



NEURO & CARDIO IMAGING

from a mathematical perspective

LUC FLORACK
BIRS 2011, Banff, Canada

Volterra's Multiplicative Calculus

Multiplicative derivative (1D): $f^*(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h)}{f(x)} \right)^{1/h}$

Antiderivative (1D): $*\int f(x)^{dt} = c F(x)$ iff $F^*(x) = f(x)$

Limiting procedure (1D): $*\int_a^b f(x)^{dx} = \lim_{\Delta x_i \rightarrow 0} \prod_{i=1}^N f(\xi_i)^{\Delta x_i}$

with $\xi_i \in [x_{i-1}, x_i]$, $x_0 = a$, $x_N = b$, and $\Delta x_i = x_i - x_{i-1}$.

Fundamental theorem of multiplicative calculus (1D):

$$*\int_a^b F^*(x)^{dx} = \frac{F(b)}{F(a)}$$

Commuting diagrams (1D):

$$\begin{array}{ccc} f^* & \xleftarrow{\text{exp}} & (\ln f)' \\ * \uparrow & & \uparrow' \\ f & \xrightarrow{\ln} & \ln f \end{array} \quad \begin{array}{ccc} *\int f(x)^{dx} & \xleftarrow{\text{exp}} & \int \ln f(x) dx \\ * \uparrow & & \uparrow f \\ f(x) & \xrightarrow{\ln} & \ln f(x) \end{array}$$

Taylor expansion (1D):

$$f(x) = \left(\prod_{k=0}^M \left(f^{(*k)}(a) \right)^{\frac{1}{k!} (x-a)^k} \right) \left(f^{*(M+1)}(\xi) \right)^{\frac{1}{(M+1)!} (x-a)^{M+1}}$$

for some ξ in-between x and a , cf. figure.



Imaging Science & Technology Eindhoven (IST/e)

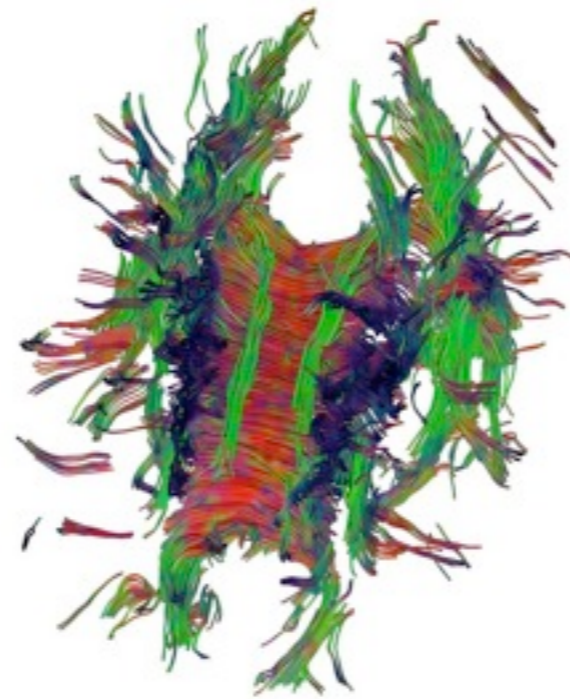
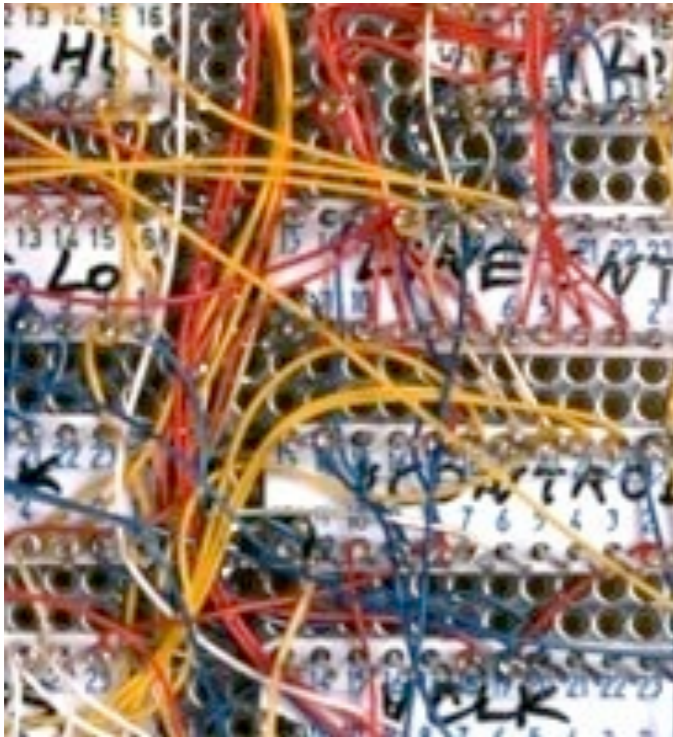


- 2-dimensional images one can look at → complex non-visual data
- calls for cross-divisional “biomaterial” approach

Imaging Science & Technology Eindhoven (IST/e)



Brain Connectomics: Grand Challenge



Anna Vilanova



Bram Platel

- connectivity + tractography = “connectome”
- macro scale

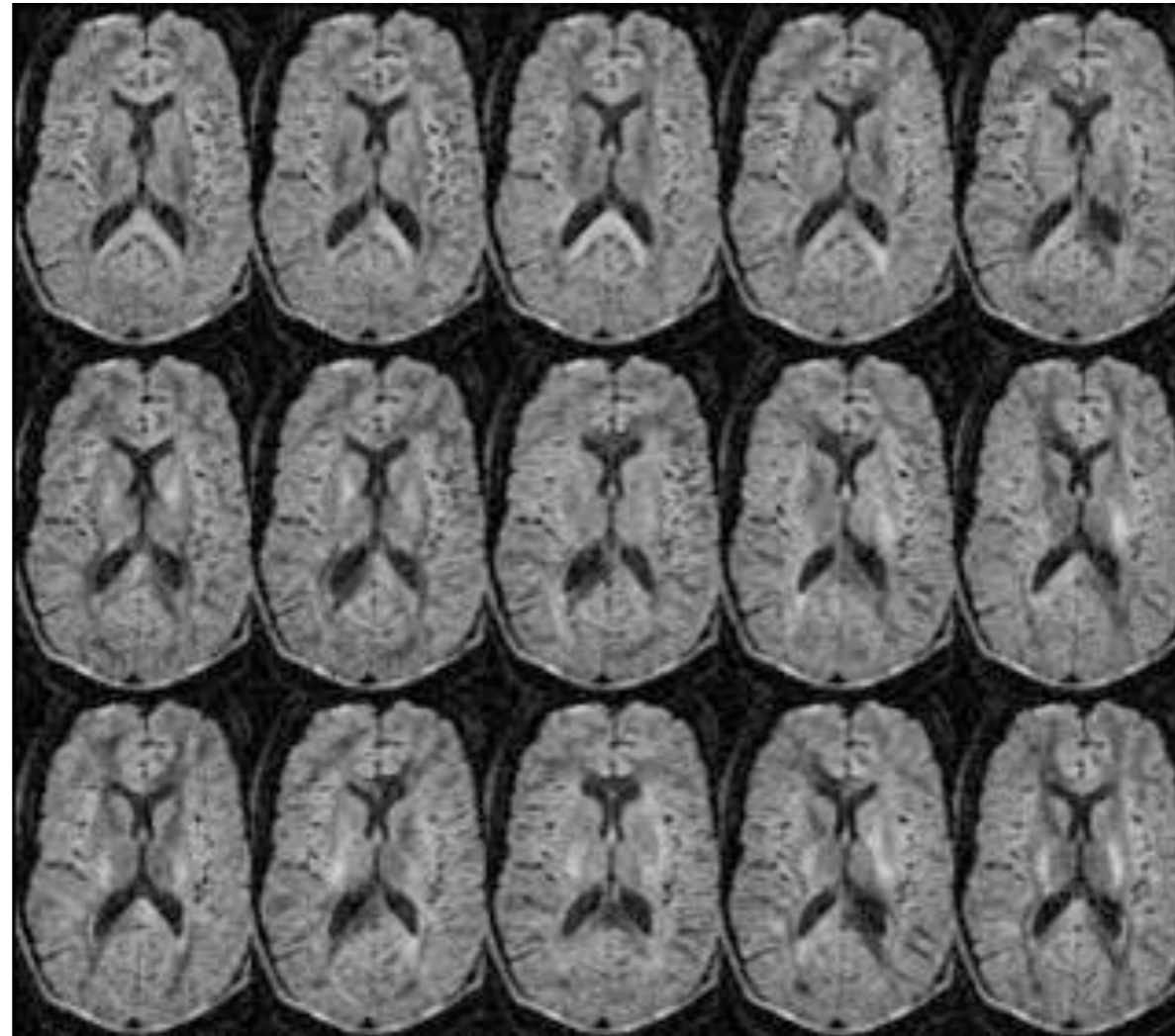


Heart Function: Grand Challenge



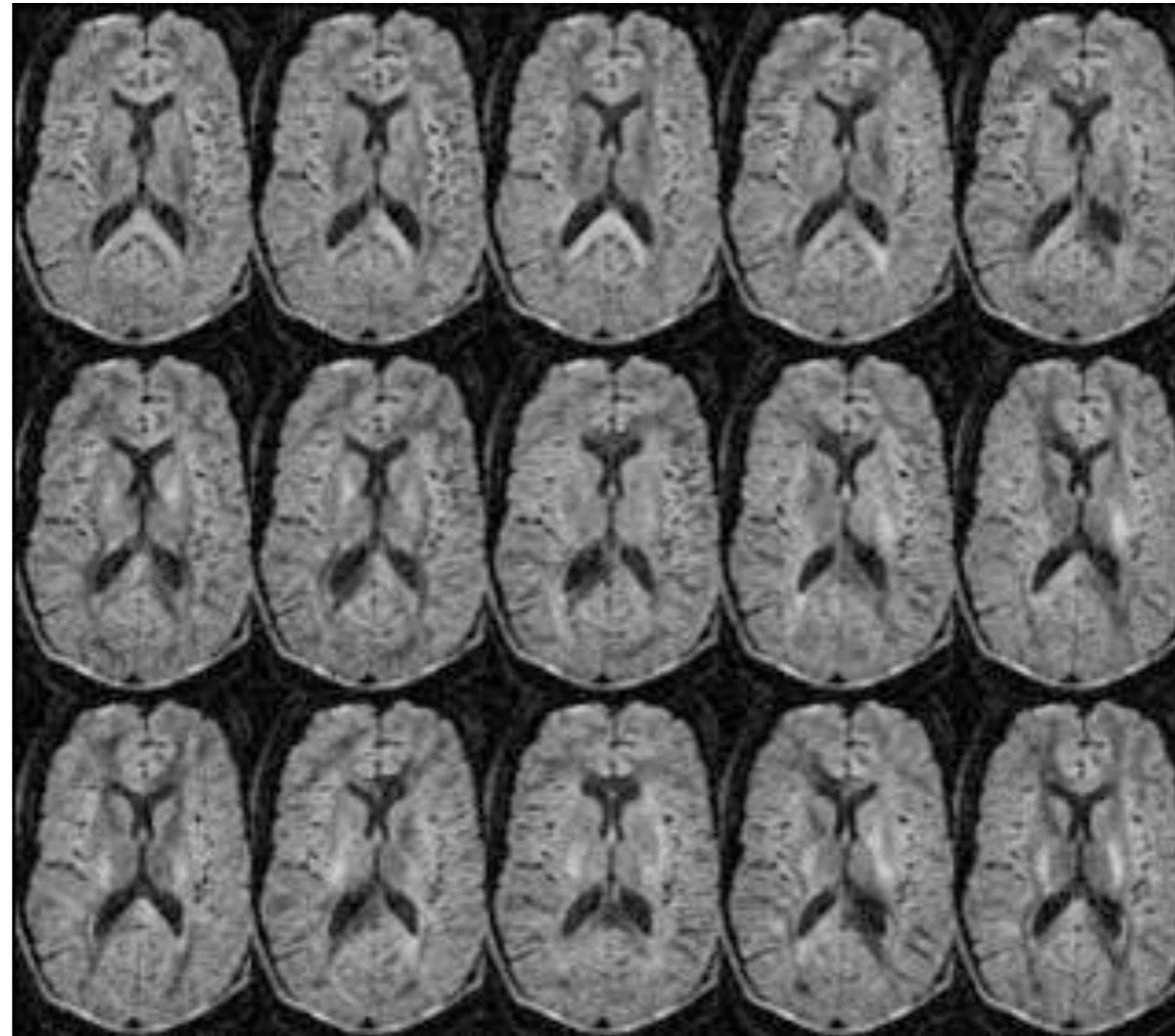
- operational definition of “(ab)normal” myocardial contractility
- diagnosis of cardiac pathology
- assessment of stem cell therapy

Brain Connectomics



- diffusion weighted MRI: DTI, HARDI, etc.

Brain Connectomics



- diffusion weighted MRI: [DTI](#), HARDI, etc.

