Approximation of Points on Low-dimensional Manifolds via Compressive Measurements

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Thursday, March 10, 2011

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Outline

- The Problem: Approximation of Points Near A Manifold
- 2 Representing the Manifold: Geometric Wavelets
- 3 Proposed Recovery Procedure
- Approximation Guarantees



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The Problem – Manifold CS [Baraniuk, Wakin, ...]

- We have an approximate representation for a compact *d*-dimensional Riemannian submanifold, \mathcal{M} , of \mathbb{R}^D
 - We expect to recover points, $\vec{x} \in \mathbb{R}^{D}$, nearly on \mathcal{M}
 - d << D
- We acquire compressed measurements of \vec{x} , $M\vec{x} \in \mathbb{R}^m$, via an $m \times D$ matrix M
 - *M* is a Johnson-Lindenstrauss embedding (also has RIP)
- Approximate the Optimal Representative for \vec{x} on \mathcal{M} ,

$$\vec{x}_{\text{opt}} = \operatorname*{arg\,min}_{\vec{y} \in \mathcal{M}} \|\vec{x} - \vec{y}\|.$$

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$$ec{x}_{ ext{opt}} = rgmin_{ec{y} \in \mathcal{M}} \|ec{x} - ec{y}\|.$$

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Geometric Wavelets [Allard, Chen, Maggioni]

Built in two stages:

• Create a Dyadic Partition of Samples from $\mathcal{M} \subset \mathbb{R}^D$



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• Estimate Tangent Space within Each Dyadic Cube

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Example: Swiss Roll



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Coarse Scale Approximation of Swiss Roll



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Dyadic Structure for Swiss Roll Approximation



M.A. Iwen, M. Maggioni Manifold CS

Geometric Wavelets Give Us

At each Scale $j \in [J] = \{1, \ldots, J\}$ we get:

- A set of dyadic "centers" denoted by $\vec{c}_{j,k}$ for $k \in [K_j]$
- A set of orthogonal $d \times D$ matrices, $\Phi_{j,k}$, for $k \in [K_j]$
- An affine projector, $\mathbb{P}_{j,k}(\vec{x})$, for $k \in [K_j]$ defined as

$$\mathbb{P}_{j,k}\left(ec{x}
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The Scale *j* Approximation of $\vec{x} \in \mathbb{R}^{D}$ near \mathcal{M} is ...



• $k_j(\vec{x}) = \arg\min_{k \in [K_j]} \|\vec{x} - \vec{c}_{j,k}\|$ • $\vec{x} \approx \mathbb{P}_{j,k_j(\vec{x})}(\vec{x}) = \Phi_{j,k_j(\vec{x})}^T \Phi_{j,k_j(\vec{x})}(\vec{x} - \vec{c}_{j,k_j(\vec{x})}) + \vec{c}_{j,k_j(\vec{x})}$

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Scale *j* Approximation Guarantee

• Recall that
$$\vec{x}_{\text{opt}} = \underset{\vec{y} \in \mathcal{M}}{\arg\min} \| \vec{x} - \vec{y} \|.$$

Theorem

Let \mathbb{P}_j be a scale *j* Geometric Wavelet representation for a compact smooth submanifold, \mathcal{M} , of \mathbb{R}^D . Then, for *j* sufficiently large, we will have

$$\left\|\vec{x} - \mathbb{P}_{j,k_{j}\left(\vec{x}\right)}\left(\vec{x}\right)\right\| \leq 4\left\|\vec{x} - \vec{x}_{\text{opt}}\right\| + O\left(2^{-j}\right)$$

for all $\vec{x} \in \mathbb{R}^D$.

• Can we achieve similar approximation guarantees using only compressed measurements of \vec{x} (i.e., $M\vec{x}$)???

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Approximate $\mathbb{P}_{j,k_j(\vec{x})}(\vec{x})$ using only $M\vec{x} \in \mathbb{R}^m$

Input: Measurements $M\vec{x} \in \mathbb{R}^m$, Approximation to manifold $\mathcal{M} \subset \mathbb{R}^D$, $\mathbb{P}_j = \{\mathbb{P}_{j,k} \mid k \in [K_j]\}$.

