Grid-Free Denoising of Point-Cloud Data via Non-Local Regularization[1]

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Workshop on Sparse and Low Rank Approximation

Banff International Research Station

March 7, 2011

[1]Support provided by NGA

Outline

Introduction and Problem Description

- Typical *Light Detection and Ranging* (LiDAR) systems transmit a laser pulse and receive a **deluge of data** in the form of a three-dimensional (3D) **point-cloud**.

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- Unfortunately, there are **significant noise** issues, e.g., due to sensitivity of photodiodes.
- State-of-the-art 3D surface reconstruction algorithms require the **use of a grid** to determine measures of gradient and how "close" points are.
- We propose a **3D grid-free** generalization of regularization with **non-local methods**, which is already used extensively to denoise 2D images.

Review of LiDAR

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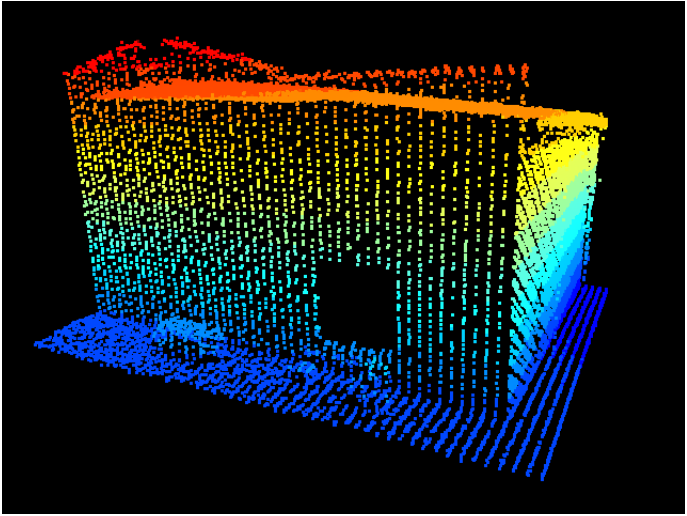
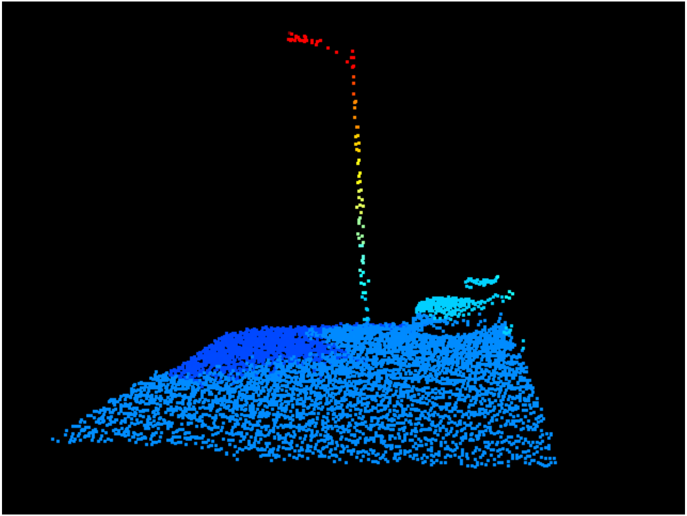
Denoised LiDAR Scene, Low-Altitude (22,120 points)



[Courtesy of AFRL/MNG VEAA Data Set #1]

Denoised LiDAR Scene, Low-Altitude (5,928 points)



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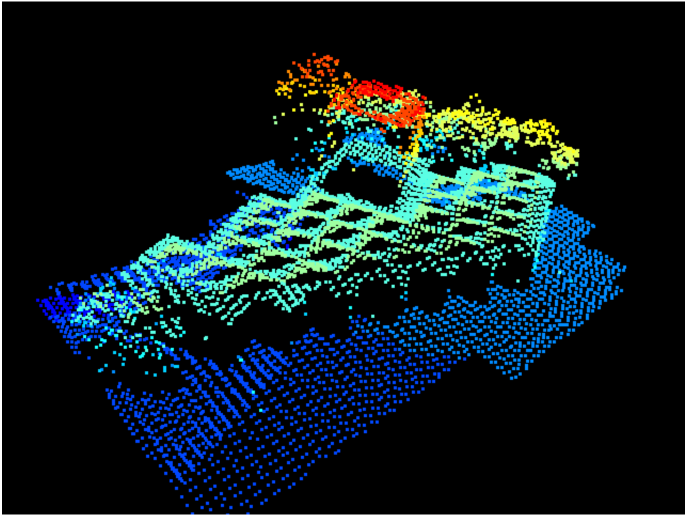
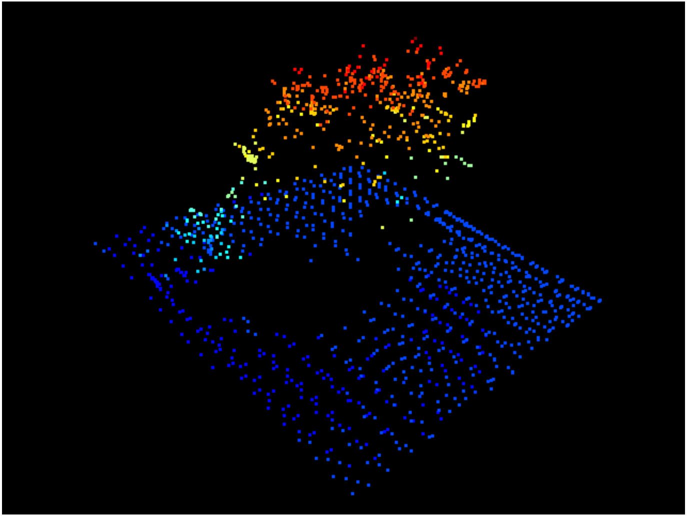
Denoised LiDAR Scene, High-Altitude (10,118 points)



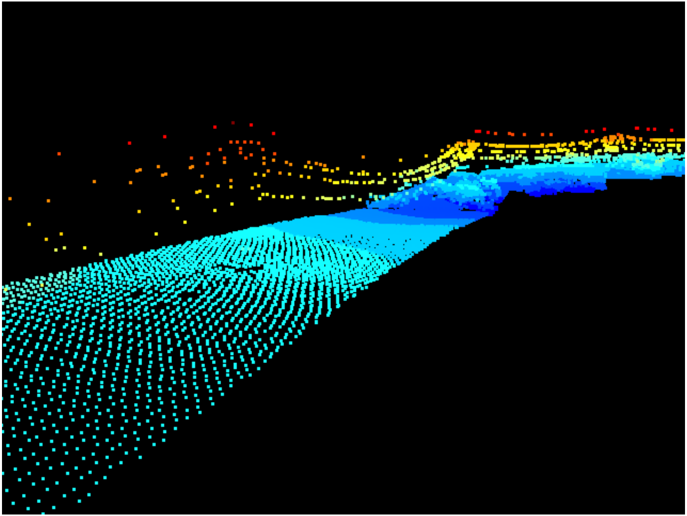
[Courtesy of ITC Data Set]

Denoised LiDAR Scene, High-Altitude (1,467 points)



[Courtesy of ITC Data Set]

Denoised LiDAR Scene, High-Altitude (24,942 points)