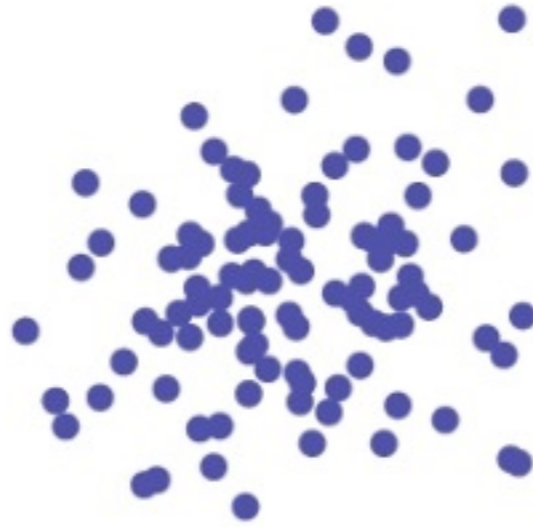
Methodology



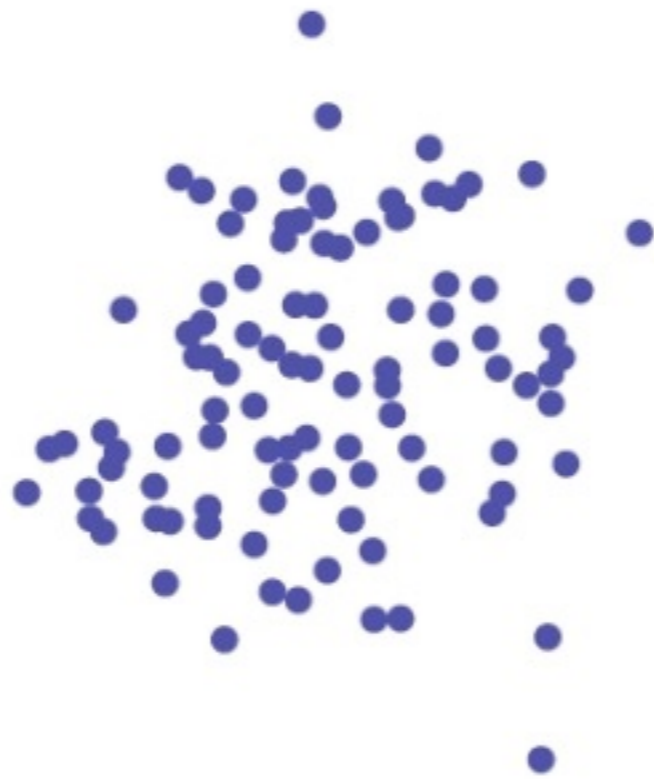
Methodology



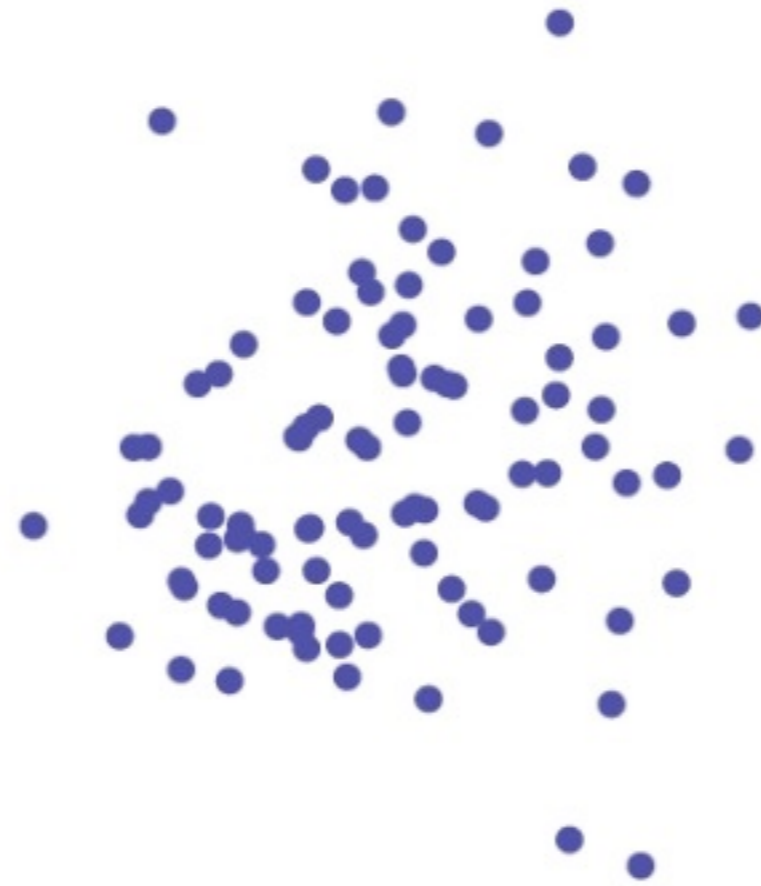
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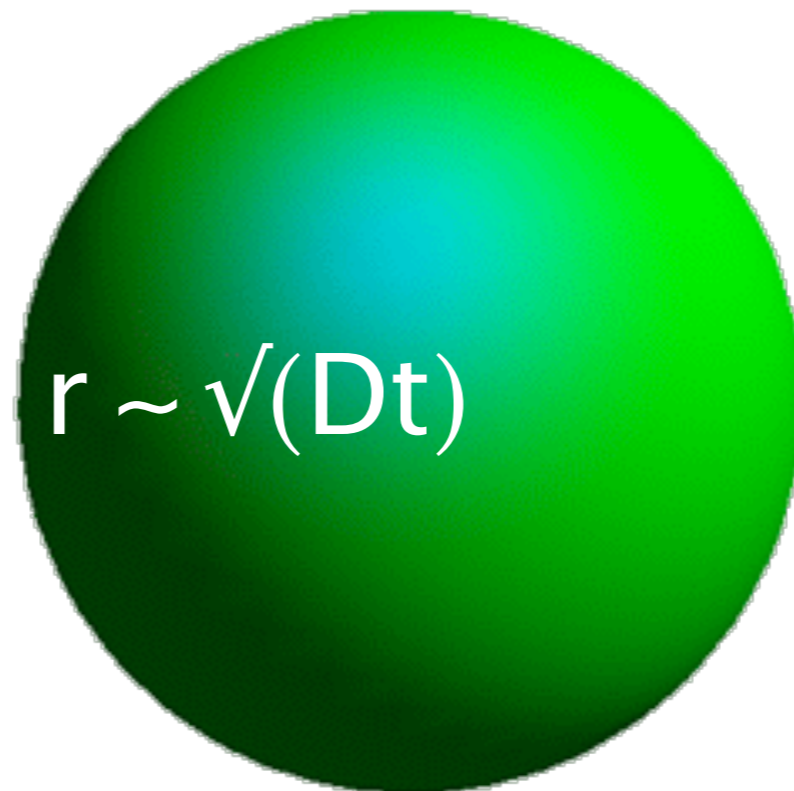
Methodology



Methodology

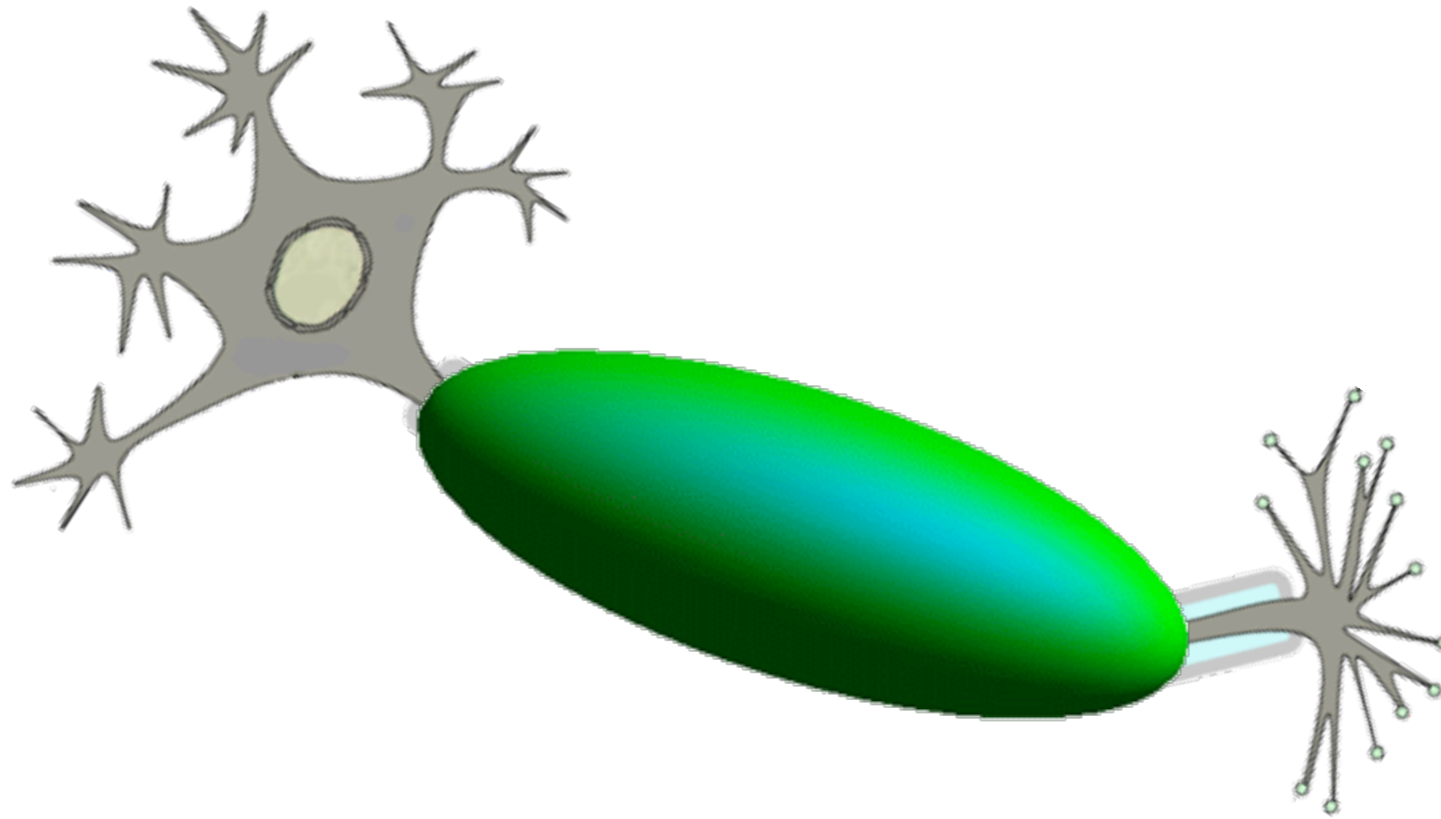


Methodology



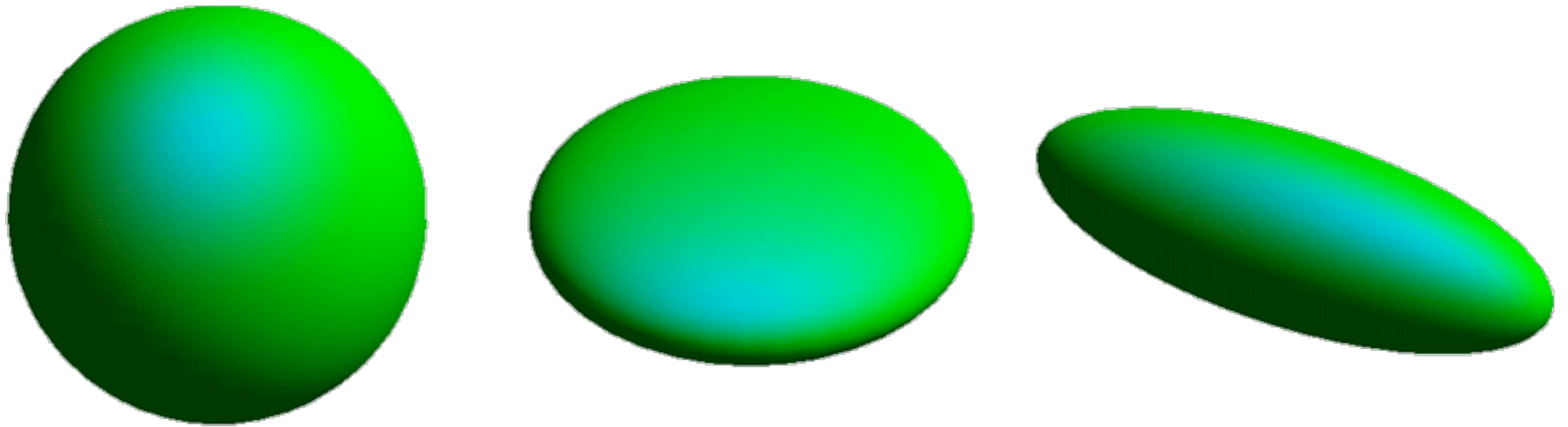
- isotropic medium ~ isotropic diffusion ~ spherical gauge figure

Methodology



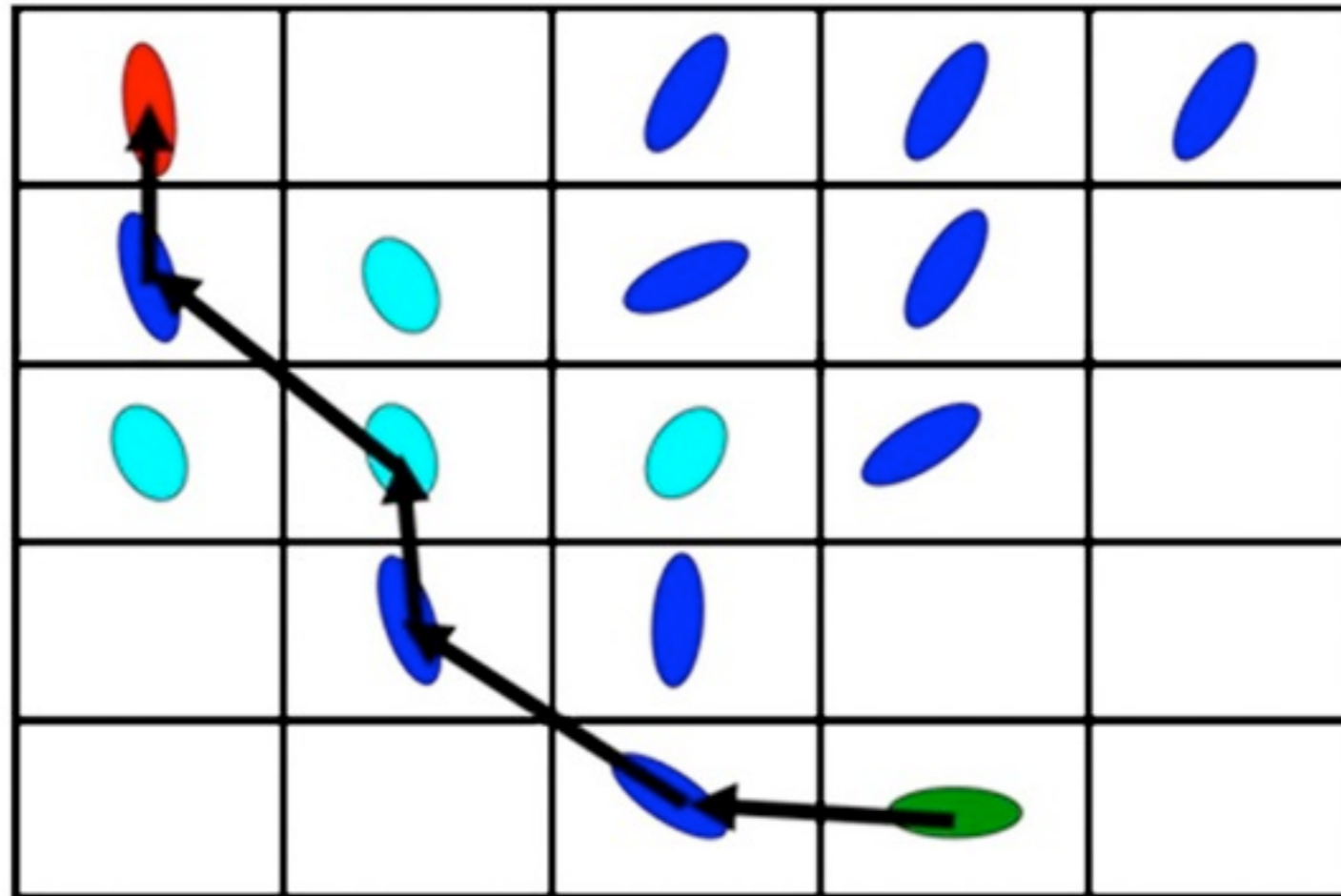
- fibrous tissue ~ anisotropic diffusion ~ ellipsoidal gauge figure

