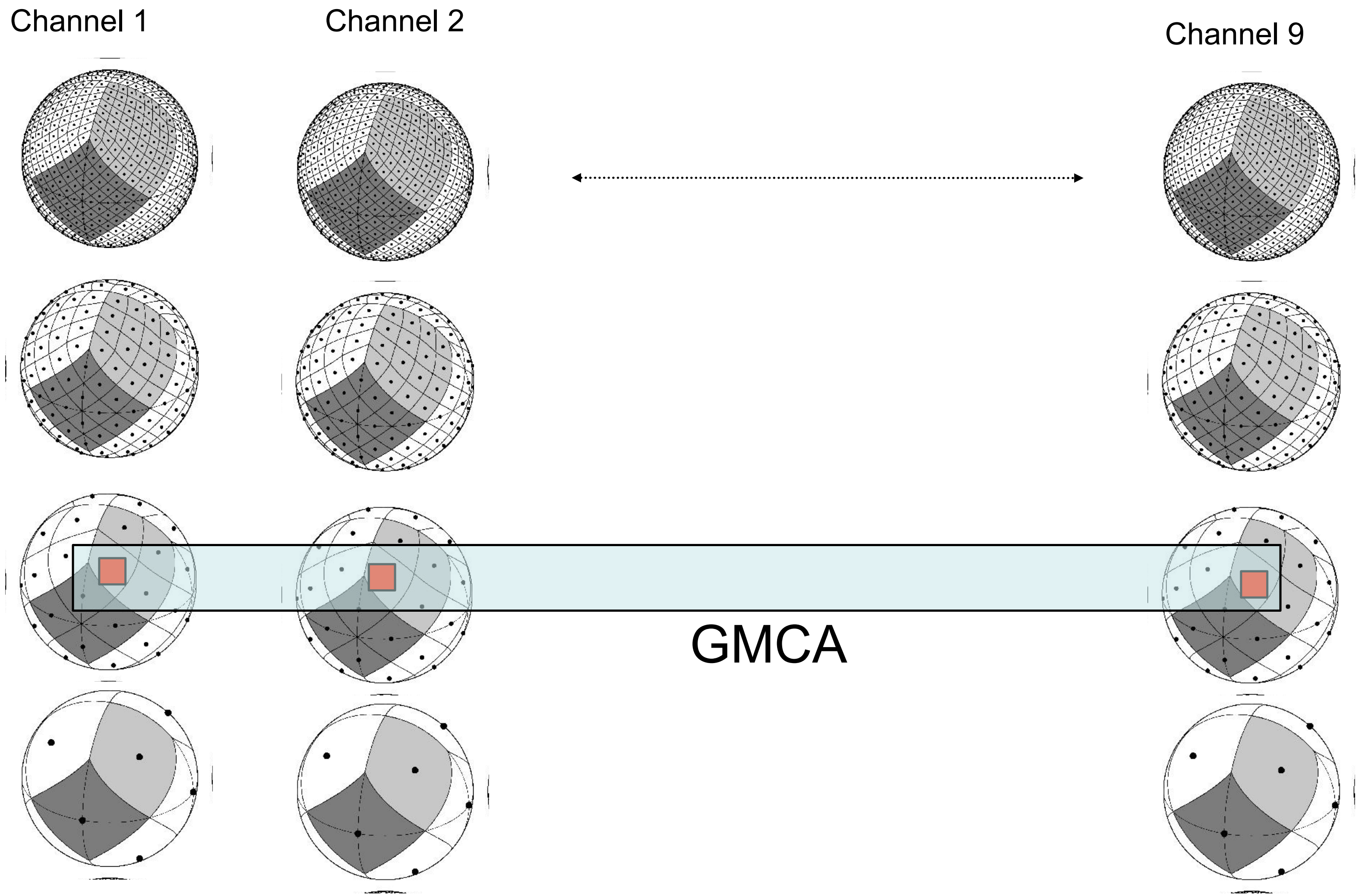
The sparse GMCA solution is obtained by minimizing:

$$\min_{\alpha_j, A_j} \sum_j \frac{1}{2\sigma^2} \left\| \beta_j - A_j \alpha_j \right\|^2 \quad \text{s.t.} \quad \alpha \quad \text{is sparse}$$