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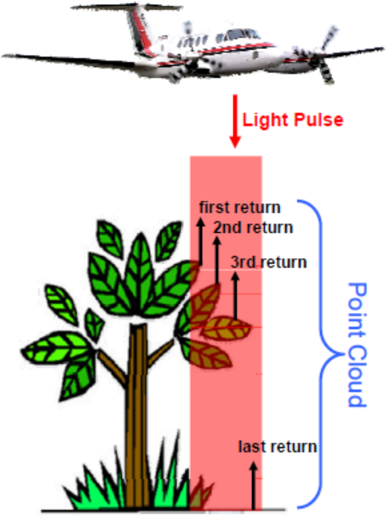
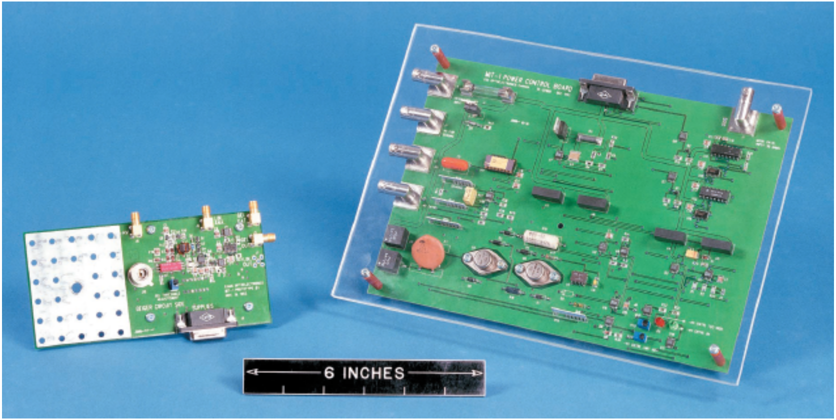
LiDAR Remote Sensing Data Collection



[Image from presentation by Myron Brown, Johns Hopkins University, Applied Physics Lab. Duke University 2009]

Early Version of a 6×5 GmAPD Detector Array



[Image from "Three-Dimensional Imaging Laser Radars with Geiger-Mode Avalanche Photodiode Arrays",

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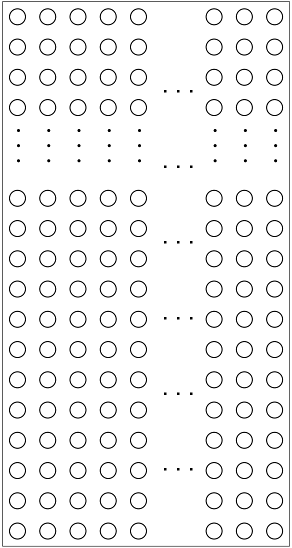
More Laser and Detector Array Details



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  $128 \times 32 =$ **4,096 elements**

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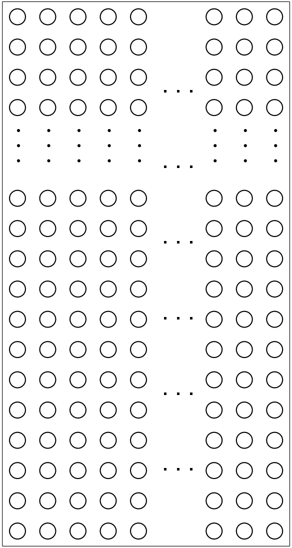

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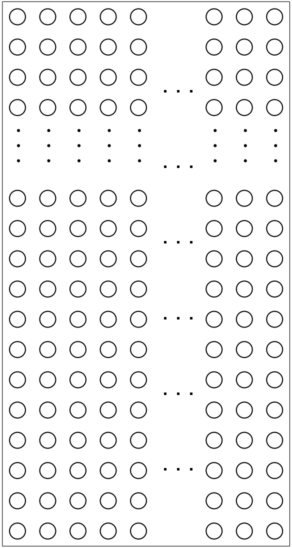

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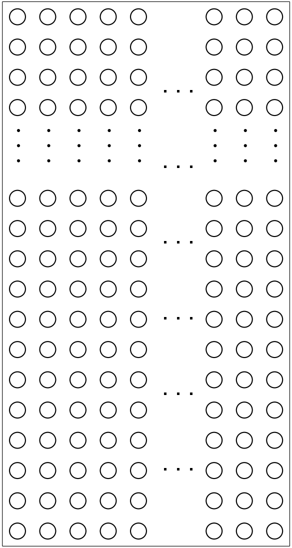

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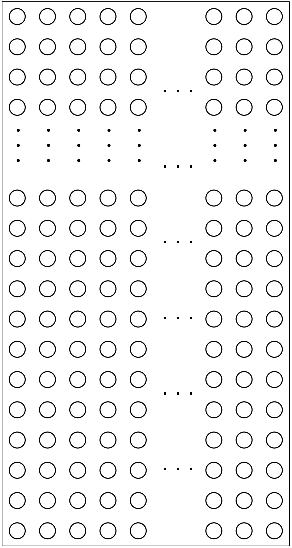

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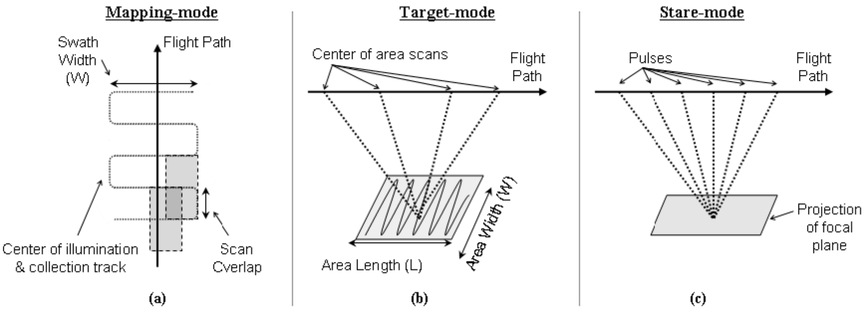
Noise in measurements due to:

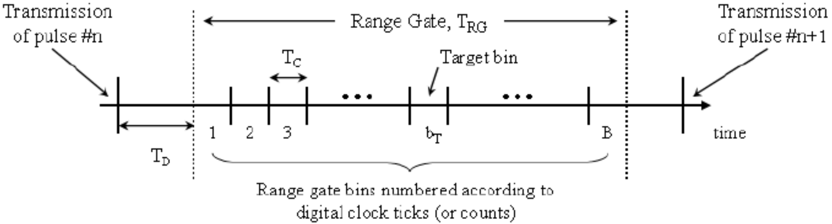
- Dark current, internal thermal energy
- Other light sources (solar, lunar, etc.)
- Cross-talk between elements

LiDAR Data Collection Scan Patterns



[Image from "Redundancy Analysis of Raw Geiger-mode Laser Radar Data", N. Lopez *et al.*, *SPIE* 2010]

Timing Model for Single GmAPD Detector Element



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Raw Point Cloud Formulation from GmAPD Counts