Methodology



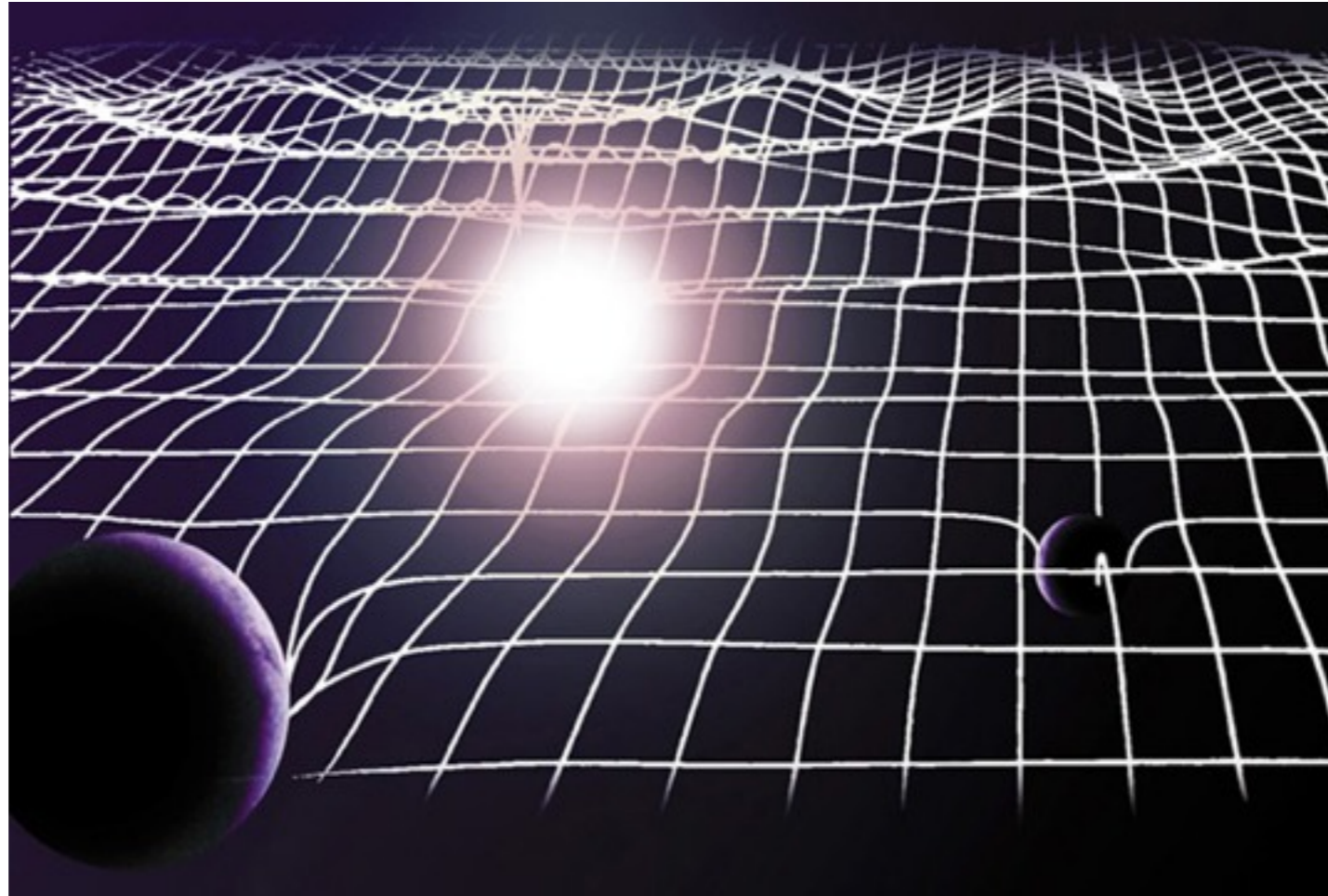
- DTI ~ ellipsoidal gauge figure ~ Riemann metric (6 d.o.f.'s)

Methodology



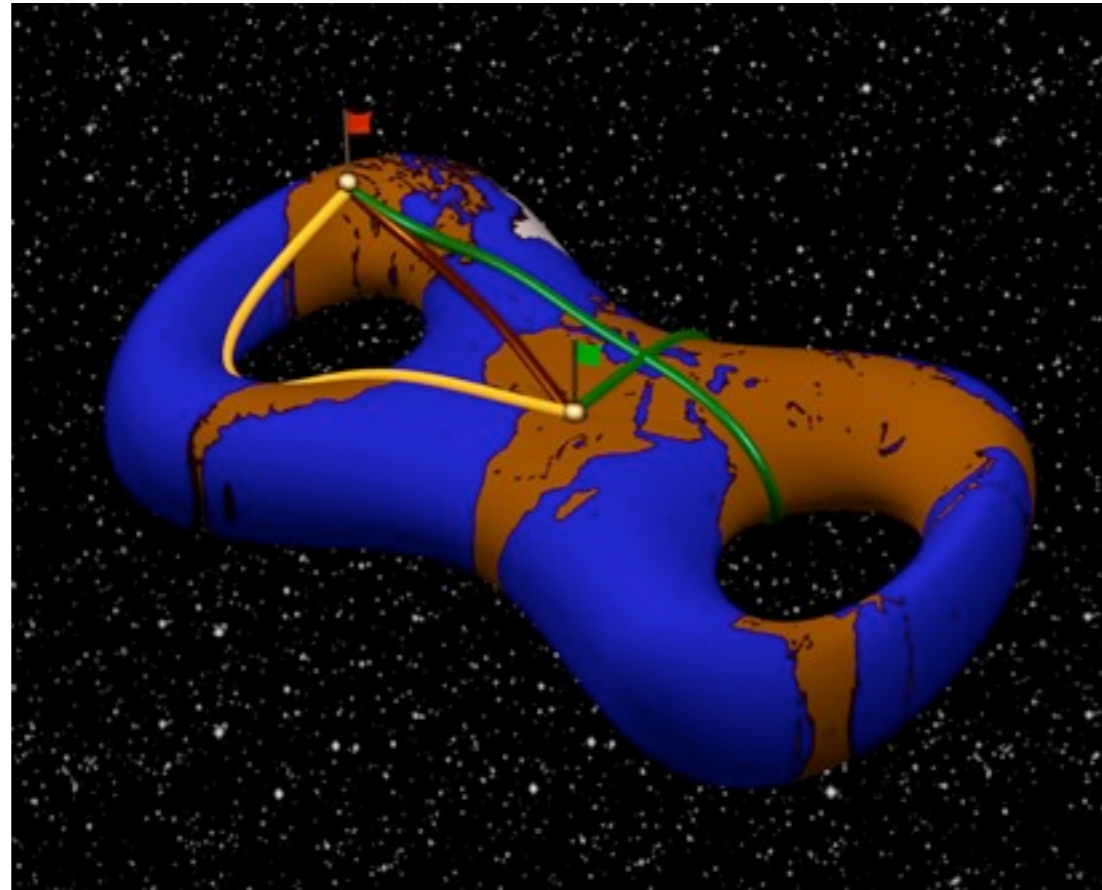
- tractography = geodesic path problem on a Riemannian manifold

Methodology



- analogy with GRT: DTI image ~ inhomogeneous gravitational field

Methodology



- there are infinitely many geodesics through any given point
- distant points may be connected by multiple geodesics
- conjecture: neural tracts \subset geodesics
- probabilistic criterion: scale invariant connectivity measure

Work in Progress

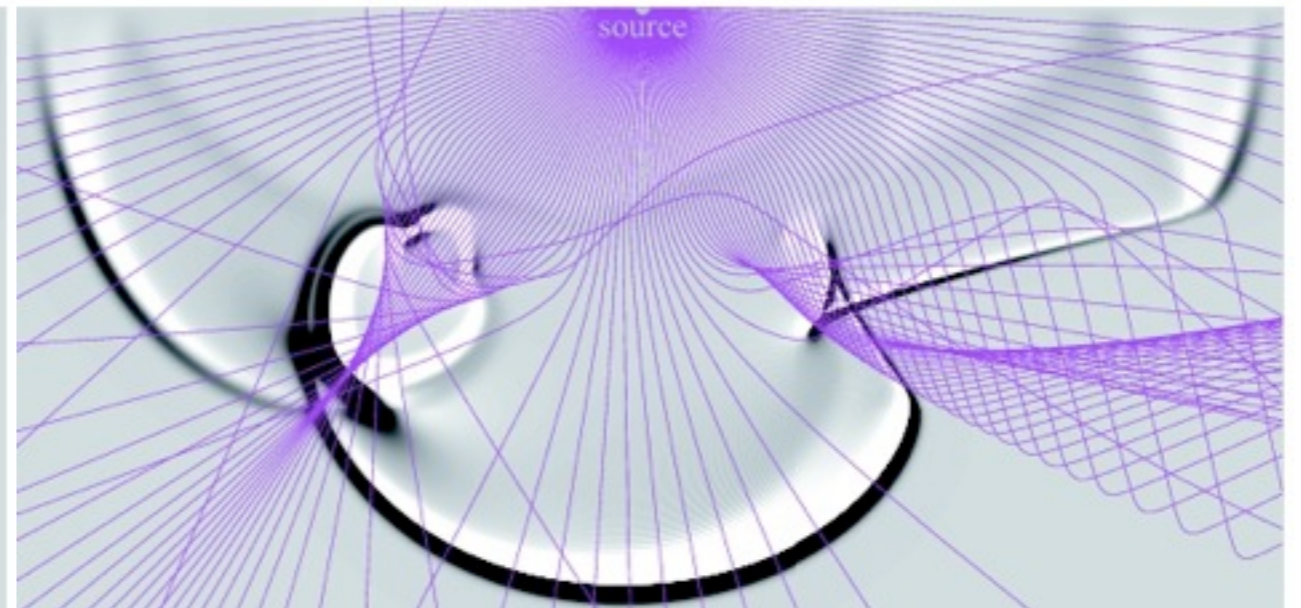
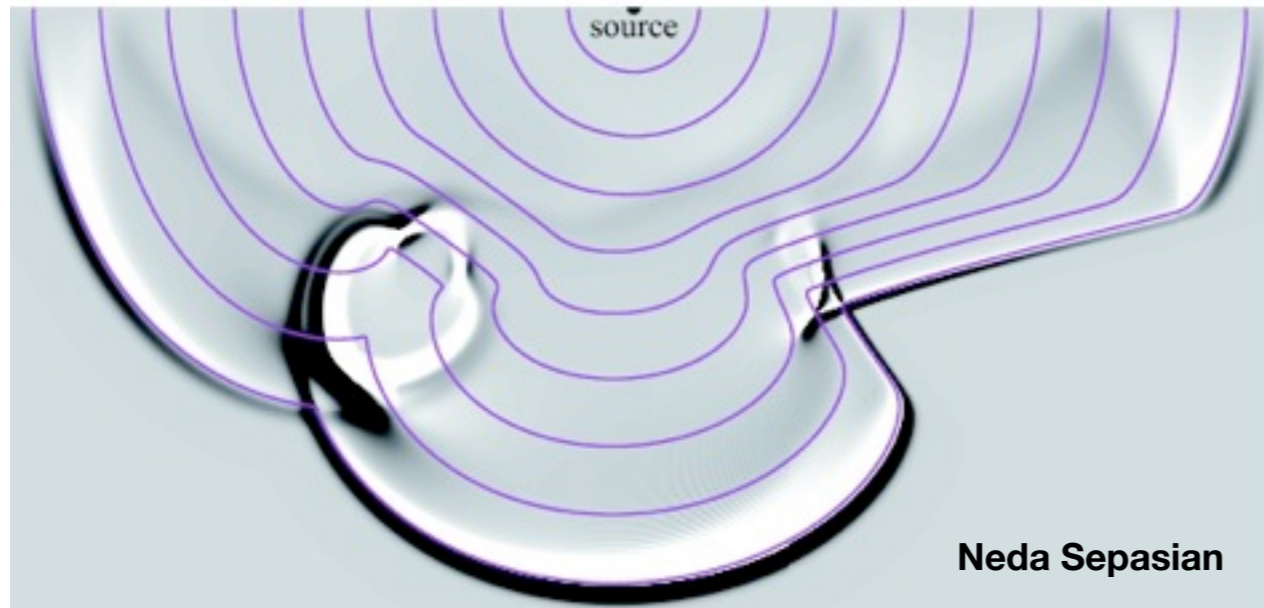
- Constantin Carathéodory (1873-1950): “der Königsweg”
 - general formalism for the calculus of variations
 - velocity driven geodesic congruences versus momentum driven distance fronts