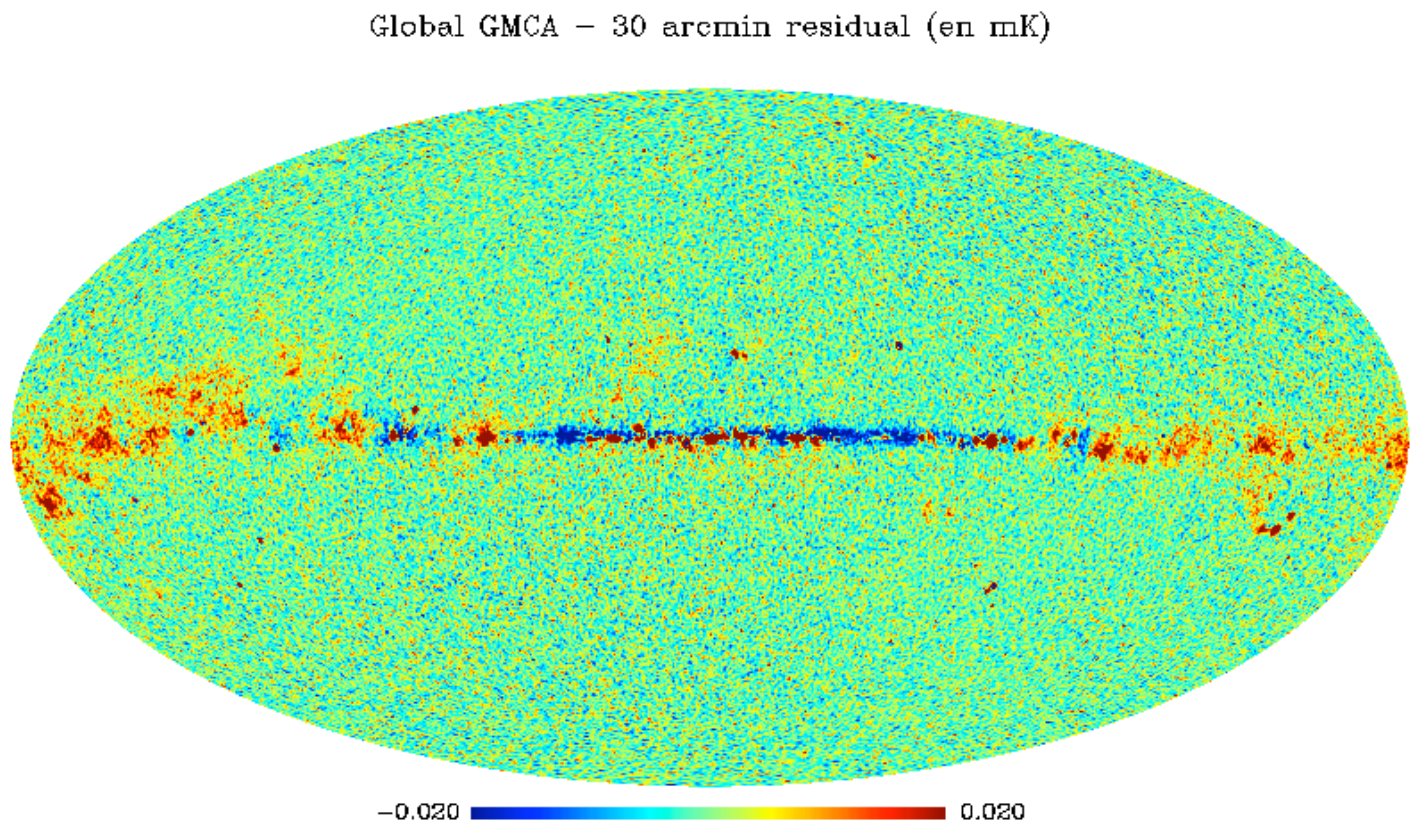
LOCAL GMCA

Undecimated Isotropic Wavelet Transform + Block Partitioning