For each laser pulse, and each GmAPD element the value of b_T yields the range-to-target distance

$$R_T = \left(\frac{b_T T_C + T_D}{2} \right) \cdot c$$

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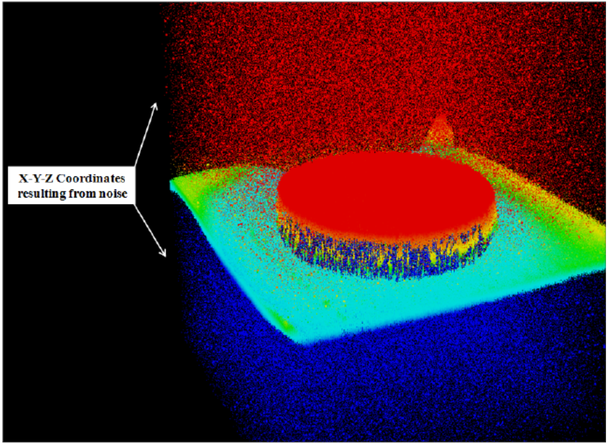
$$\begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} = \begin{bmatrix} X_P \\ Y_P \\ Z_P \end{bmatrix} + R_T \cdot (M_P M_R M_H)(M_{IT} M_{CT})(M_{row} M_{col}) \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

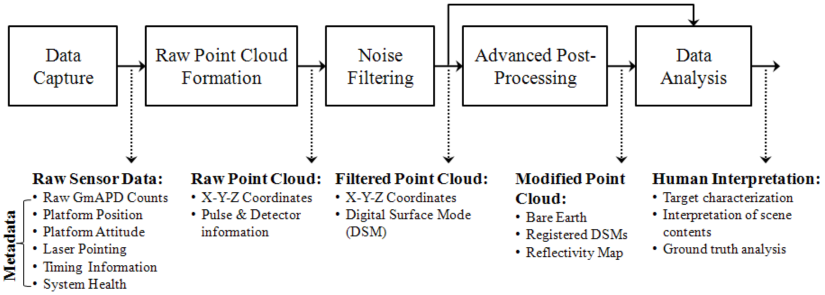
- $[X_P, Y_P, Z_P]$ is the platform position
- M_P, M_R, M_H are rotational matrices for the pitch, roll and heading (platform attitude)
- M_{IT}, M_{CT} are rotational matrices for the in-track and cross-track laser pointing angles
- M_{row}, M_{col} are rotational matrices calculated from angles related to a detector's position from the surface normal

Simulation of Raw LiDAR Data Point Cloud



[Courtesy of Myron Brown, Johns Hopkins University, Applied Physics Lab. Image from "Product Chain Analysis of Three-Dimensional Imaging Laser Radar Systems employing Geiger-mode Avalanche Photodiodes", N. Lopez *et al.*, *SPIE* 2010]

Product Chain for 3D imaging LiDAR Systems



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Review of 2D Non-Local Ideas
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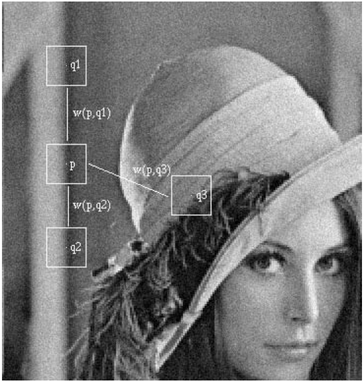
Background: 2D Non-Local Patches

Noisy Image f



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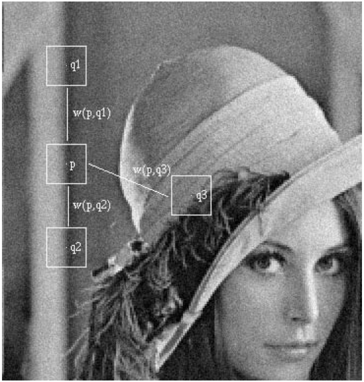

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Consider **pixels** p, q_1, q_2, q_3 and their respective **patches** $\mathcal{N}_p, \mathcal{N}_{q_1}, \mathcal{N}_{q_2}, \mathcal{N}_{q_3}$.

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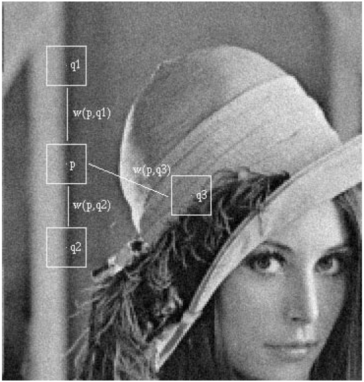

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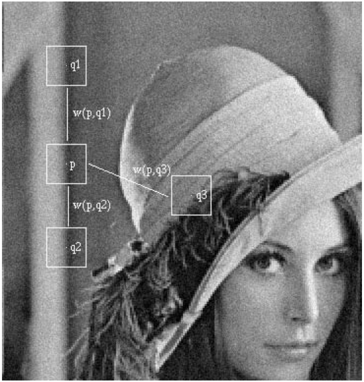

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The **weight** between pixels p, q is

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with normalization factor Z_p.

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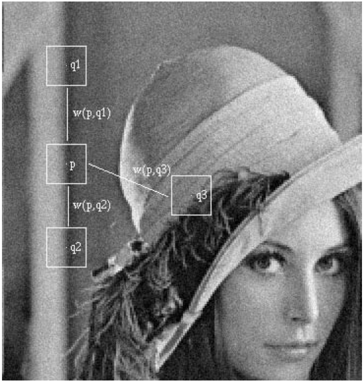

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Thus, the weights $w(p, q_1)$ and $w(p, q_2)$ are relatively large compared to $w(p, q_3)$.

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- **We want to use NL-TV to denoise point cloud data**!!!

Non-Local Ideas in 3D
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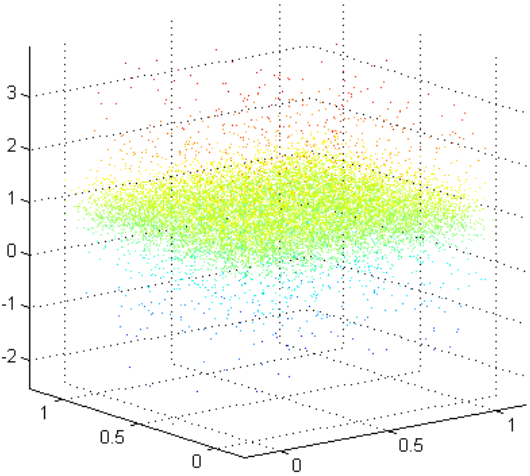
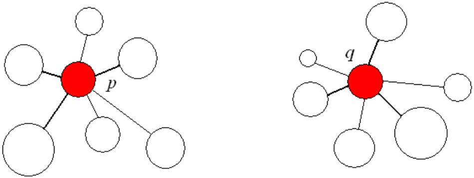
How to apply the NL concept to noisy 3D Cloud Data?

