Laser-Driven Quantum Control of Nuclear Excitations

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Quantum Control Workshop
BIRS

4 April 2011

Outline

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Introduction

- There is significant interest in the feasibility of controlled nuclear dynamics driven by high-intensity lasers (Bürnevich, J. Evers, and C. H. Keitel, 2006)
- Objective : determine whether coherent light-matter driven dynamics, analogous to those of atomic and molecular quantum optics, are feasible for nuclear systems
- recent experiments have relied upon *indirect* methods of nuclear excitation
  - bombardment of nuclei with laser-accelerated electron and proton beams
  - the use of high-energy bremsstrahlung photons produced from interactions between laser-generated plasma electrons and matter (H. Schwoerer, J. Magill, and B. Beleites, 2006)
- *Direct* laser-nuclear excitation has thus far been prohibitive due to
  - lack of coherent photon sources matching typical nuclear transition energies
  - the enormous laser intensities required
- with the future development of high-intensity, coherent x-ray sources, the potential for direct laser-nuclear interaction deserves attention

Outline

Laser nuclear interaction dynamics

- the Hamiltonian describing the atomic nucleus and its interaction with an external electromagnetic field may be written as

  $$H(t) = H_0 + H_1(t)$$

  where H_0 represents the internal Hamiltonian of the nucleus and $H_1(t)$ describes its coupling with the applied laser field
- the interaction Hamiltonian $H_1(t)$ is then written within the dipole approximation as

  $$H_1(t) = -\mathbf{p} \cdot \mathbf{E}(t) - \mathbf{m} \cdot \mathbf{B}(t),$$

  where the electric $\mathbf{p}$ and magnetic $\mathbf{m}$ dipoles interact, respectively, with the electric $\mathbf{E}(t)$ and magnetic $\mathbf{B}(t)$ components of the applied laser field.
- electric and magnetic dipole-allowed transitions to a given excited nuclear eigenstate are mutually exclusive

Laser nuclear interaction dynamics

- the time evolution of a pure nuclear state is described by the Schrödinger equation

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = \left(H_0 - \mathbf{p}\cdot\mathbf{E}(t) - \mathbf{m}\cdot\mathbf{B}(t)\right)|\psi(t)\rangle,$$

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = \left(H_0 - \mu_1\epsilon_1(t) - \mu_2\epsilon_2(t)\right)|\psi(t)\rangle,$$

  - μ_1 is the electric dipole that interacts with the electric component of the field $\epsilon_1(t)$
  - μ_2 is the analogous magnetic dipole that responds to the magnetic field component $\epsilon_2(t)$

- we consider the structure of the nucleus as a finite N-level quantum system

$$H_0 = \sum_{n=1}^{N} \lambda_n |n\rangle\langle n|$$

with eigenstates $|n\rangle : n = 1, \ldots, N$ and corresponding energy spectrum $\{\lambda_n\}$.

Outline

Controllability analysis

- establish the existence of a set of laser control fields $\epsilon_1(t)$ and $\epsilon_2(t)$ that are capable of steering the quantum system from an initial state $|\psi(0)\rangle = |\psi_i\rangle$ to a user-defined final state $|\psi(T)\rangle = |\psi_f\rangle$ at a target time T

Non-oriented graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, called the connectivity graph.

- The set of vertices $\mathcal{V}$ of the graph is defined as the collection of eigenstates $|n\rangle$ of the field-free Hamiltonian H_0
- the set of edges $\mathcal{E}$ is comprised of all pairs of field-free eigenstates coupled by the dipole elements μ_1 or μ_2 :

$$\mathcal{G} = (\mathcal{V}, \mathcal{E}) : \quad V = \{|n\rangle, i = 1, ..., N\}, \quad \mathcal{E} = \{(|a\rangle, |b\rangle); |a\rangle \neq |b\rangle, \mu_{ab} \neq 0 \text{ for some } \mu \in \{\mu_1, \mu_2\}\}.$$