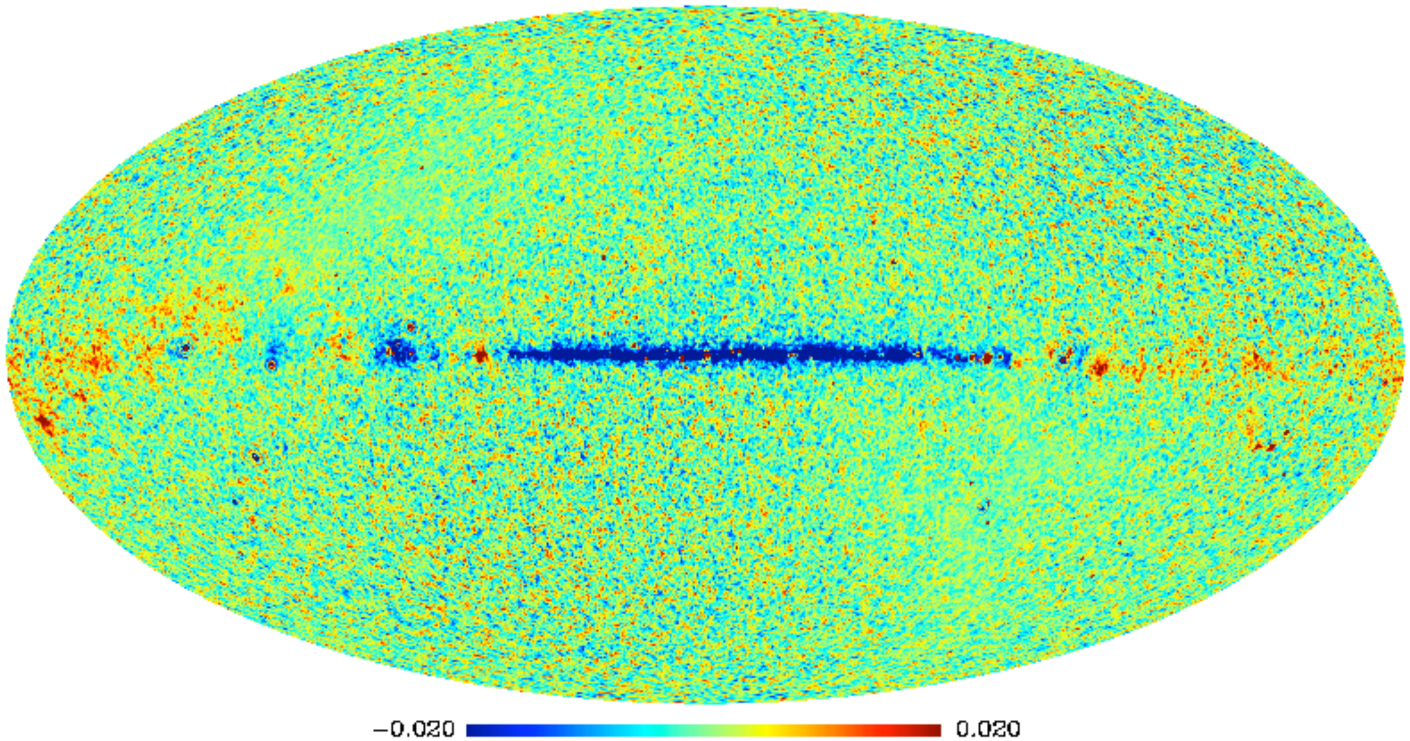


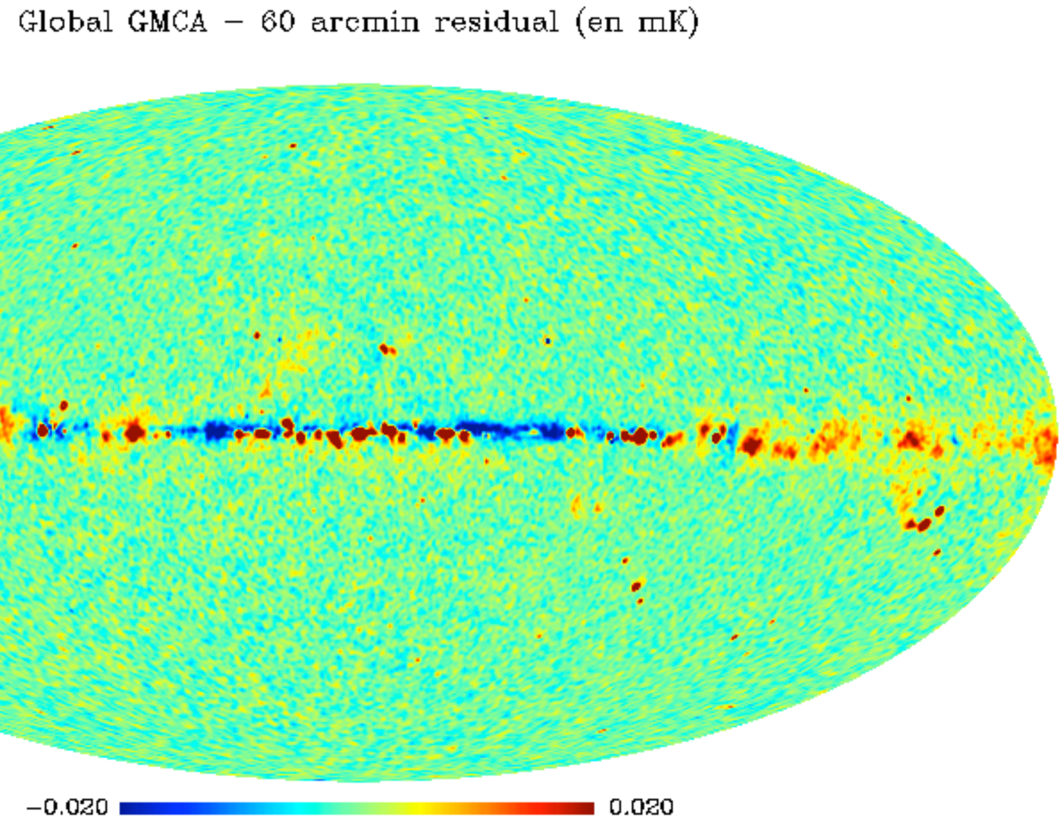

Channel 1
Channel 2
Channel 9
GMCA

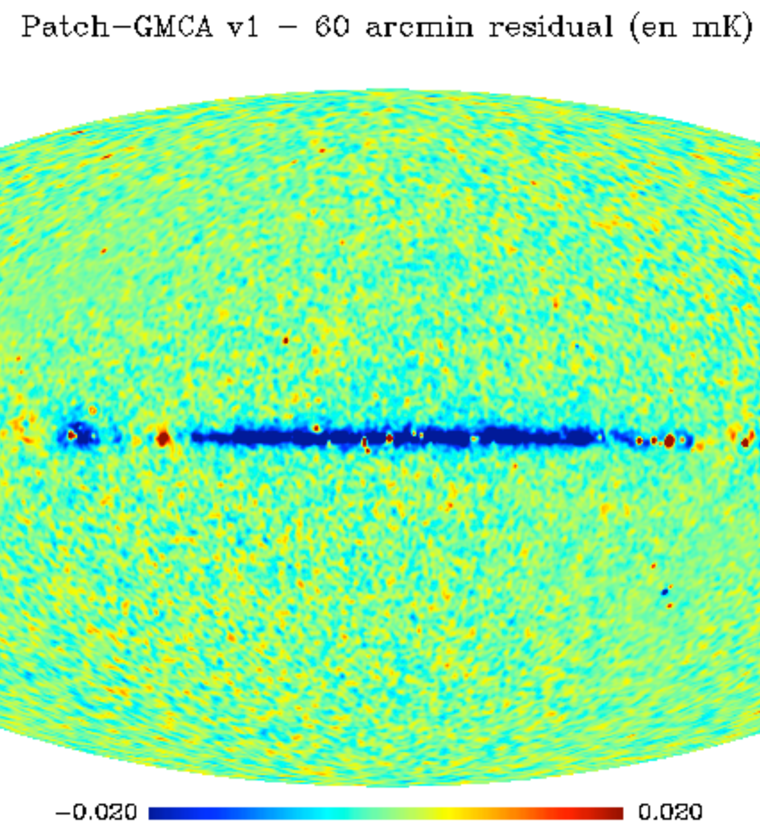