Simulated point cloud data of a noisy unit-plain



New Idea: "Patch Clouds" around points p and q



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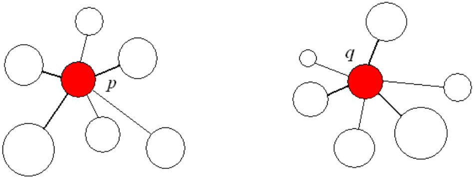

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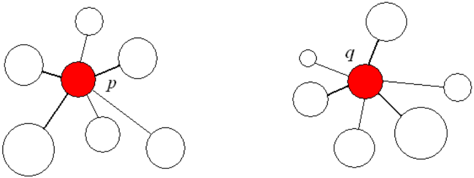

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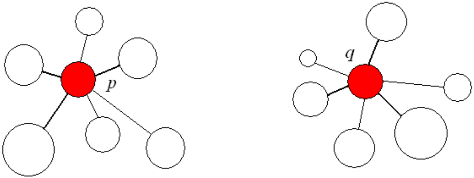

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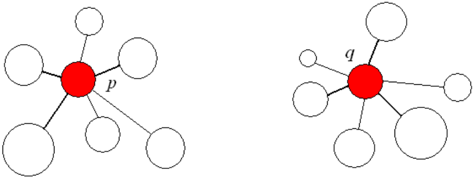

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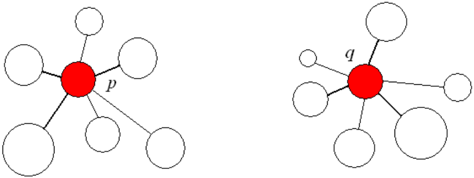

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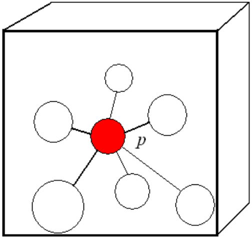
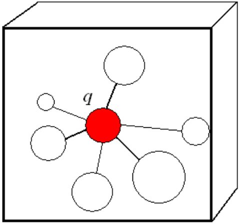
- But if no grid, then what/where are the **reference points**?!?!

“Centered” and Normalized Patch Clouds





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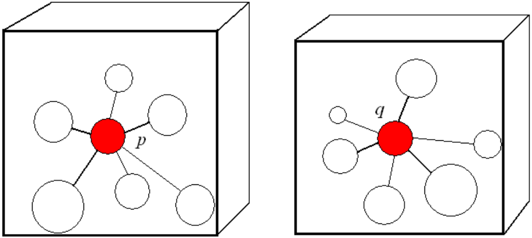

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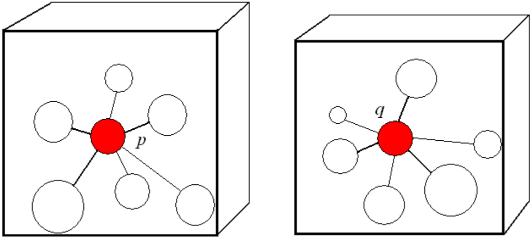

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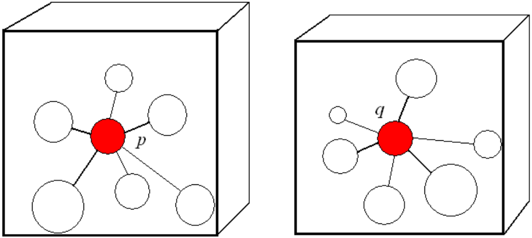

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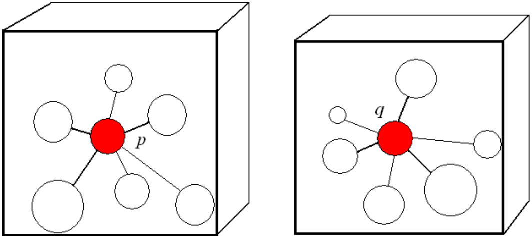

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- Next, **normalize** by dividing by the largest component (i.e., $\max\{|x|_{\max}, |y|_{\max}, |z|_{\max}\}$) in the patch cloud.

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  (Question: Is there a better way to do this?)
- Now the weight between points p and q in point cloud f is

$$w_f(p, q) := \frac{1}{Z_p} \mathrm{e}^{-\frac{\|\mathcal{G}_f(q) - \mathcal{G}_f(p)\|_2^2}{h^2}}.$$

How to Compare Geometries of Patch Clouds?

- For each corner of the centered-normalized cube, find the Euclidean distance to the **nearest point**.
- The function $\mathcal{G}(p) \in \mathbb{R}^8_+$ **encodes the geometry of the patch cloud** associated with point p.
  (Question: Is there a better way to do this?)
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- Note: Number of points in each cloud need **not** be the same.
  (Consider a noisy plain with different densities of points.)

- Now we can apply the NL regularization techniques w.r.t. the **characteristic** $\mathcal{C}$ and **geometry** $\mathcal{G}$ of the patch clouds.

Grid-Free Denoising of Point Clouds via Non-Local Reg.

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$$\min_u \left(\int \mu |\nabla_w u| + \frac{1}{2} \|u - f\|_2^2 \right),$$

and **NL-TVL1** solves:

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where

$$\Big(\nabla_w u\Big)(p, q) \ := \ \Big(\mathcal{C}_u(q) - \mathcal{C}_u(p)\Big) \sqrt{w_f(p, q)}.$$

Numerical Simulations

Numerical Simulations
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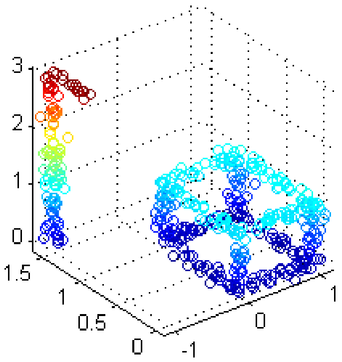
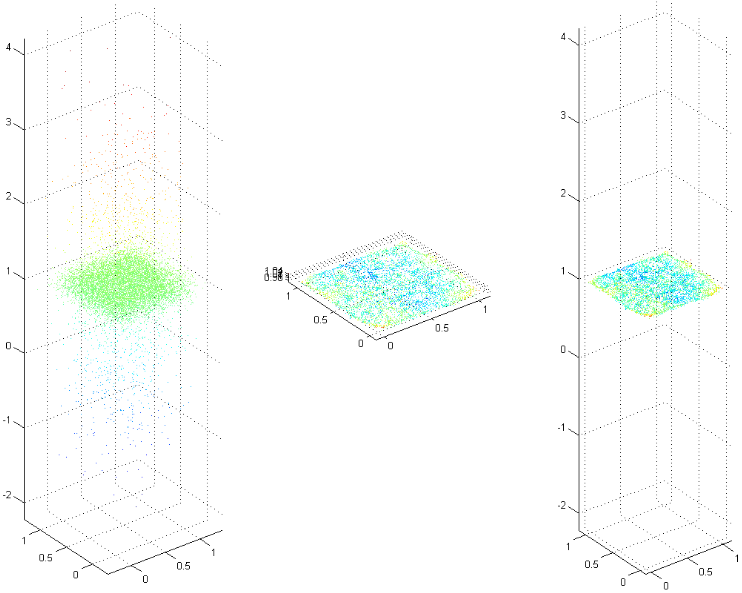
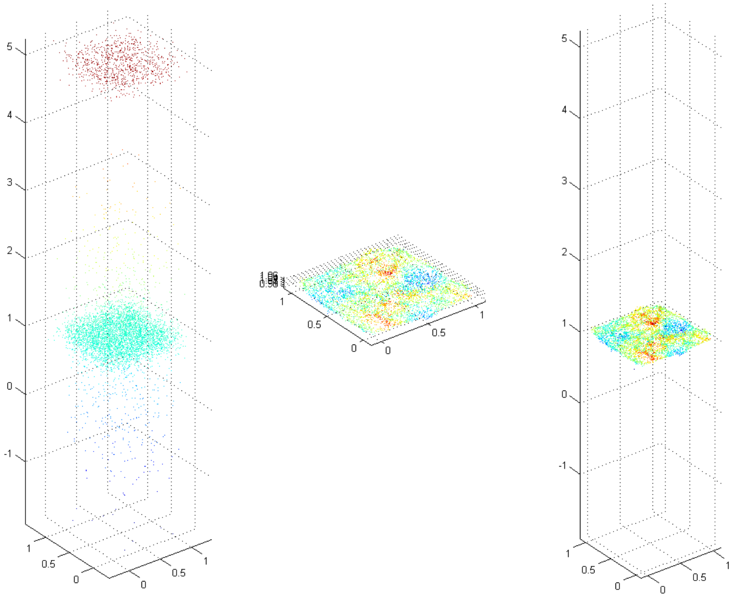
Using NL-TVL2