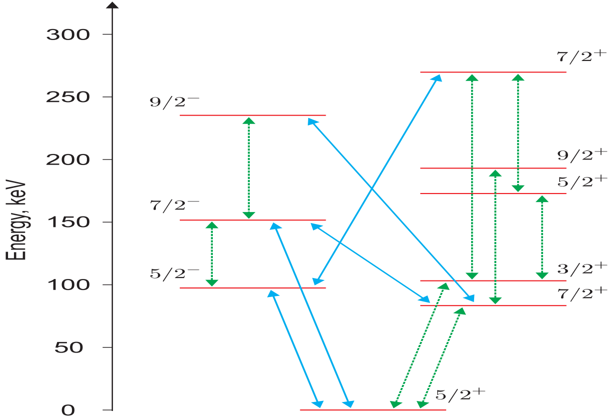

FIGURE: Representation of the connectivity graph for eight excited eigenstates of $^{153}_{63}$Eu. The vertices V correspond to the considered nuclear eigenstates and are represented by horizontal lines along with their associated energy. Arrows connecting the various eigenstates are the graph edges E; electric-dipole allowed transitions (E1) are shown by a solid arrow, and magnetic-dipole allowed transitions (M1) are depicted with a dashed arrow

- transition energy between the eigenstates $|i\rangle$ and $|j\rangle$ is denoted as $\omega_{ij} = \lambda_i - \lambda_j$, $i, j = 1, ..., N$,

Theorem

Under the hypotheses :

$\mathcal{H}_1$: *the graph $\mathcal{G}$ is connected,*

$\mathcal{H}_2$: *the graph $\mathcal{G}$ does not have degenerate transitions, i.e for every $(i, j) \neq (a, b)$ $i \neq j$, $a \neq b$ such that $\mu_{ij} \neq 0$, $\mu_{ab} \neq 0$, for some $\mu \in \{\mu_1, \mu_2\}$: $\omega_{ij} \neq \omega_{ab}$.*

the nuclear system dynamically governed by equation is controllable

the proof

the proof of this controllability statement follows the same steps as that for atomic and molecular situation in which only the electric-dipole interaction is considered (Turinici, Rabitz 2001)

Outline

Control Landscape Topology

- when a nuclear system is state controllable,
  - manipulate its dynamical evolution by appropriately varying external control fields $\epsilon_1(t)$ and $\epsilon_2(t)$
  - steer the probability $P_{i\to f}$ of a transition from an initial pure state $|\psi_i\rangle$ to a target pure state $|\psi_f\rangle$
- maximize the transition probability $P_{i\to f}$, is expressed as a functional of the control fields

$$P_{i\to f} = P_{i\to f}[\epsilon_1(t), \epsilon_2(t)].$$

- maximization of the transition probability entails a search for the controls $\epsilon_1(t)$ and $\epsilon_2(t)$ over the landscape, with the goal of arriving at the extremal value $P_{i\to f} = 1$

- the functional is given by :

$$P_{i\to f} = |\langle\psi_f|U(T,0)|\psi_i\rangle|^2,$$

where the unitary time evolution operator $U(t,0)$ satisfies :

$$i\hbar\frac{\partial U(t,0)}{\partial t} = (H_0 - \epsilon_1(t)\mu_1 - \epsilon_2(t)\mu_2)U(t,0)$$

with the property $|\psi(t)\rangle = U(t,0)\ |\psi_i\rangle$.

- the topology of the transition probability search landscape in the case of unconstrained controls :
    - the landscape possesses critical values that correspond to perfect or null control.
    - the approach of a global maximum entails navigation of a fitness landscape with no suboptimal traps.
    - due to large Hessian null space, the control problem enjoys an intrinsic robustness in the immediate vicinity of an optimum

Outline

- the extension of coherent control methodologies to the nuclear scale offers a new approach for driving laser-induced nuclear reactions
- in addition to refining quantum control principles originally developed for atomic and molecular control, such studies will provide a means for increased understanding of the structure and dynamical evolution of the atomic nucleus
- effcient manipulation of nuclear dynamics will likely rely on an amalgam of coherent and incoherent control techniques