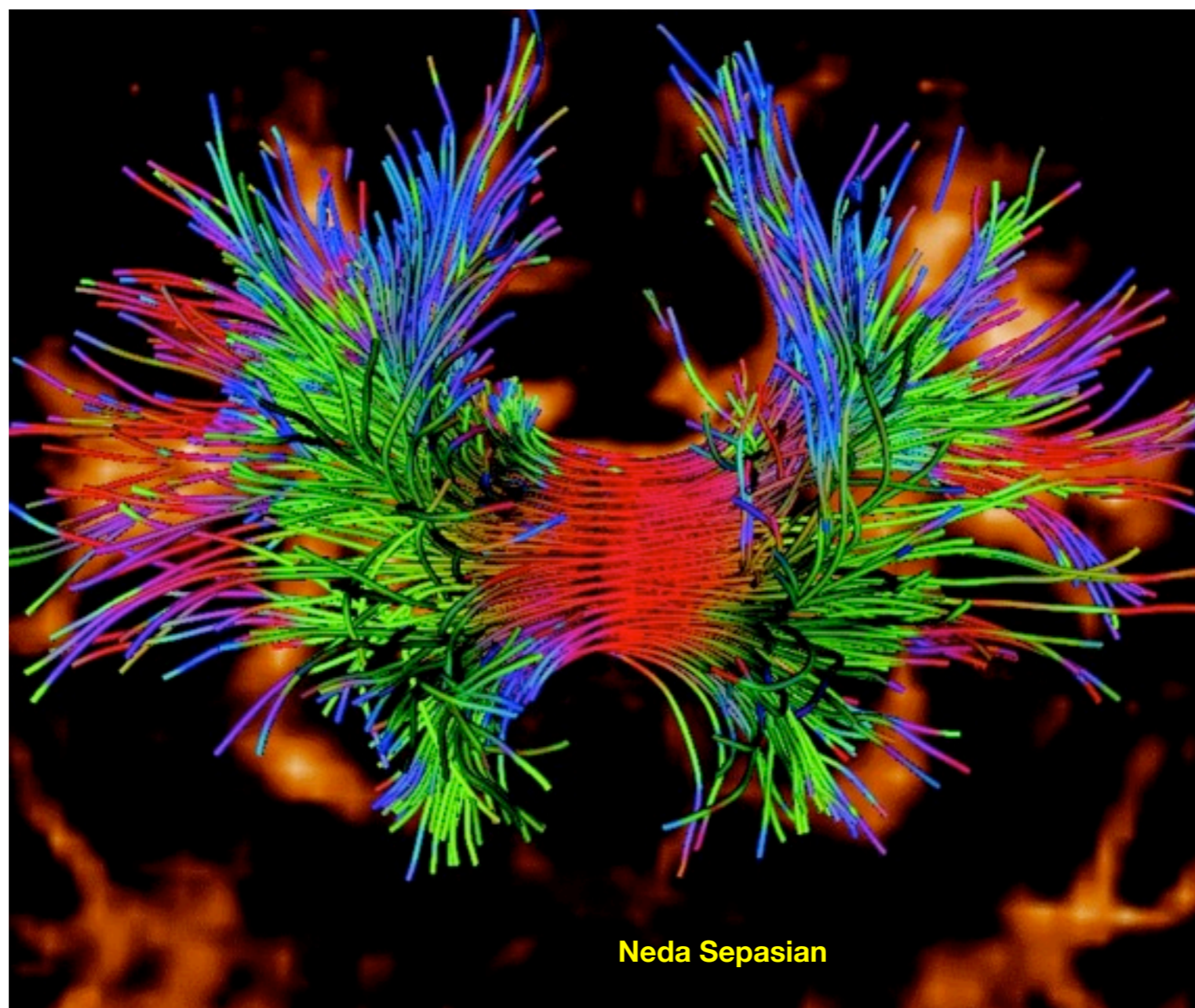
“Solutions to the Hamilton-Jacobi Equations in the Euclidean Plane” -- Sem Florack, pencil on paper (2011)

Work in Progress

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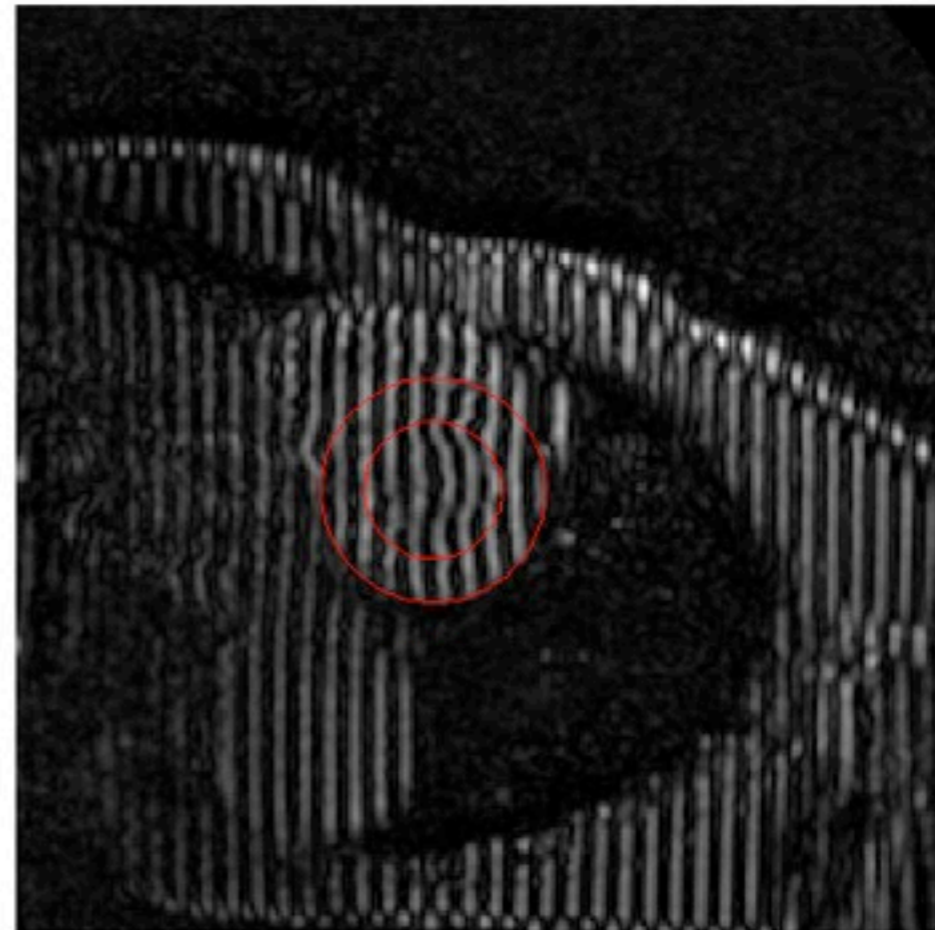
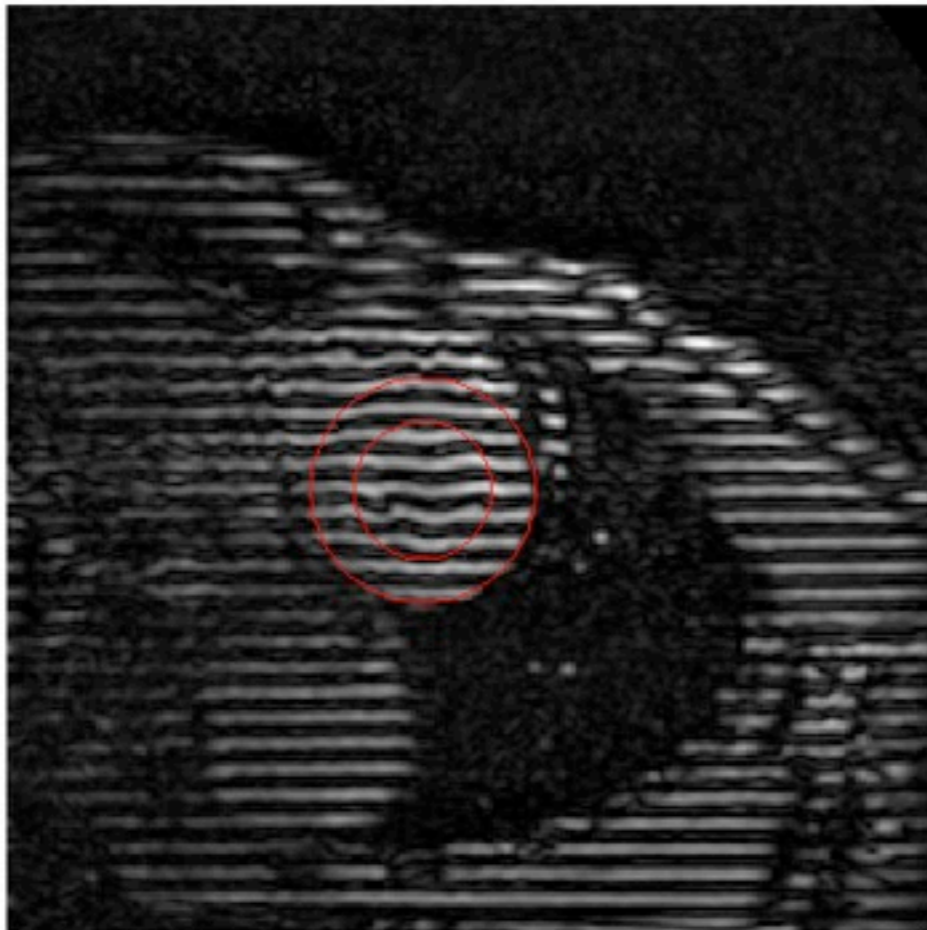


Work in Progress



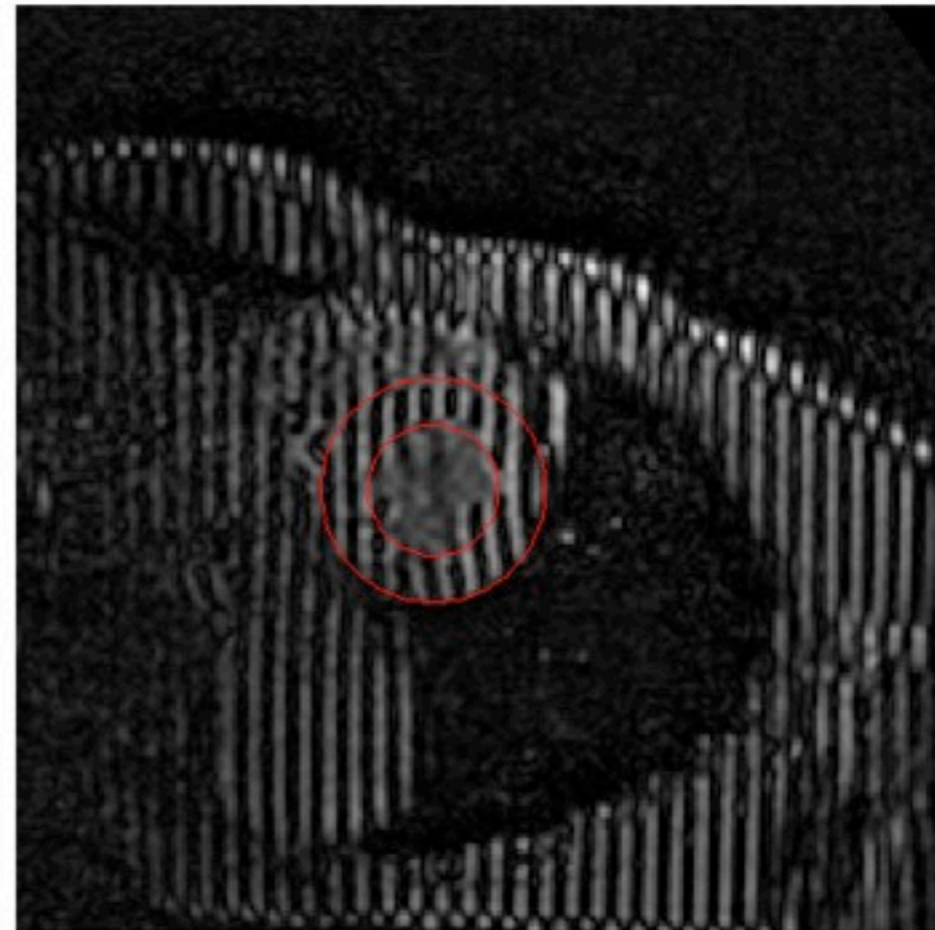
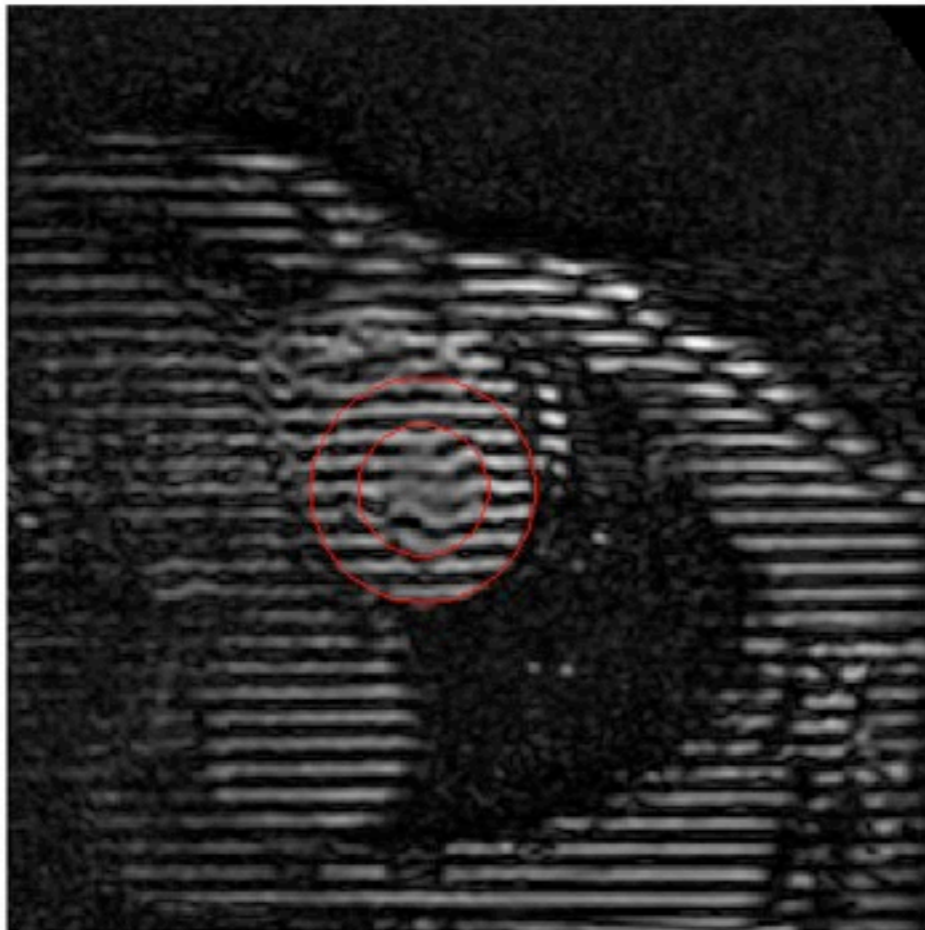
Heart Function