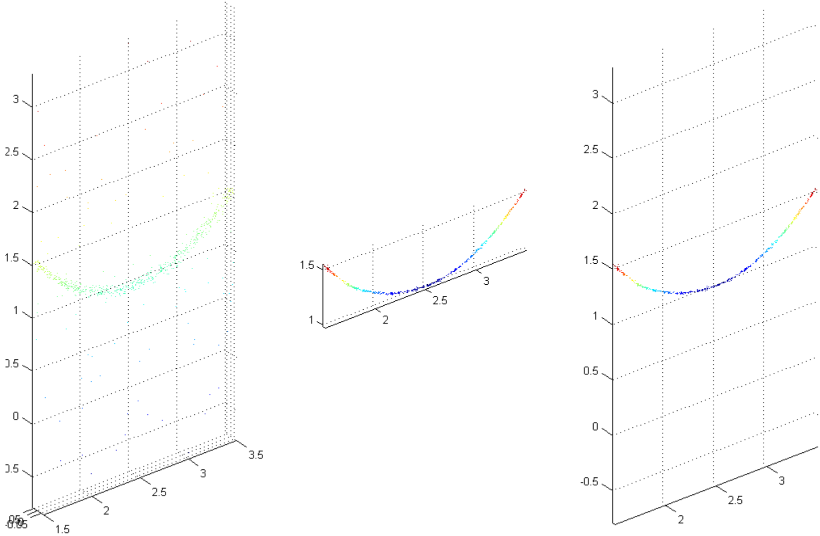
Left: Noisy Unit Plain   Center, Right: NL-TVL1 Recovery



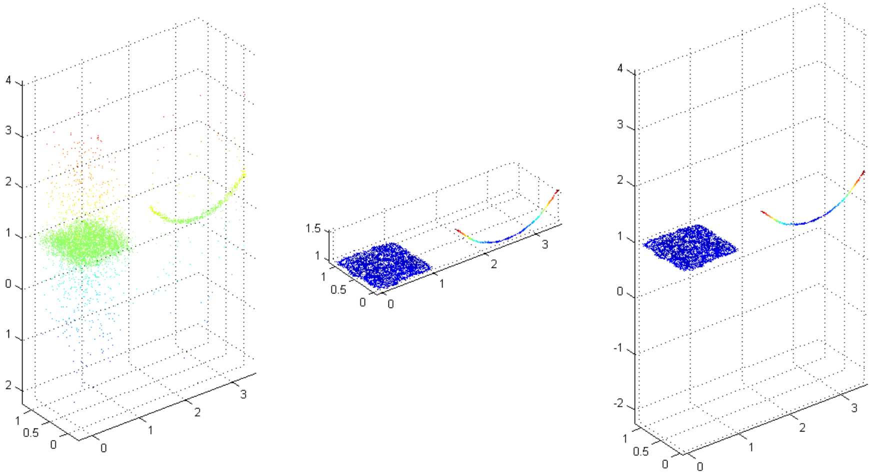
Left: Noisy Unit Plain with "Fog"   Center, Right: NL-TVL1



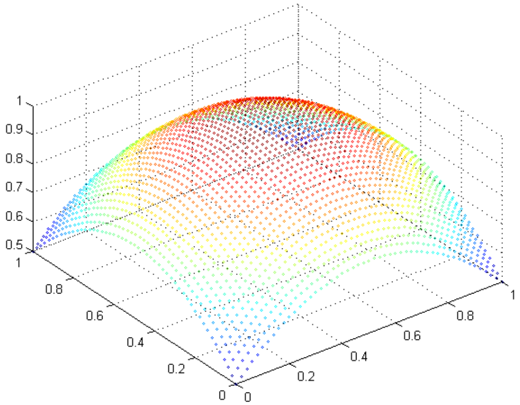
Left: Noisy Catenary Cable Center, Right: NL-TVL1



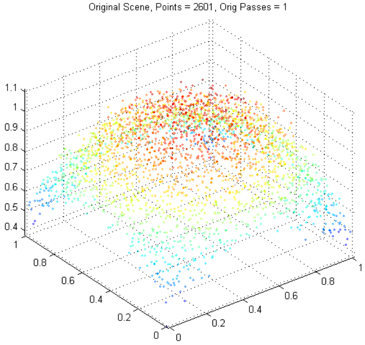
Left: Noisy Unit Plain with Cable   Center, Right: NL-TVL1