GMCA - Global vs Patch v1



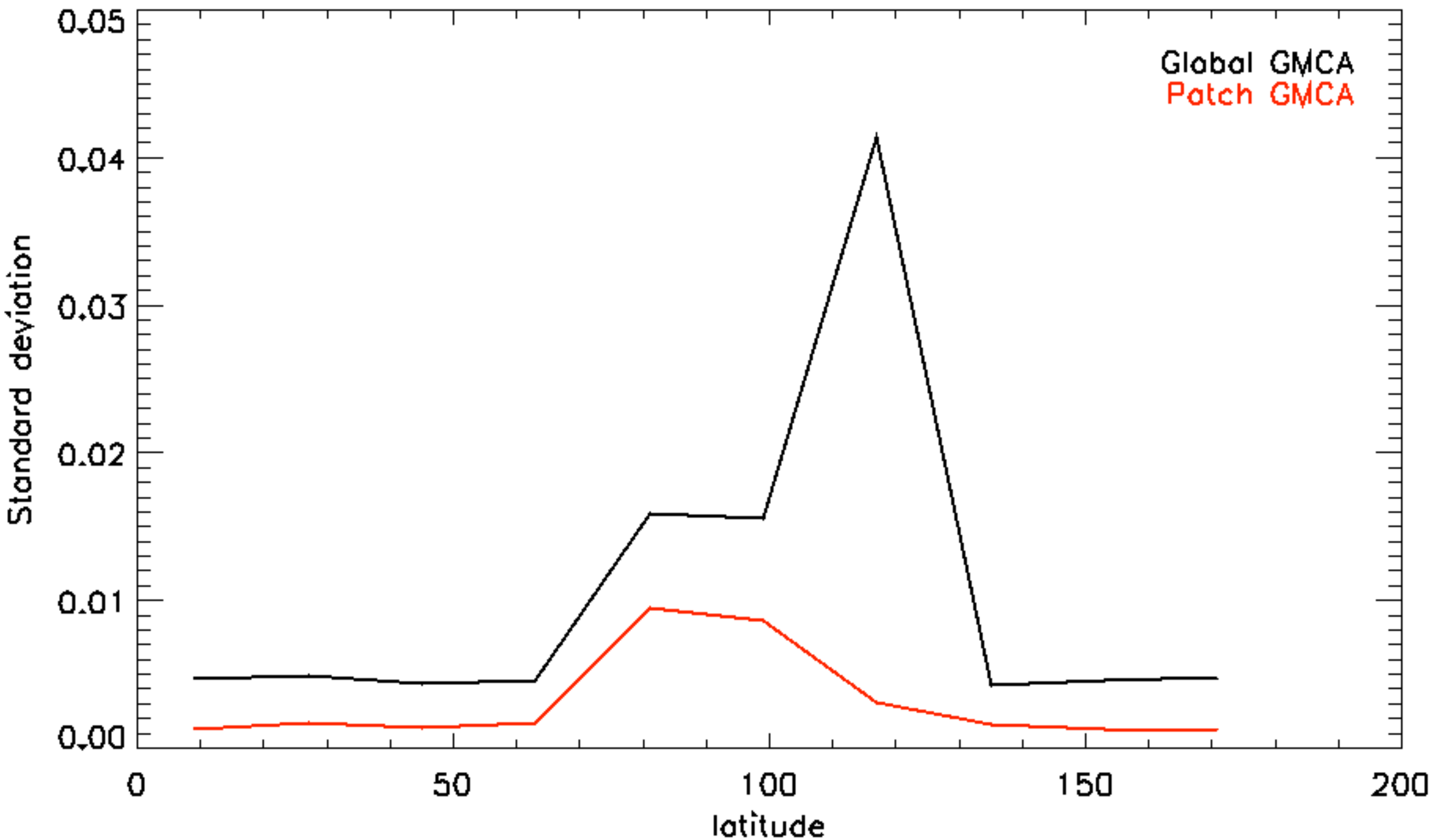
Patch-GMCA v1 - 30 arcmin residual (en mK)