- cine & tagging MRI



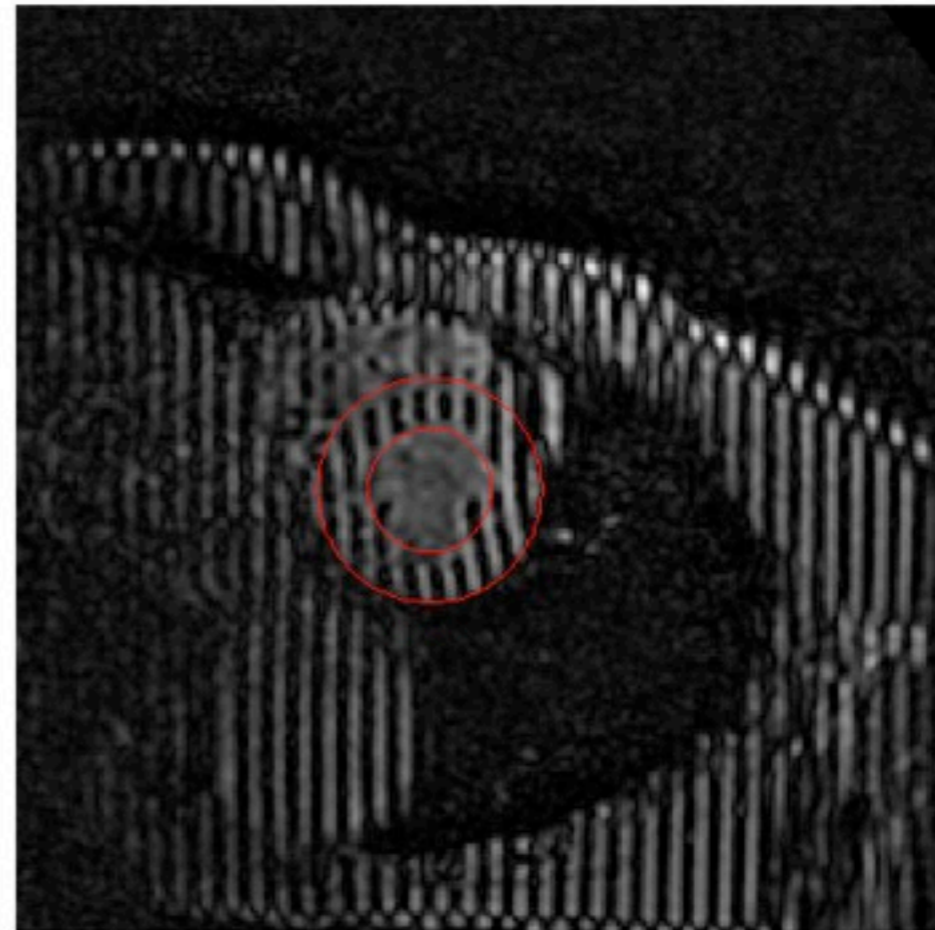
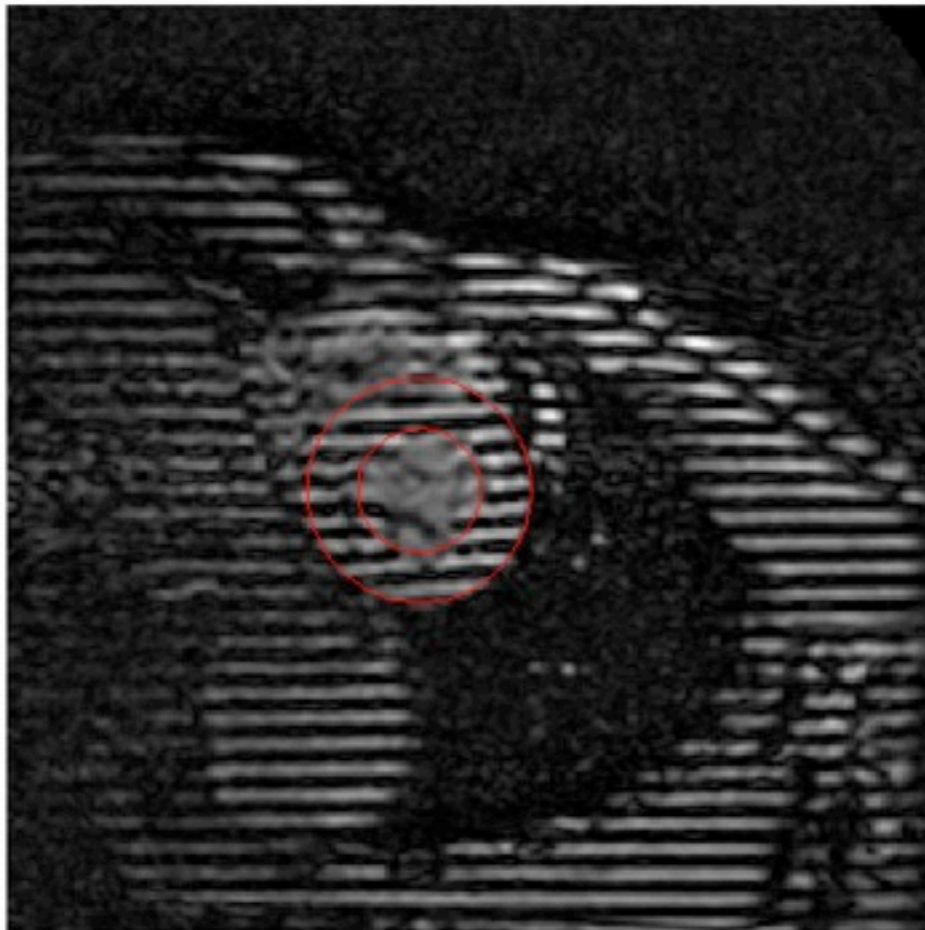
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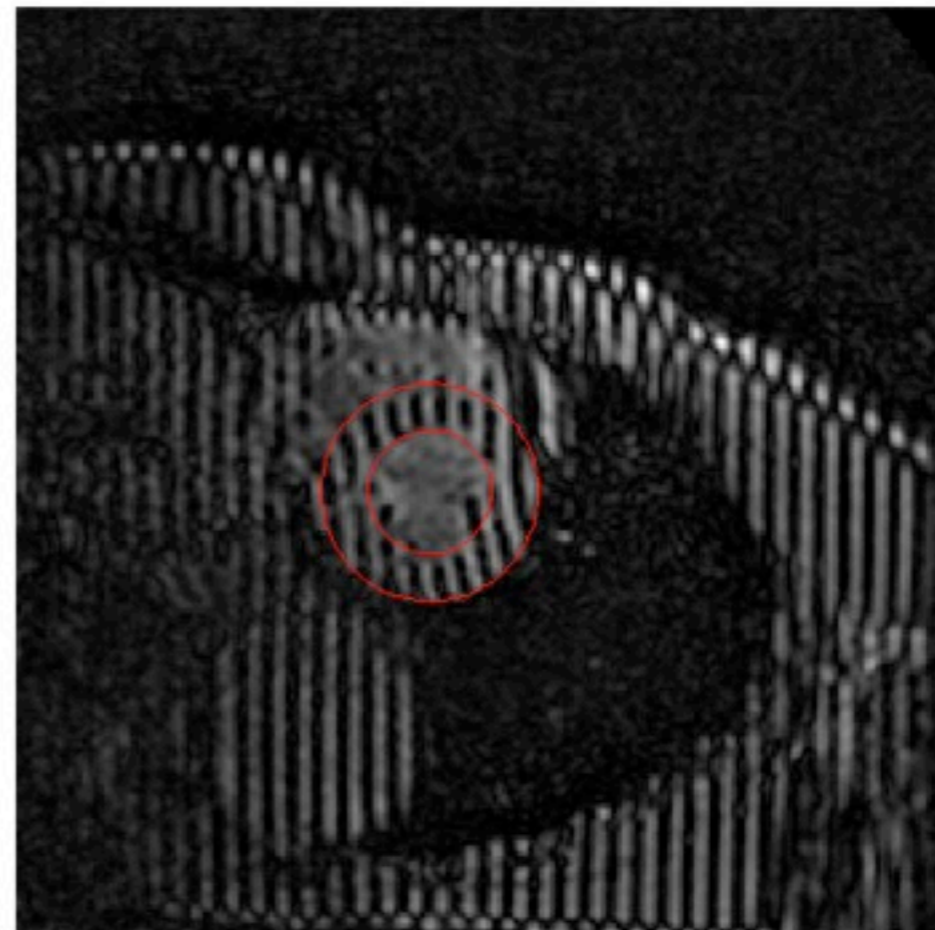
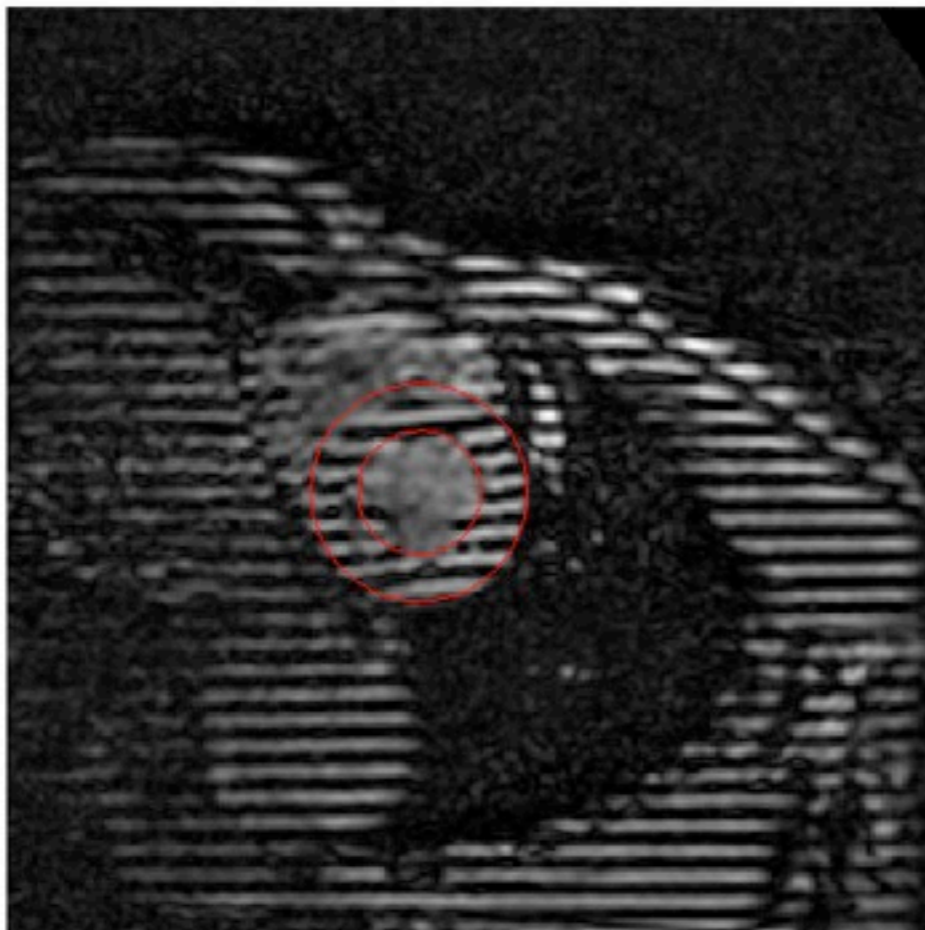
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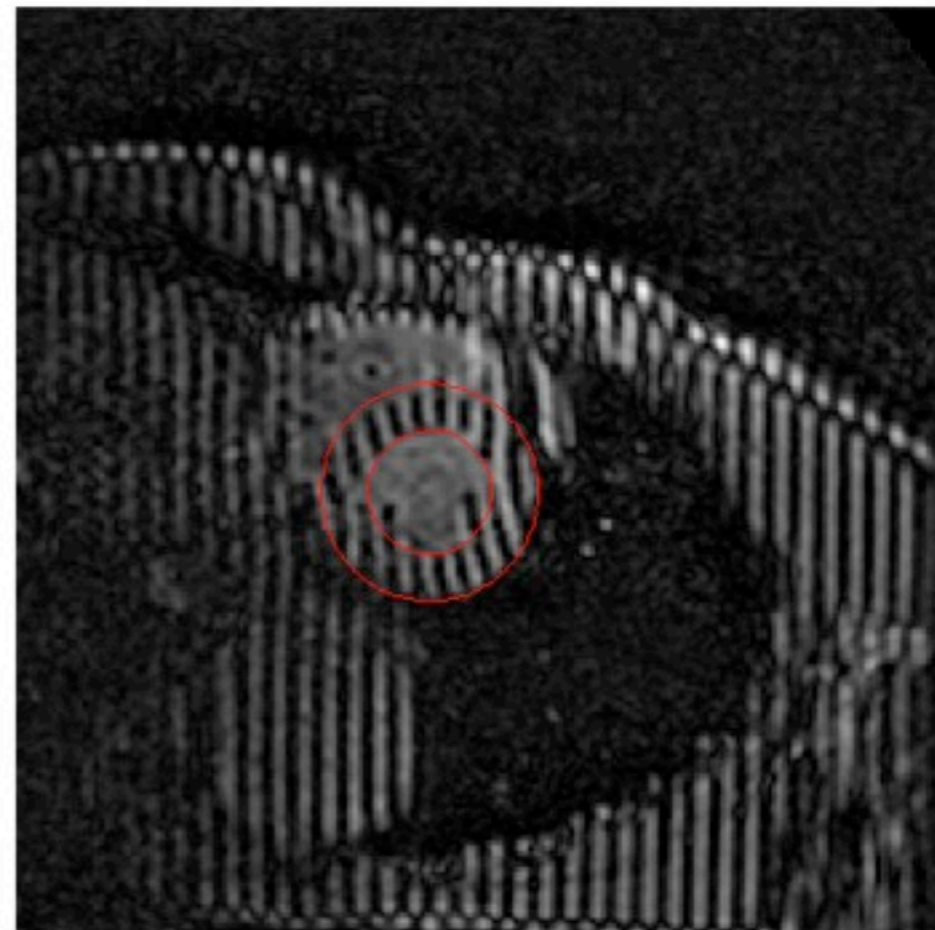