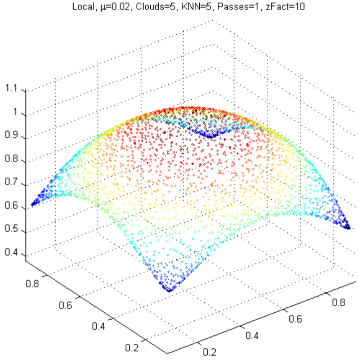


Clean and Gridded Hemisphere

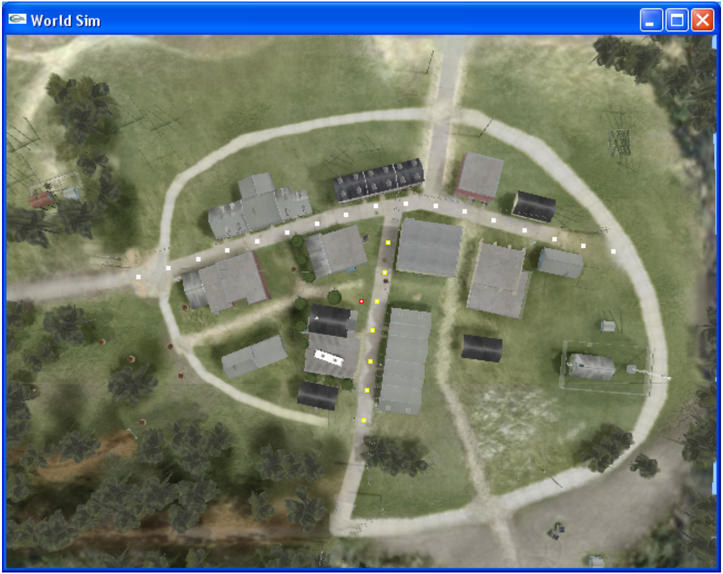


Left: Noisy Hemisphere   Right: NL-TVL1

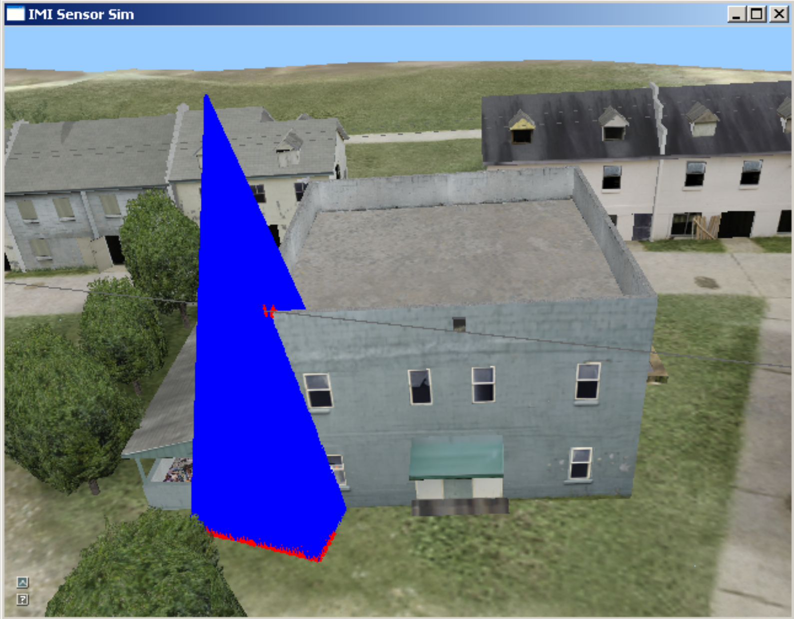


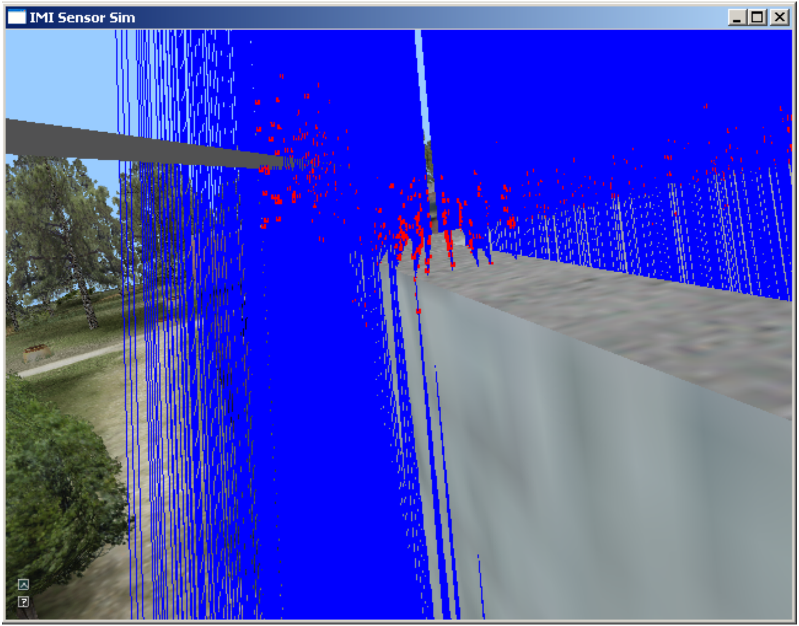


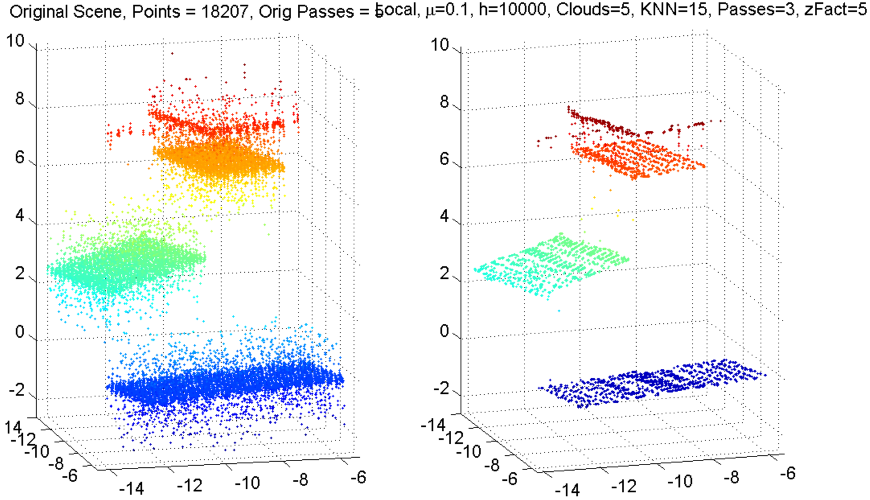
Simulated “Real” LiDAR Scene

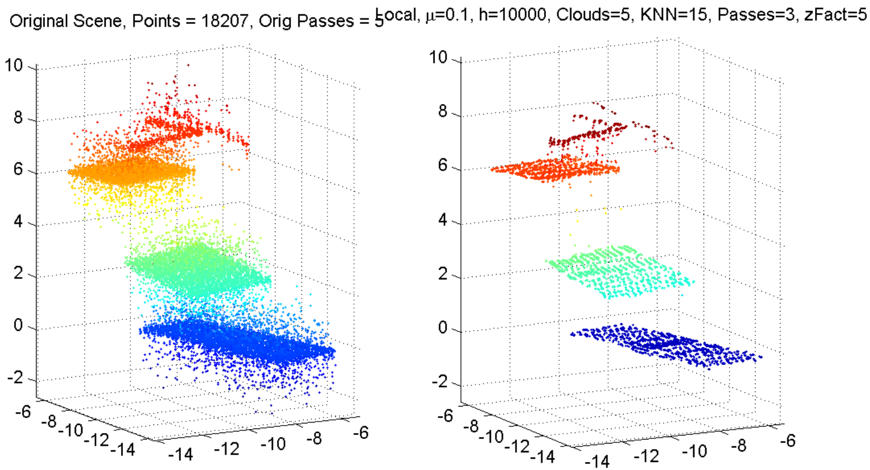



IMI Sensor Sim


IMI Sensor Sim


IMI Sensor Sim


Original Scene, Points = 18207, Orig Passes = 5
Local, μ=0.1, h=10000, Clouds=5, KNN=15, Passes=3, zFact=5