2 Nearest Neighbors:
$$k' \leftarrow$$
 arg min $_{k \in [K_i]} || M\vec{x} - M\vec{c}_{j,k} ||$.

- **3** Least Squares: $\vec{u}' \leftarrow$ arg min $_{\vec{u} \in \mathbb{R}^d} \| M \Phi_{j,k'}^{\mathrm{T}} \vec{u} M \vec{x} \|$.
- **Output:** $\mathcal{A}(M\vec{x}) \approx \mathbb{P}_{j,k_j(\vec{x})}(\vec{x})$

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- **2** Nearest Neighbors: $k' \leftarrow \arg \min_{k \in [K_i]} ||M\vec{x} M\vec{c}_{j,k}||$.

Second Least Squares: $\vec{u}' \leftarrow \arg \min_{\vec{u} \in \mathbb{R}^d} \| M \Phi_{j,k'}^{\mathrm{T}} \vec{u} - M \vec{x} \|$.

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Notes

A Nonuniform Approximation Guarantee

Theorem

Let $\mathcal{M} \subset \mathbb{R}^D$ be a compact *d*-dimensional Riemannian submanifold of \mathbb{R}^D , $\vec{x} \in \mathbb{R}^D$, $\delta \in \mathbb{R}^+$, and $p \in (0, 1)$. Then, the reconstruction algorithm on the last slide, $\mathcal{A} : \mathbb{R}^m \to \mathbb{R}^D$, is such that a random $m \times D$ measurement matrix, M, will satisfy

$$\left\| ec{x} - \mathcal{A} \left(M ec{x}
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ight\| \ \le \ \mathbf{8} \cdot \left\| ec{x} - ec{x}_{ ext{opt}}
ight\| + \delta$$

with probability at least 1 - p whenever *m* is $\Omega(d \log(d/p\delta))$. The reconstruction algorithm's runtime will be O(dmD).

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with probability at least 1 - p whenever *m* is $\Omega(d \log(d/p\delta))$.

- Ω-notation hides *m*'s dependence on properties of *M* (i.e., its *d*-dimensional volume, curvature and reach)
- But, *m* does not depend on the extrinsic dimension of \mathcal{M}

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$$\left\| \vec{x} - \mathcal{A} \left(\mathbf{M} \vec{x} \right) \right\| \leq 8 \cdot \left\| \vec{x} - \vec{x}_{\text{opt}} \right\| + \delta$$

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Uniform Approximation Guarantees

- Uniform recovery guarantees are worse, as expected [sparse recovery results, Wakin]
- Guarantees only hold when our measurement matrix has the RIP of order *r* so that $\frac{1}{\sqrt{r}} \|\vec{x} - \vec{x}_{opt}\|_1$ is $O\left(\max\left\{\|\vec{x} - \vec{x}_{opt}\|_2, 2^{-j}\right\}\right)$
 - This generally means $r = \Omega(D)$ if $\|\vec{x} \vec{x}_{opt}\|_2$ is large
 - Otherwise, $\|\vec{x} \vec{x}_{opt}\|_1$ must be $O(\sqrt{r} \cdot 2^{-j})^{-j}$
- Measurement matrix, *M*, must approximately preserve all distances between points in *M* ∪{*c*_{*j*,*k*} | *k* ∈ [*K*_{*j*}]}

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$$\left\|\vec{x} - \mathcal{A}\left(M\vec{x}\right)\right\|_{2} \leq \delta$$

for all
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 with $\left\| \vec{x} - \vec{x}_{\text{opt}} \right\|_{1} \leq \sqrt{d} \left(\delta + \left\| \vec{x} - \vec{x}_{\text{opt}} \right\|_{2} \right)$.

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Notes

- If *M* collectively spans only a small subspace of R^D,
 Geometric Wavelets will reveal it. We can then reduce the effective extrinsic dimensionality.
- If we can adaptively measure $\vec{x} \in \mathbb{R}^D$ then we can approximate $\vec{x}_{opt} \in \mathcal{M}$ by...
 - Performing *O*(*dj* log *d*) half space tests to find the proper scale *j* dyadic center.
 - Projecting \vec{x} onto the proper approximate tangent space
- Compressive measurements may not be necessary if we can collect many measurements, but only want to read a few...

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Questions?

M.A. Iwen, M. Maggioni Manifold CS

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