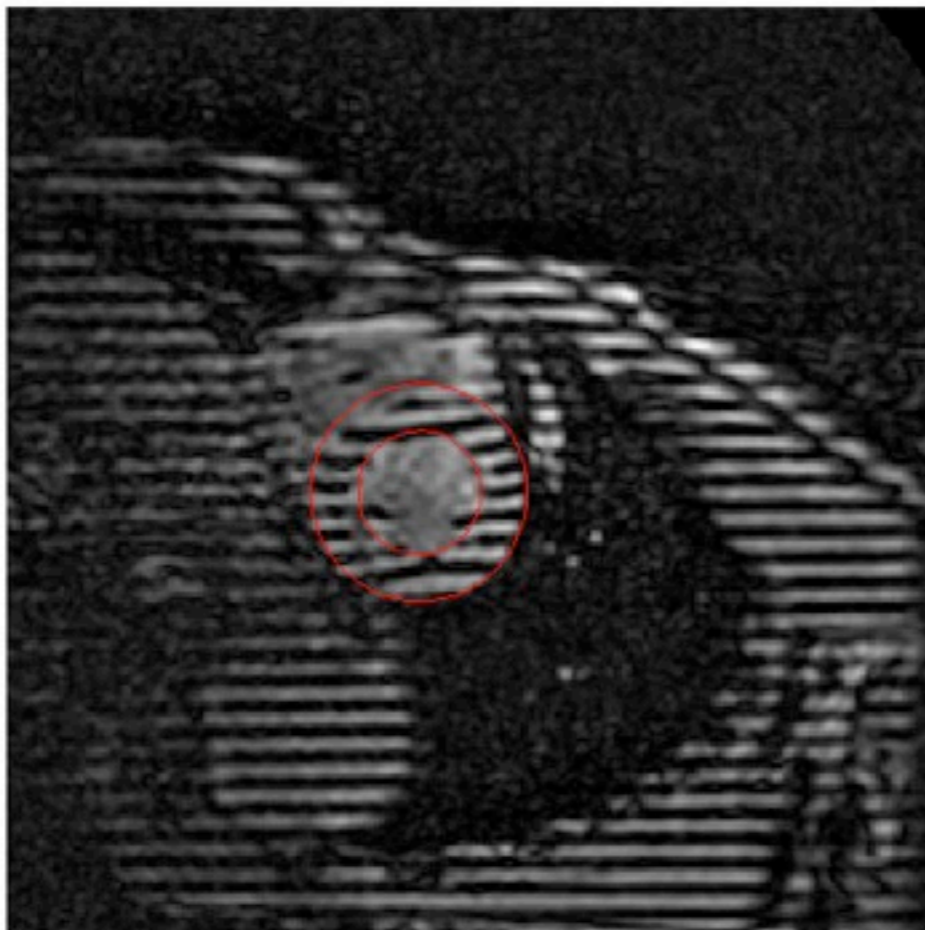
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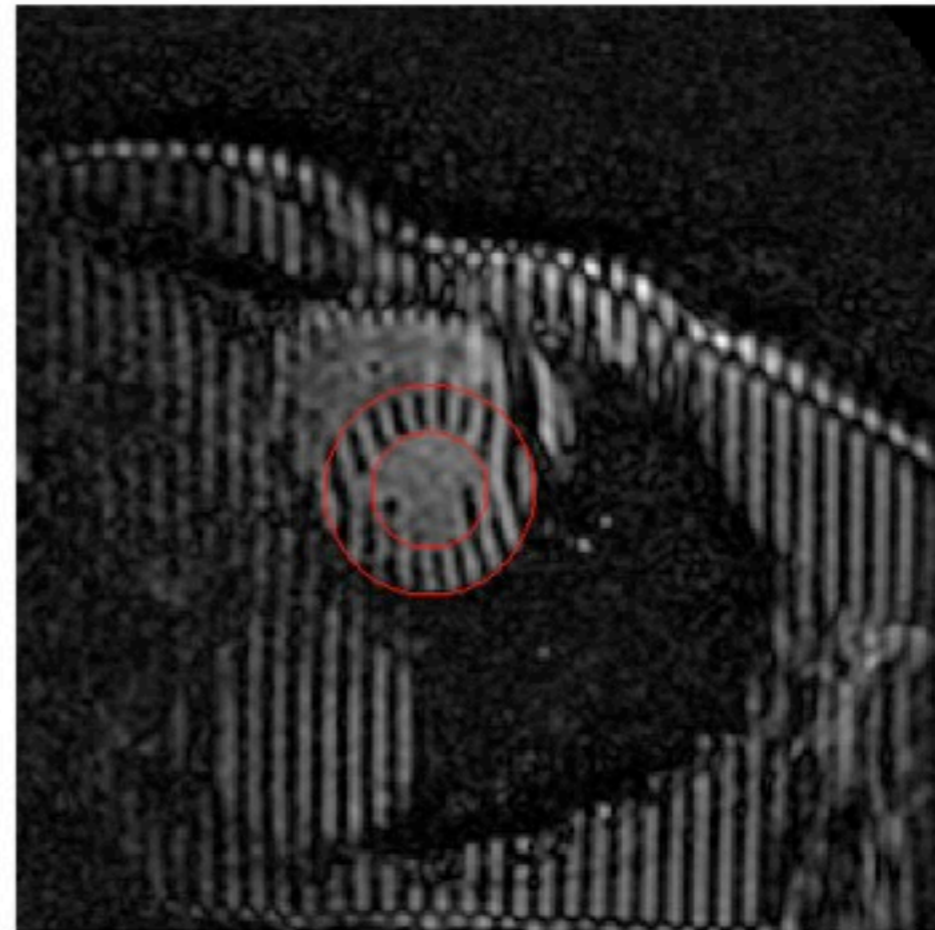
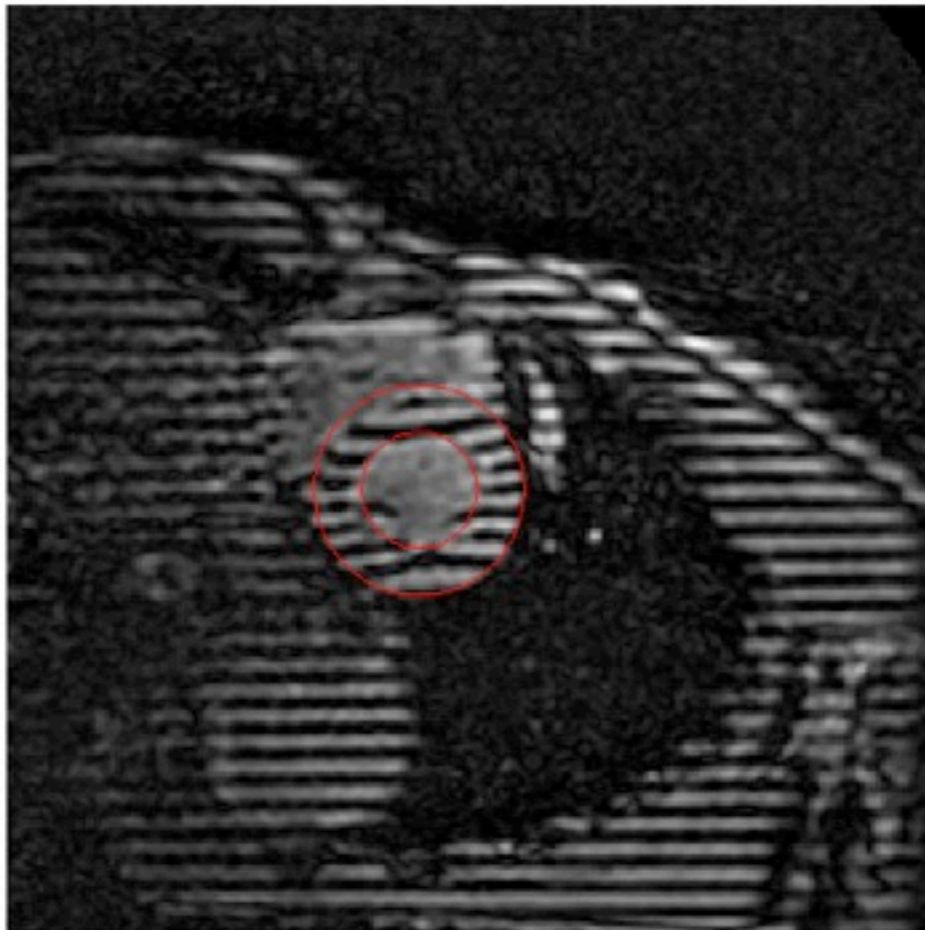
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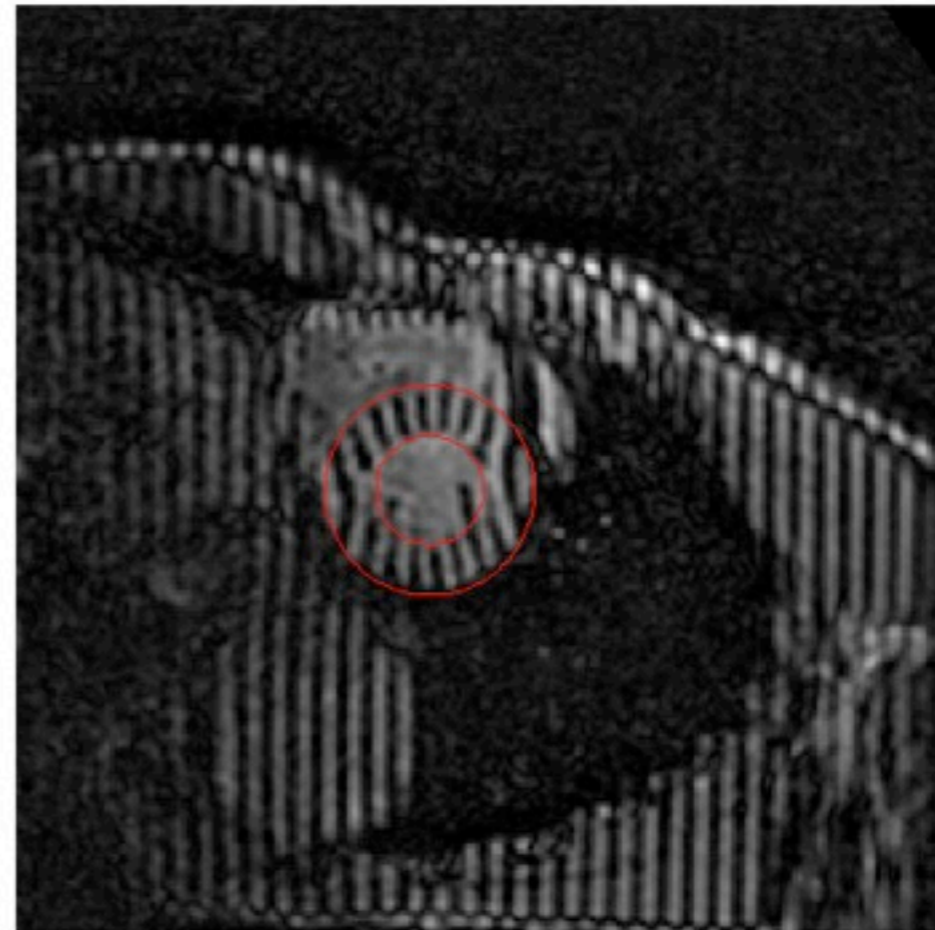
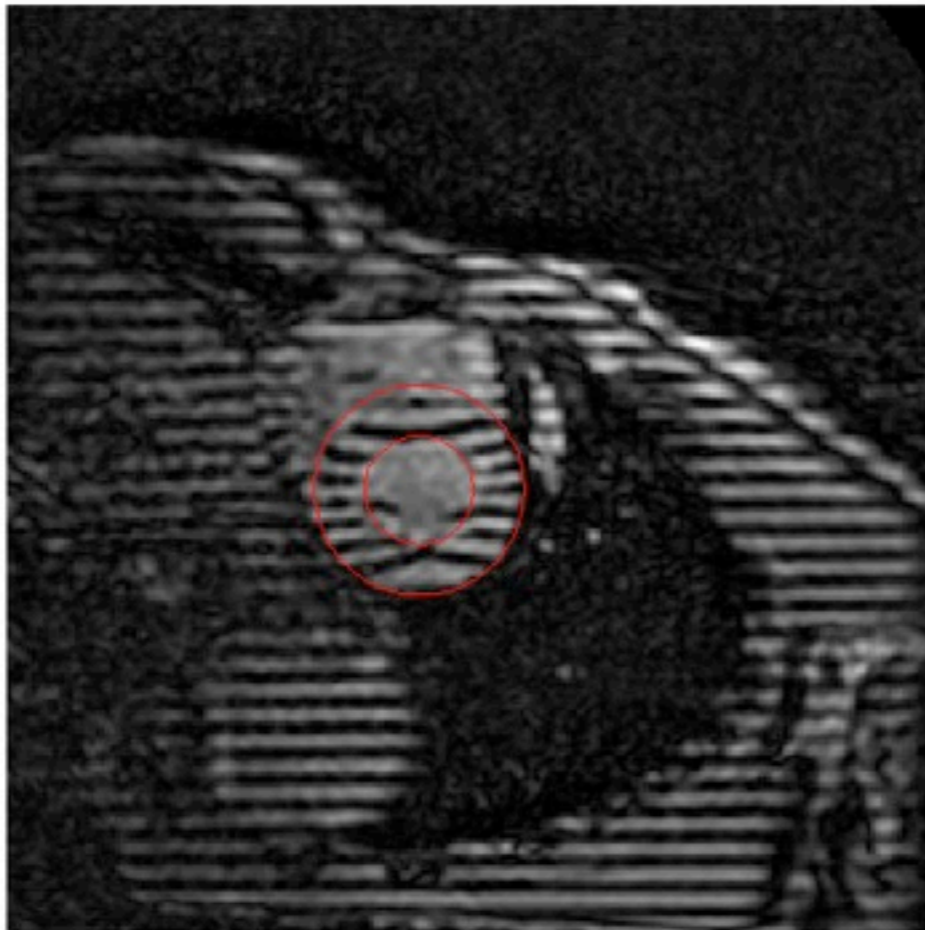
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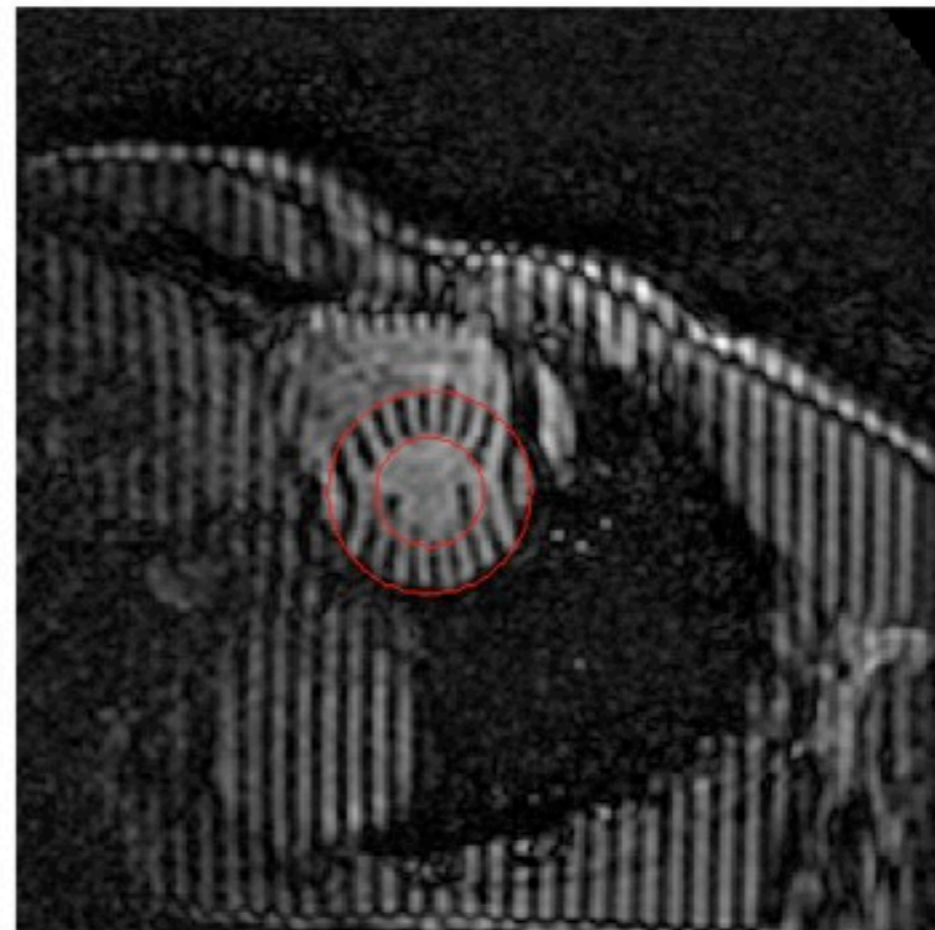