Original Scene, Points = 18207, Orig Passes = 5
Local, μ=0.1, h=10000, Clouds=5, KNN=15, Passes=3, zFact=5
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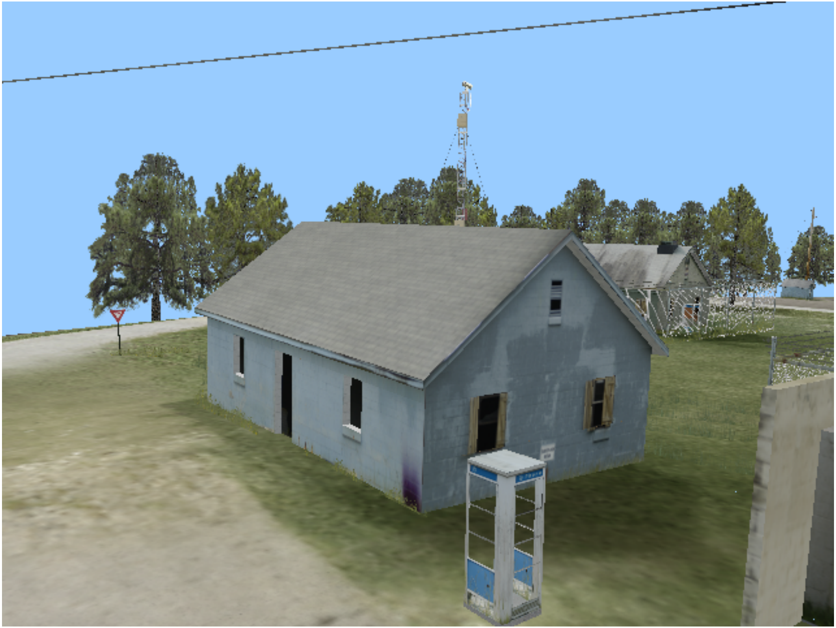
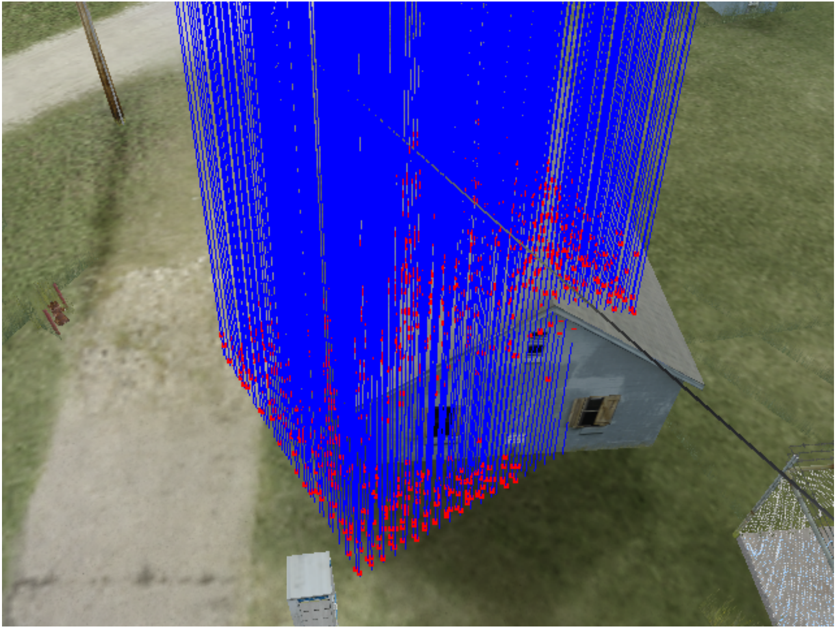
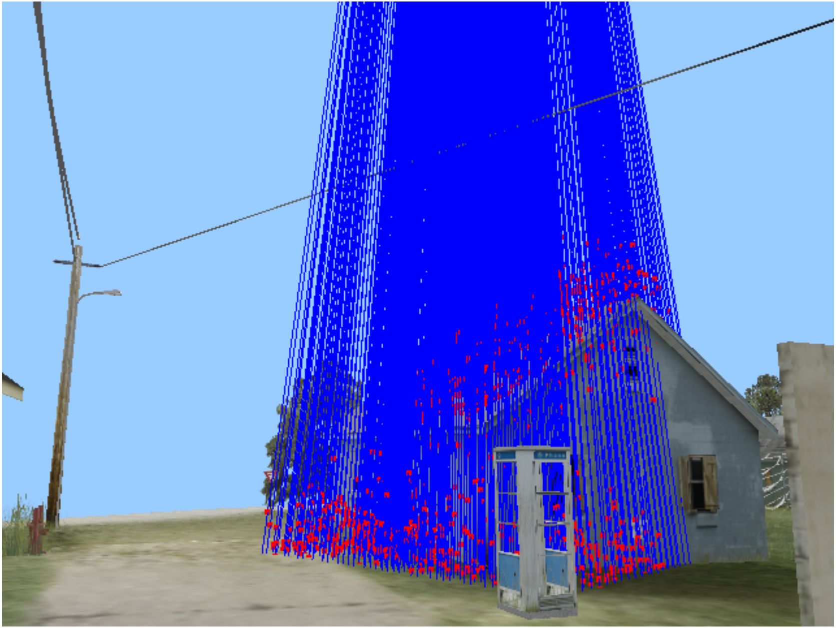
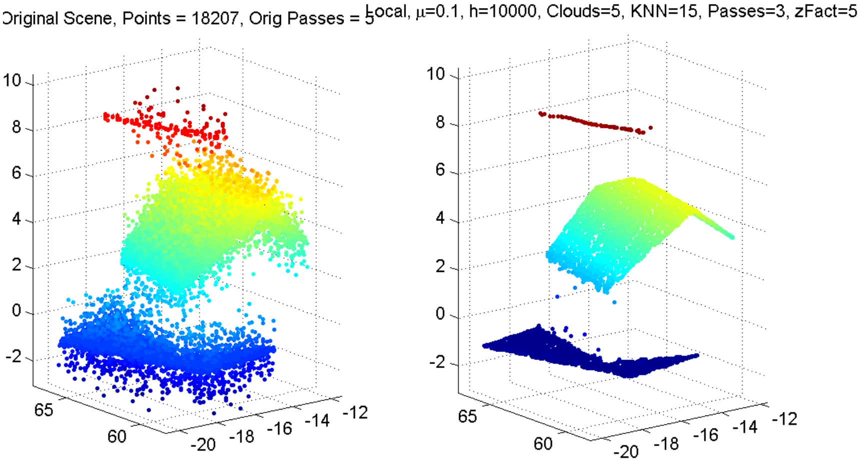
-8
-6





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Local, μ=0.1, h=10000, Clouds=5, KNN=15, Passes=3, zFact=5
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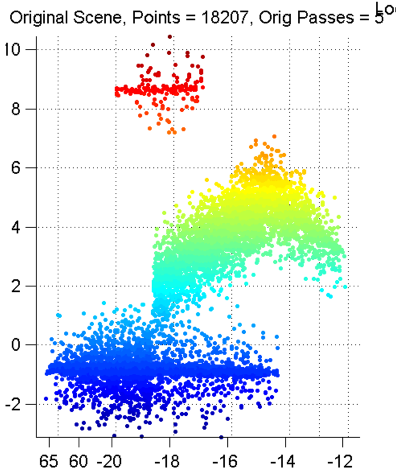