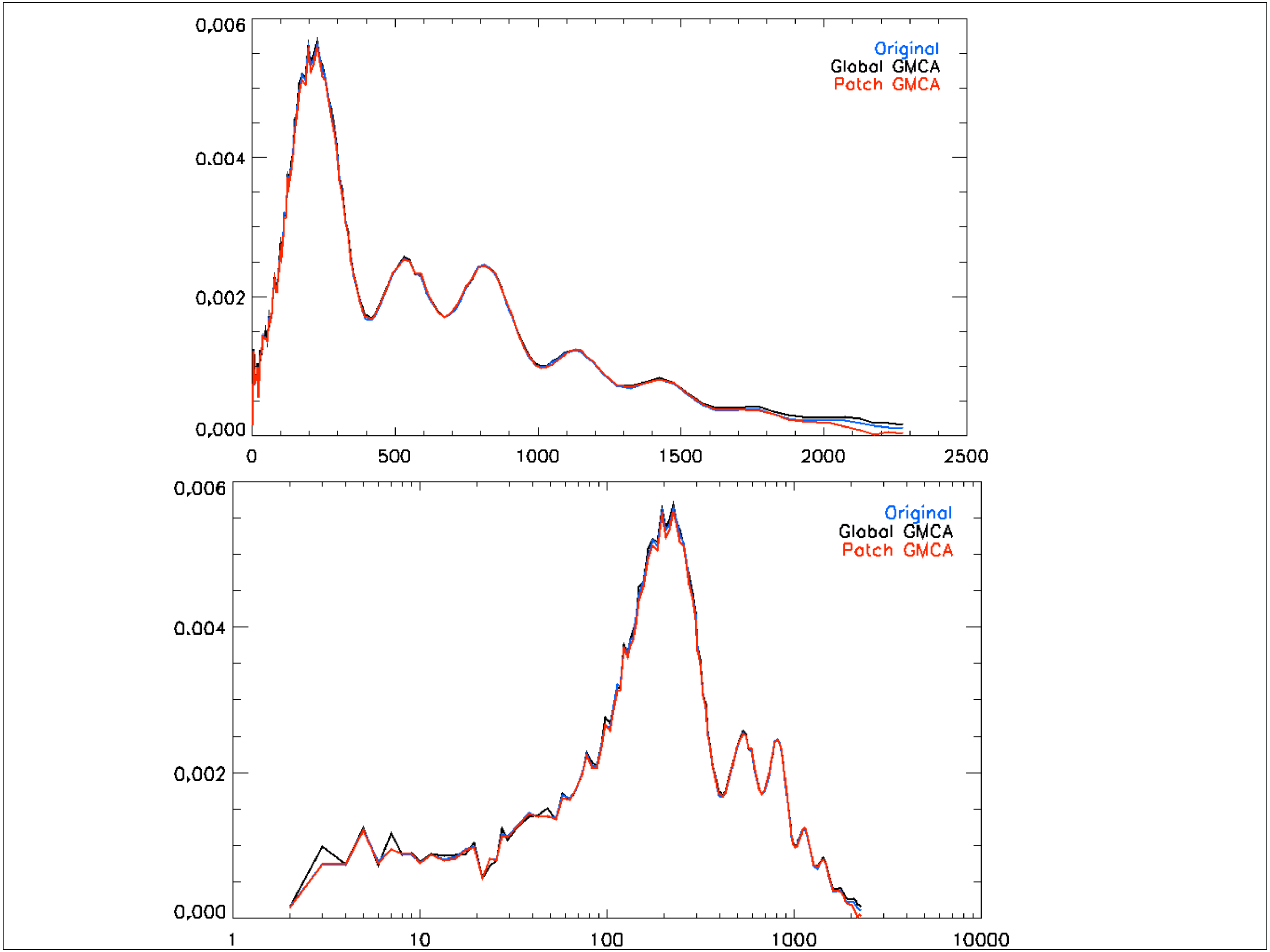



Global GMCA – 60 arcmin residual (en mK)
−0.020
0.020

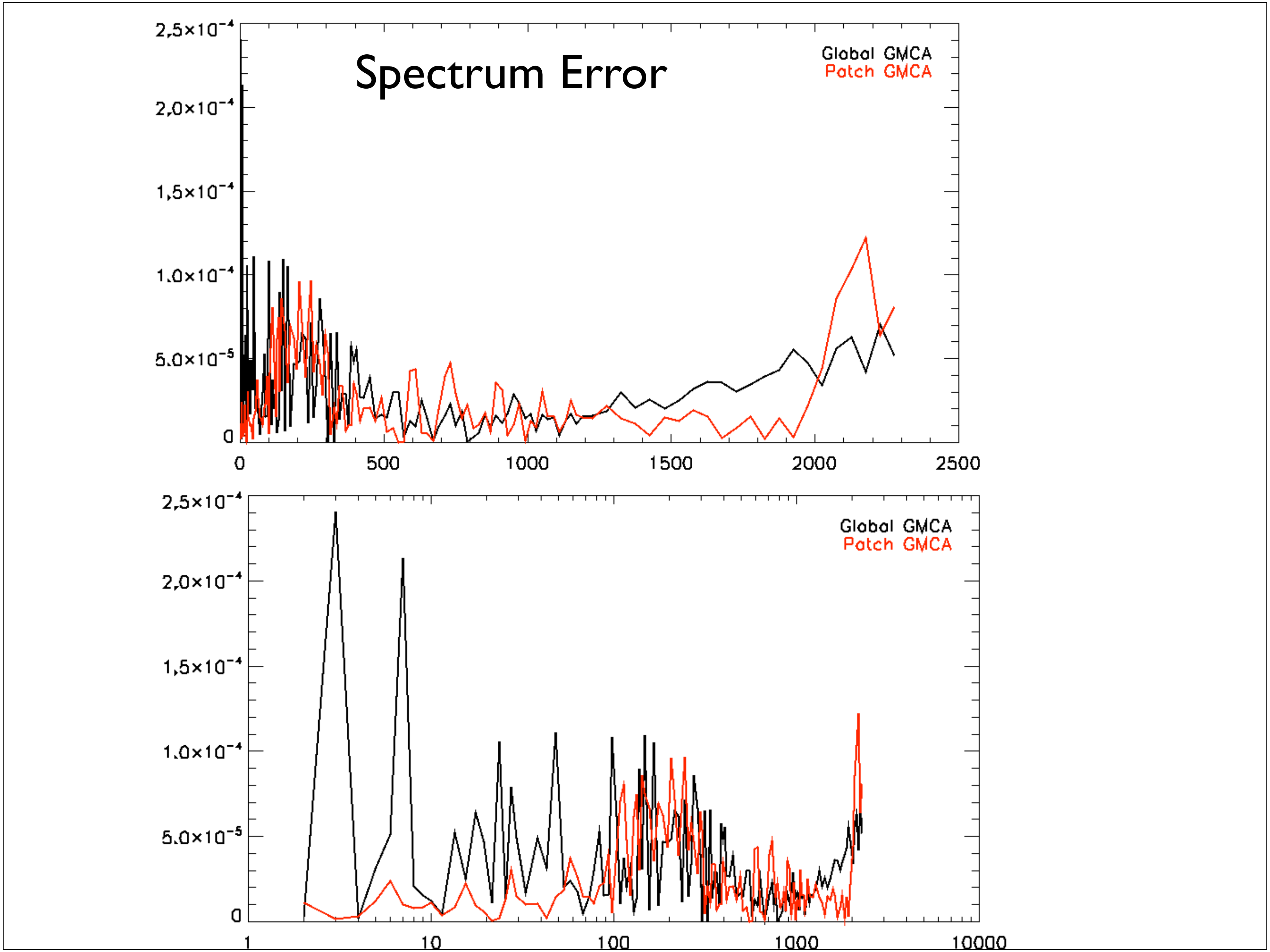

Patch-GMCA v1 – 60 arcmin residual (en mK)
−0.020
0.020

Quadratic Error per Latitude Band





0,006
0,004
0,002
0,000
0
500
1000
1500
2000
2500
Original
Global GMCA
Patch GMCA
0,006
0,004
0,002
0,000
1
10
100
1000
10000
Original
Global GMCA
Patch GMCA



Spectrum Error
Global GMCA
Patch GMCA
2,5×10⁻⁴
2,0×10⁻⁴
1,5×10⁻⁴
1,0×10⁻⁴
5,0×10⁻⁵
0
0
500
1000
1500
2000
2500
Global GMCA
Patch GMCA
1
10
100
1000
10000

PSM Sparse Reconstruction

Flexible Framework in Convex Optimization

- **Flexible Framework** developed in Convex Optimization

$$\underset{x \in \mathbb{R}^n}{\text{minimize}}\, f_1(x) + \ldots + f_m(x)$$