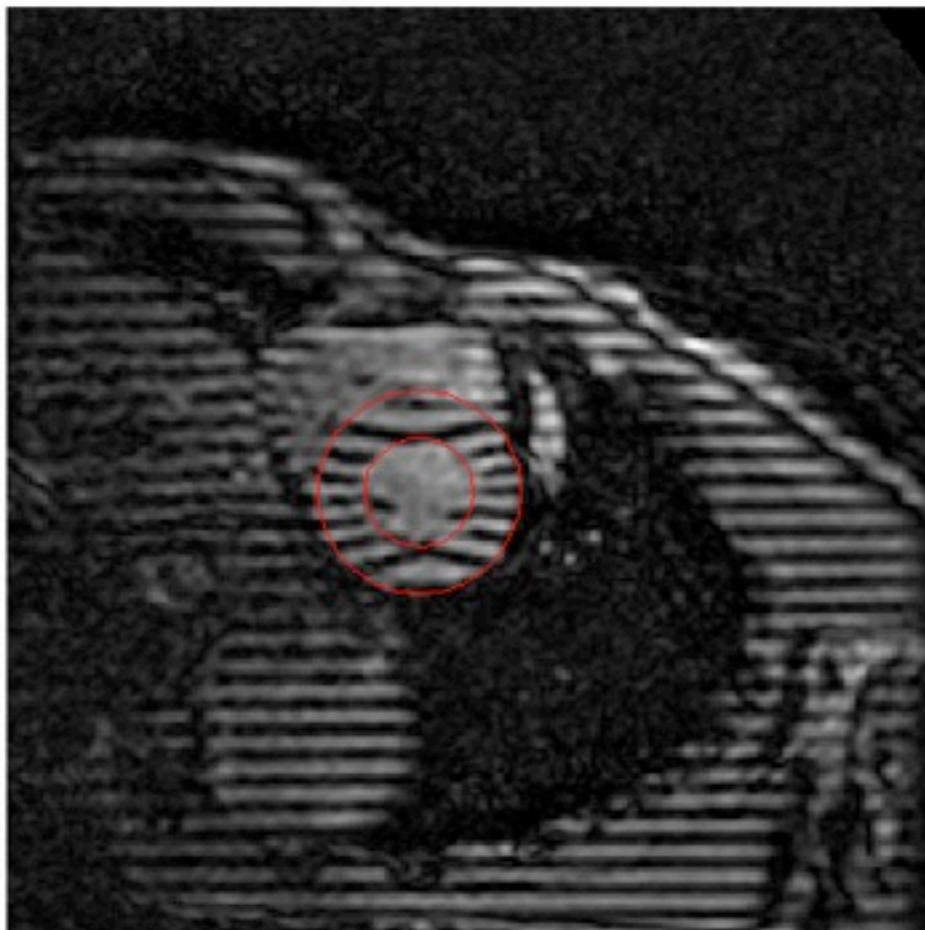
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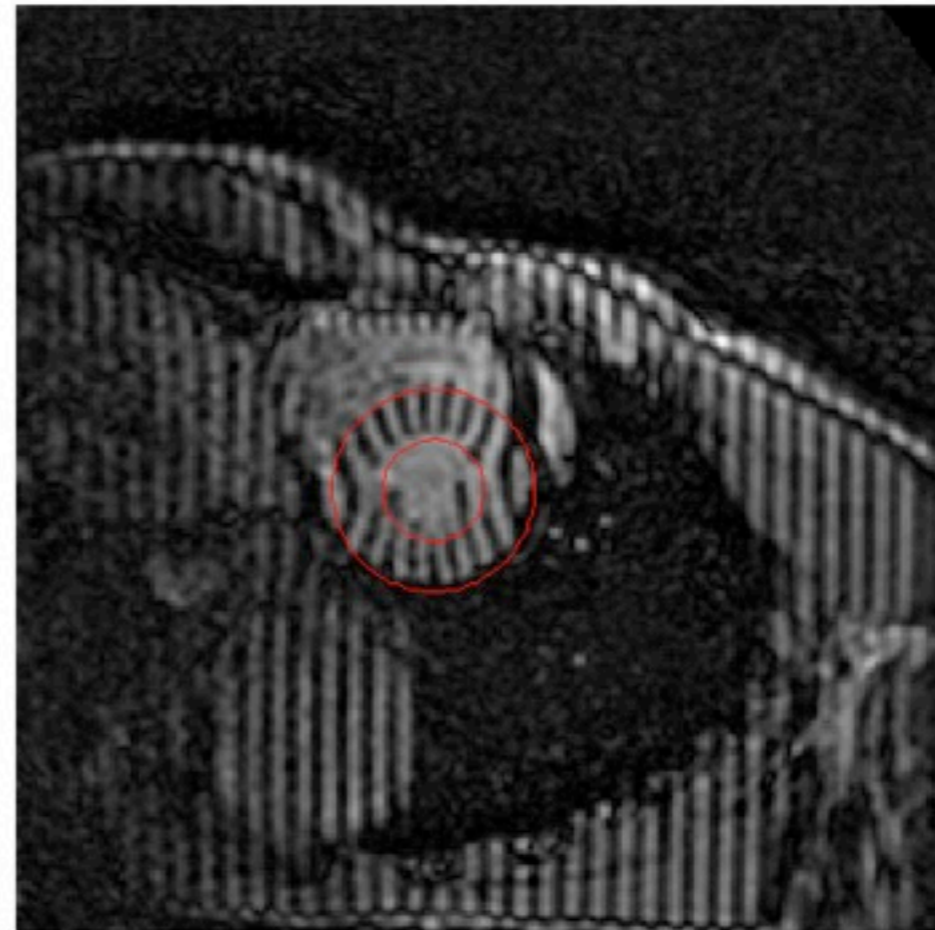
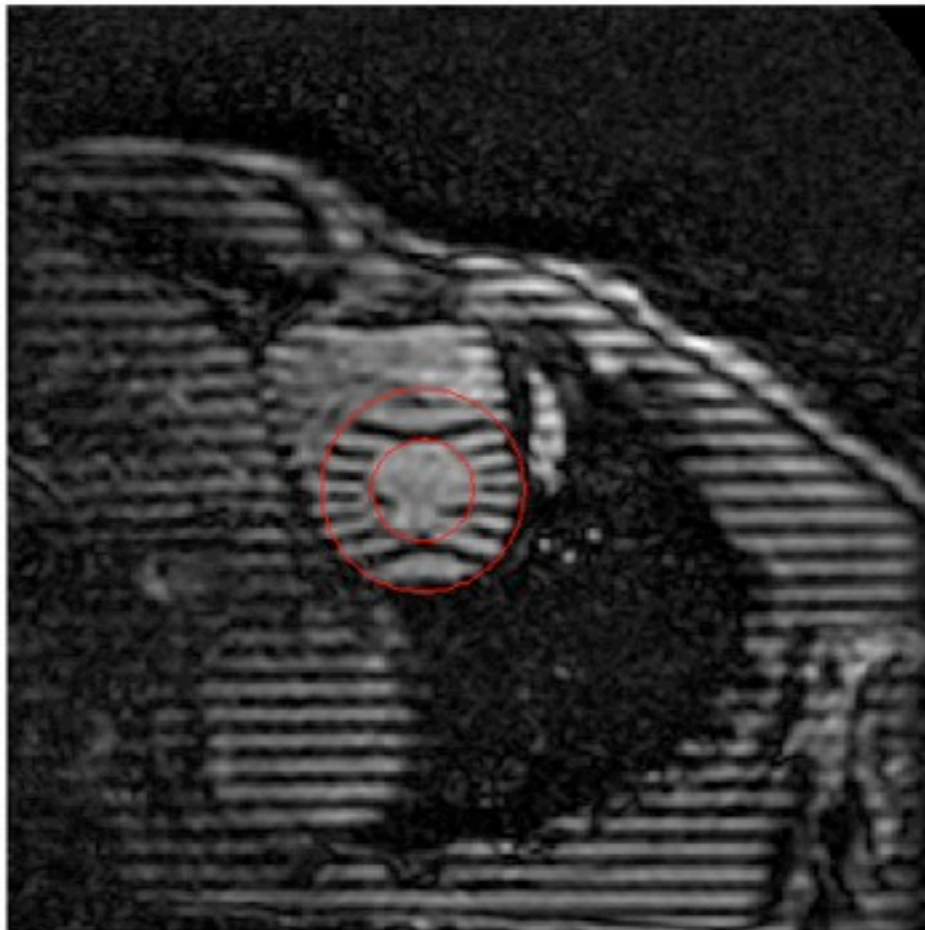
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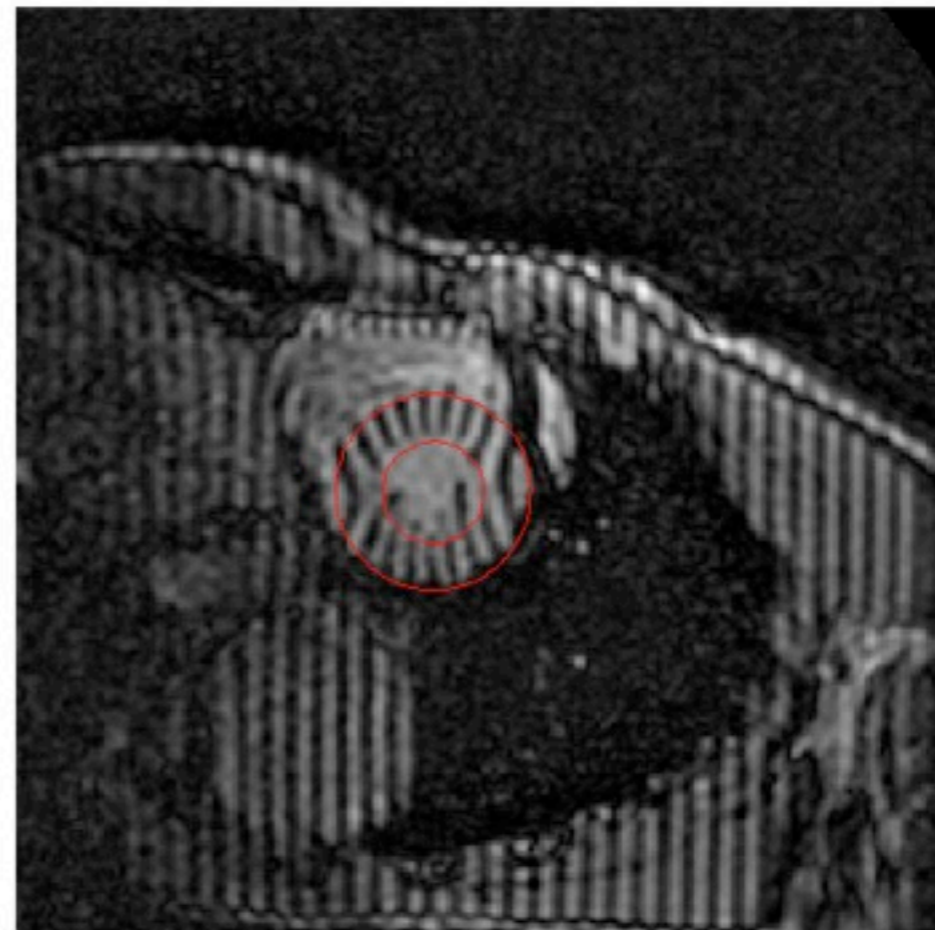
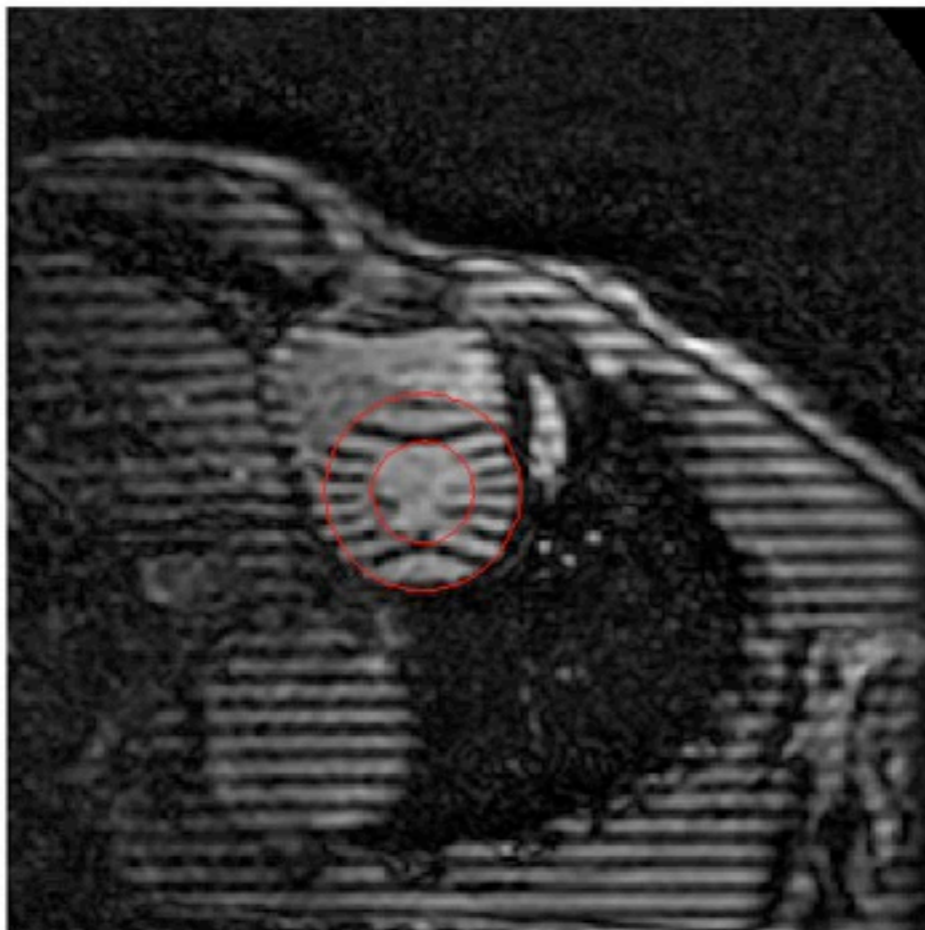