Original Scene, Points = 18207, Orig Passes = 5
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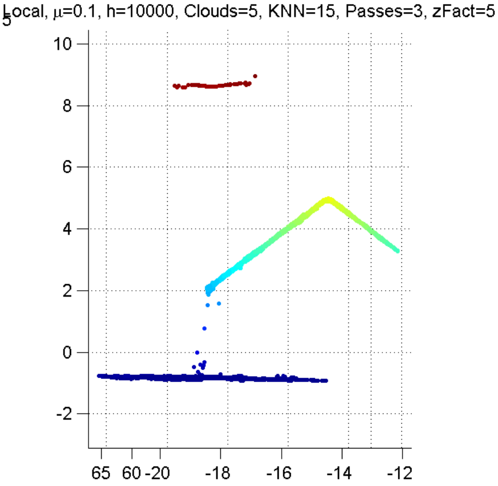

Local, μ=0.1, h=10000, Clouds=5, KNN=15, Passes=3, zFact=5
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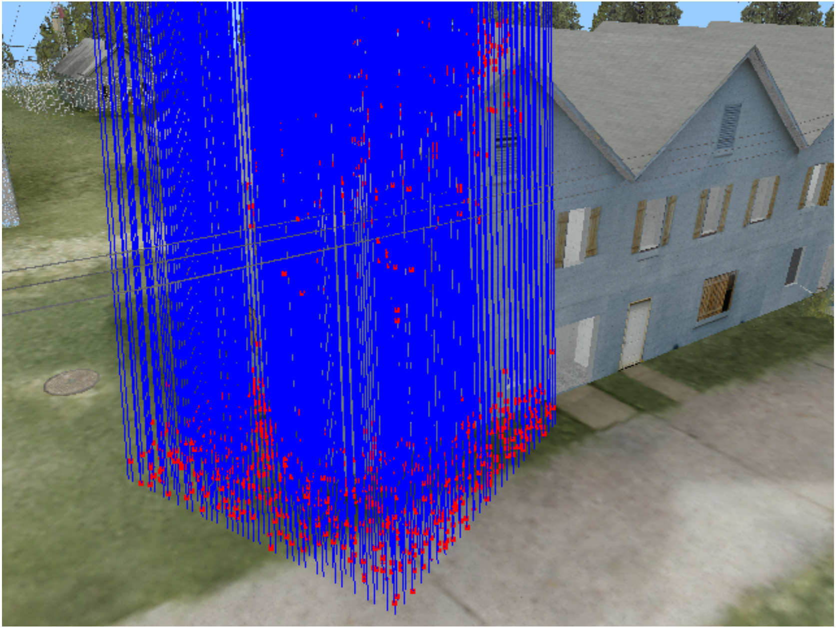

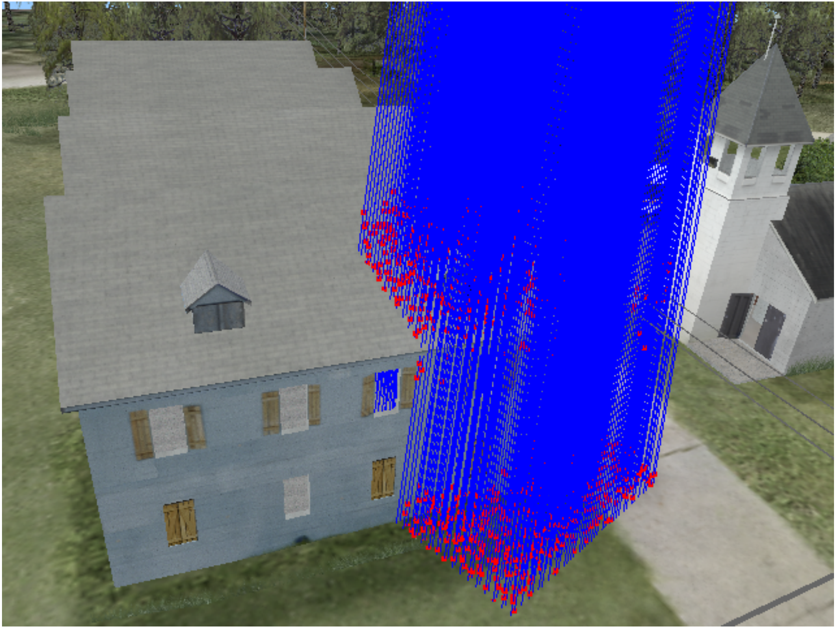

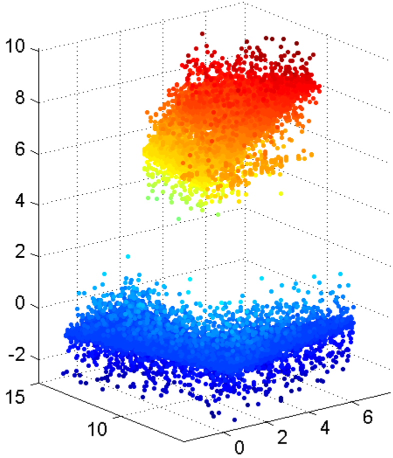
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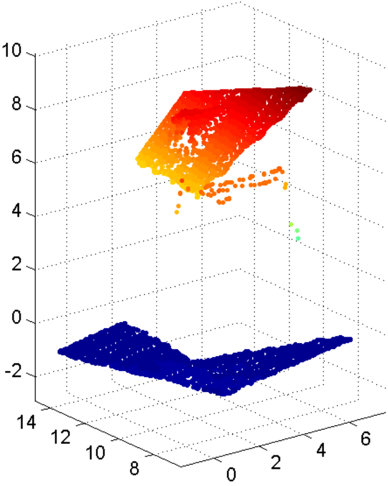
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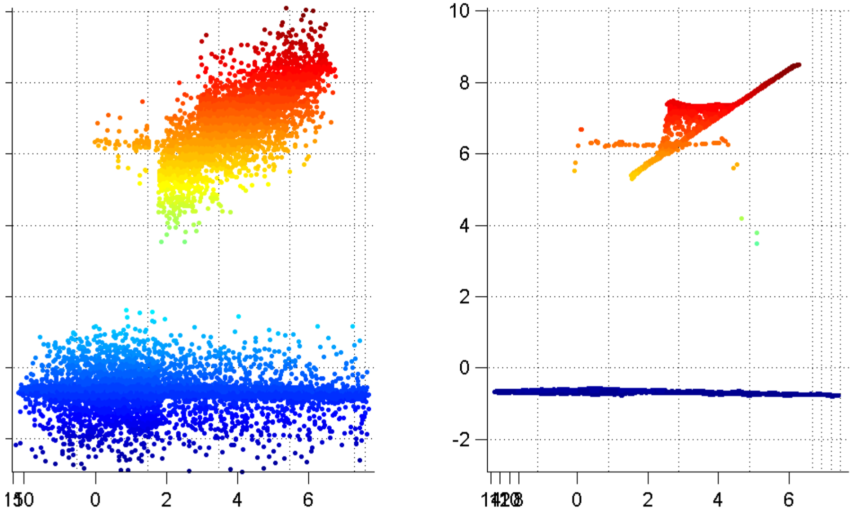
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Comments about Simulations

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- Similar to non-local methods for 2D images, determining the weights has a huge impact.

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- It may be possible to use the geometry encoder $\mathcal{G}$ to train a dictionary and obtain a sparse representation. This opens the possibility of obtaining further compression.

Thank you.

Questions?