P. L. Combettes and V. R. Wajs, "Signal recovery by proximal forward-backward splitting", Multiscale Modeling and Simulation, vol. 4, no. 4, pp. 1168-1200, November 2005

F.X. Dupe, M.J. Fadili, J.-L. Starck, " A proximal iteration for deconvolving Poisson noisy images using sparse representations", IEEE Transactions on Image Processing, Vol. 18, No. 2

·C. Chaux, J.-C.Pesquet, N. Pustelnik, "Nested iterative algorithms for convex constrained image recovery problems", SIAM Journal on Imaging Sciences, Vol. 2, No. 2, Jun. 2009, pp. 730-762

- **Convex function to minimize:** F. Sureau, O. Fourt, and J.L Starck, ADA 6, may 2010.

$$\hat{\alpha}_c^{(n)} = \arg\min_{\alpha_c} \underbrace{||Y - A\Phi\alpha_c||^2_\Sigma}_{f_1} + \underbrace{\lambda||\alpha_c||_1 + i_C(\Phi\alpha_c)}_{f_2}$$

$$i_C(s) = \begin{cases} 0 \text{ if } s \in C \\ +\infty \text{ if } s \notin C \end{cases}$$ to enforce constraints (positivity, bounds, support...)

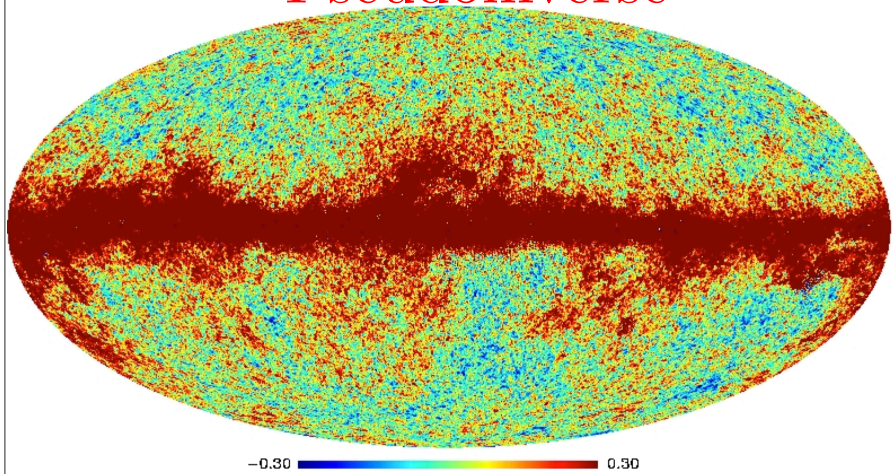
Toy Model Experiment

- **Toy model to evaluate if this approach is robust to error in A**
- Linear Model based on maps from the Planck Sky Model (CMB, Free/Free, Synchrotron, Spinning and Thermal Dust)
- No spatial variance of spectral indices
- Gaussian Beam and i.i.d noise according to Planck specifications
- Approximate mixing matrix by uniform error of 5% on A except for CMB (known)
- Sparsity constraint (Wavelets)
- Bounds on min/max values for each component, map Nside=512
- Constraints in image space and wavelet space.

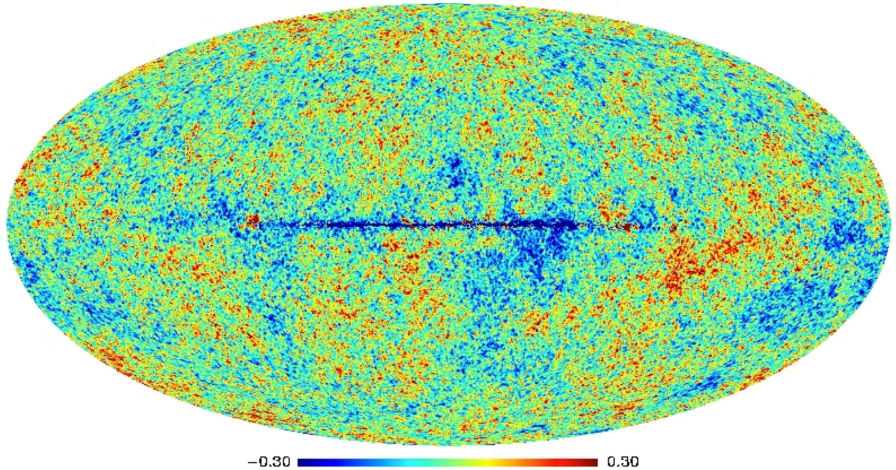
Model: $Y = L\,X+N$, and we have an approximation of L

Results

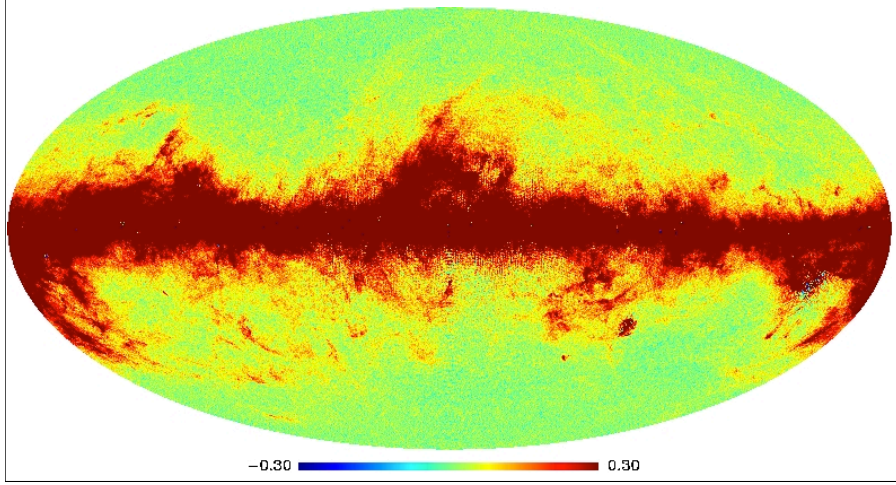
Pseudoinverse



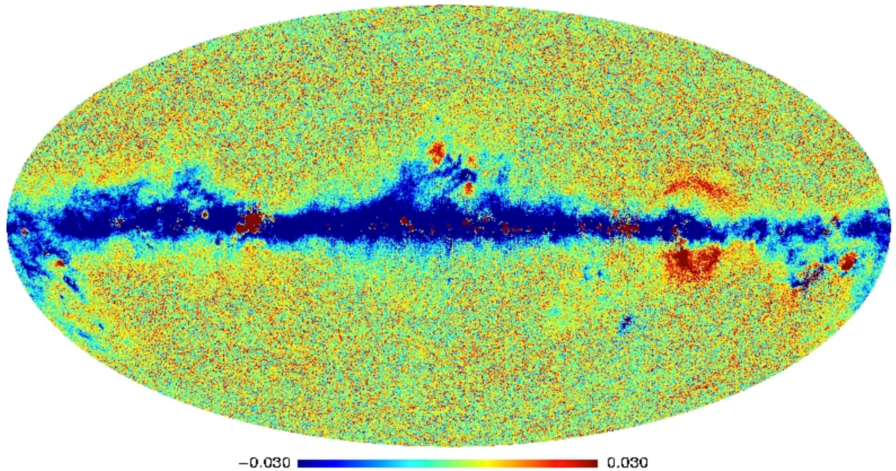
Constrained restoration



Residual Pseudoinverse



Residual Constrained Rest.



Conclusions

•We have investigated another approach (i.e. sparsity) for PLANCK data analysis.
•Software available for sparse representation for both temperature and polarized maps.
•GMCA, the sparse component separation method, gives very interesting results a low l, with a simple model (a single matrix).

Perspectives

•We expect a strong improvement by using a more complex model (i.e. local GMCA, waguelet-GMCA decomposition).
•We need to extend it to take into account the polarization.
•We need to exploit better our knowledge of the galactic emission ==> MISTIC project.