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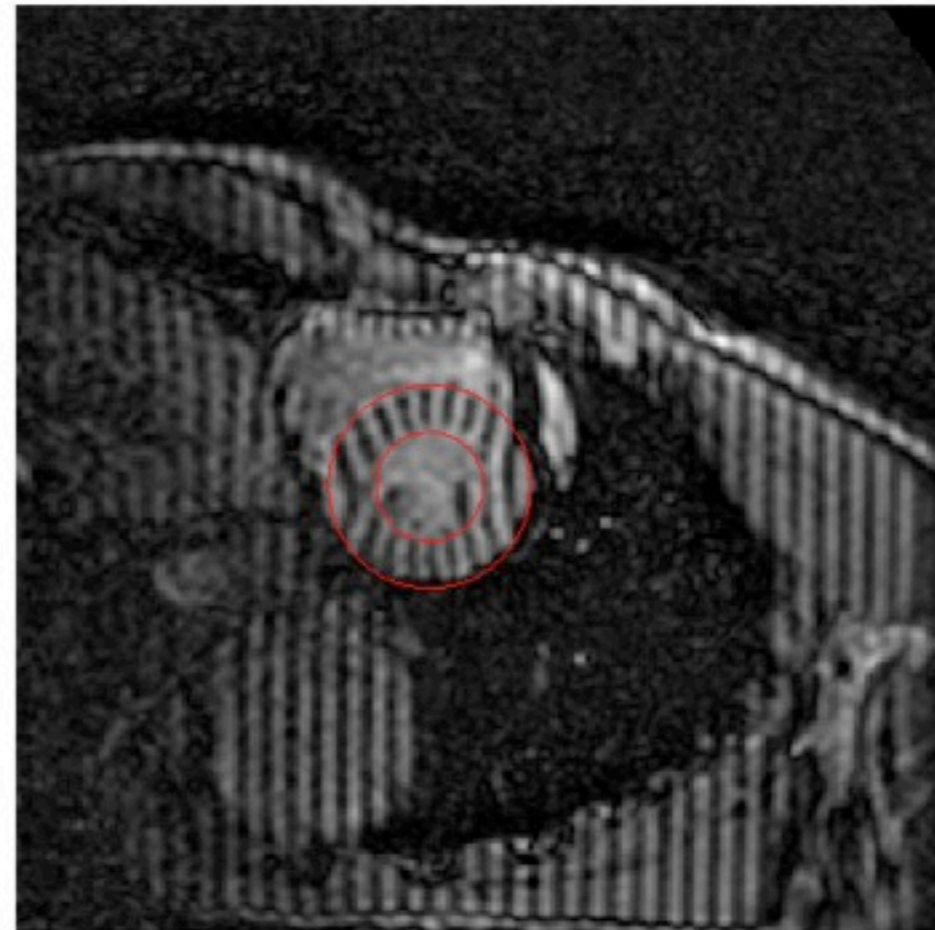
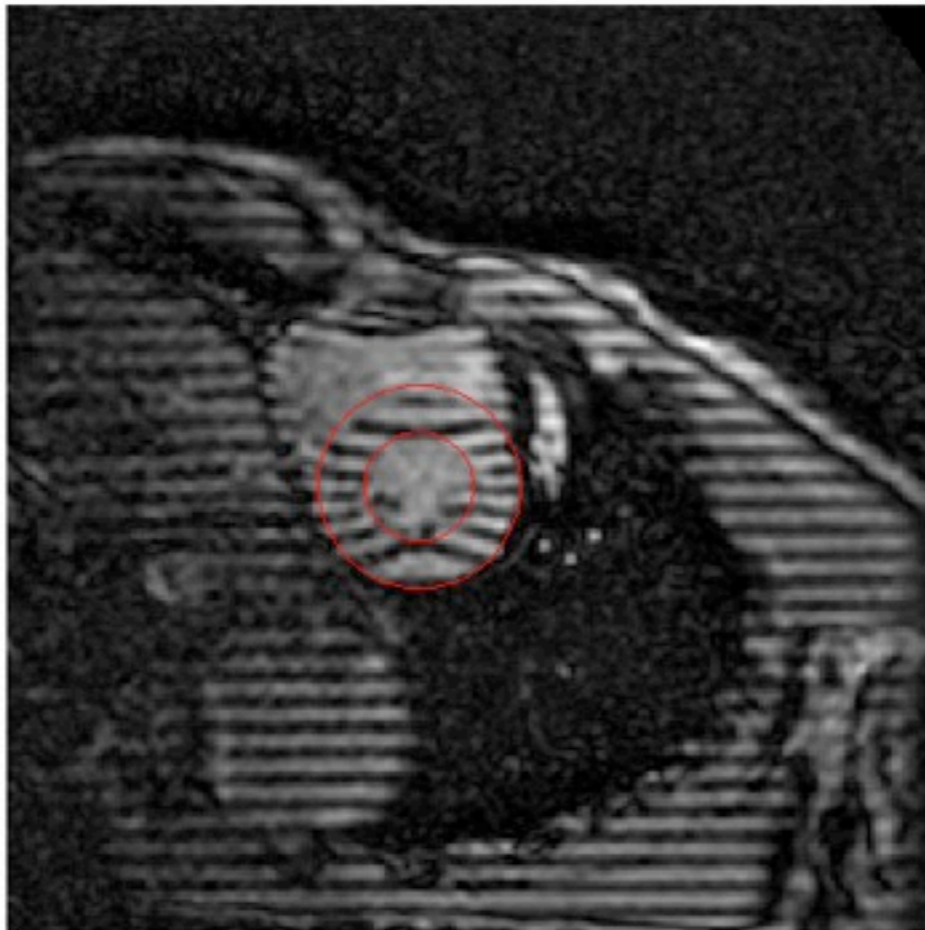
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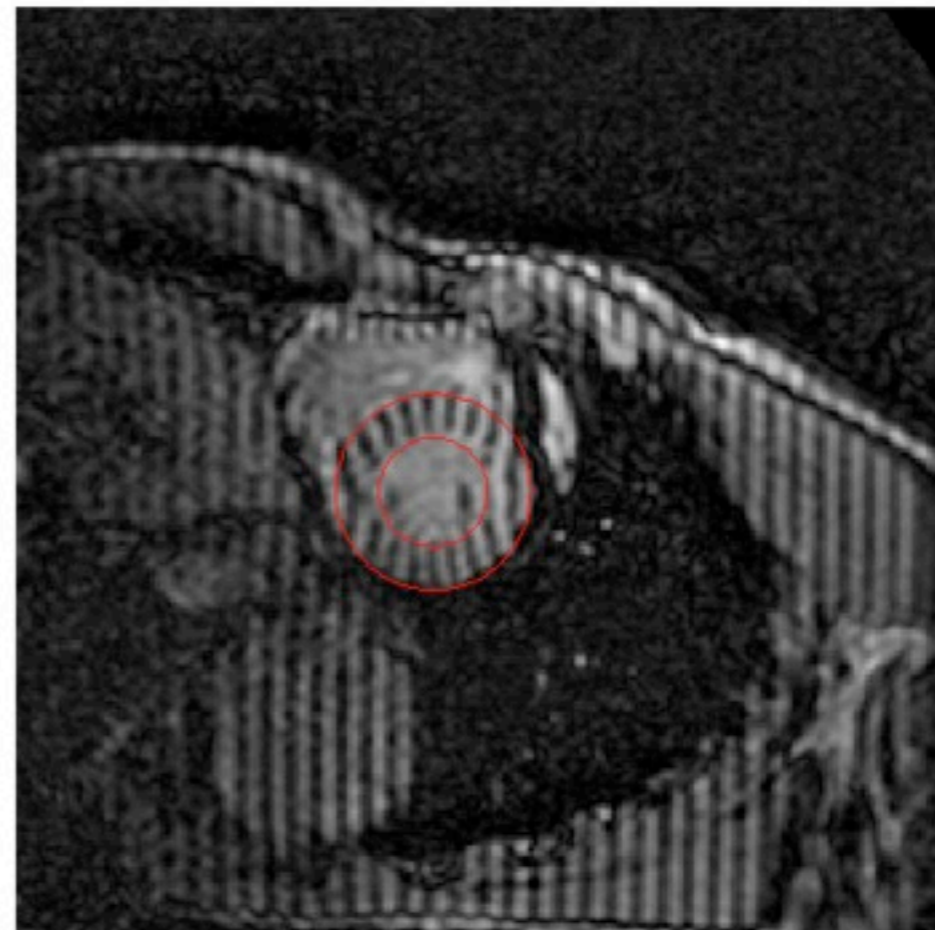
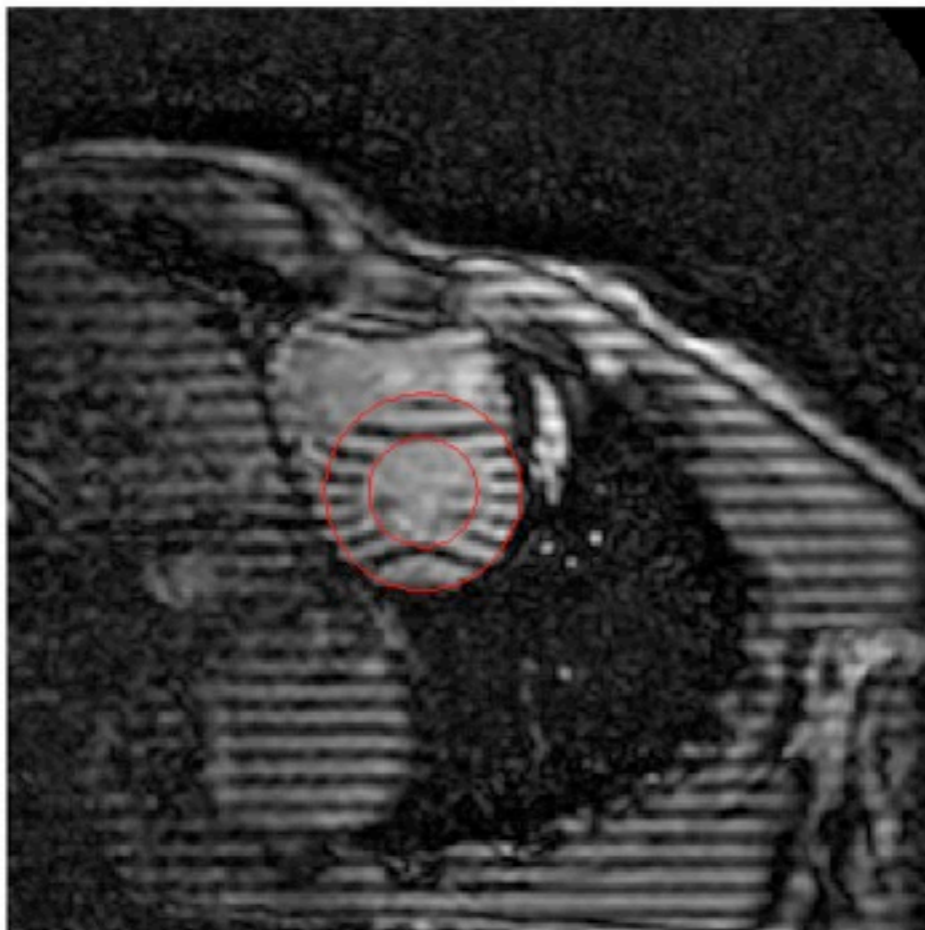
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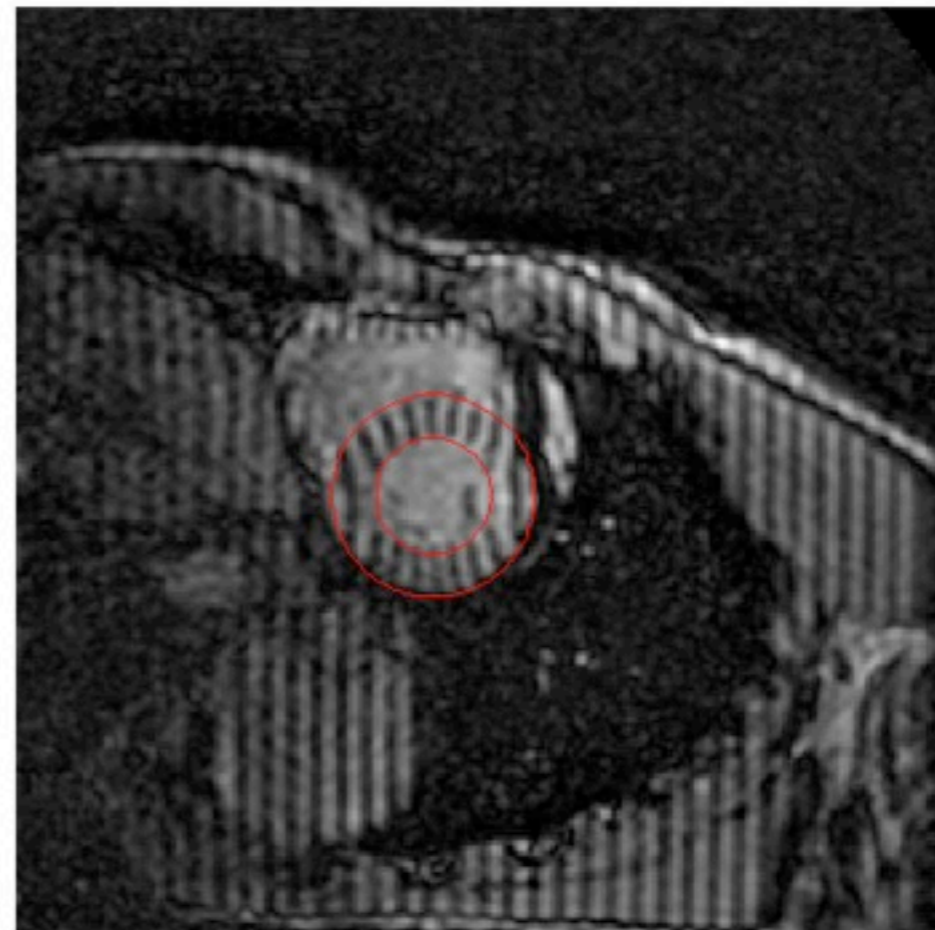
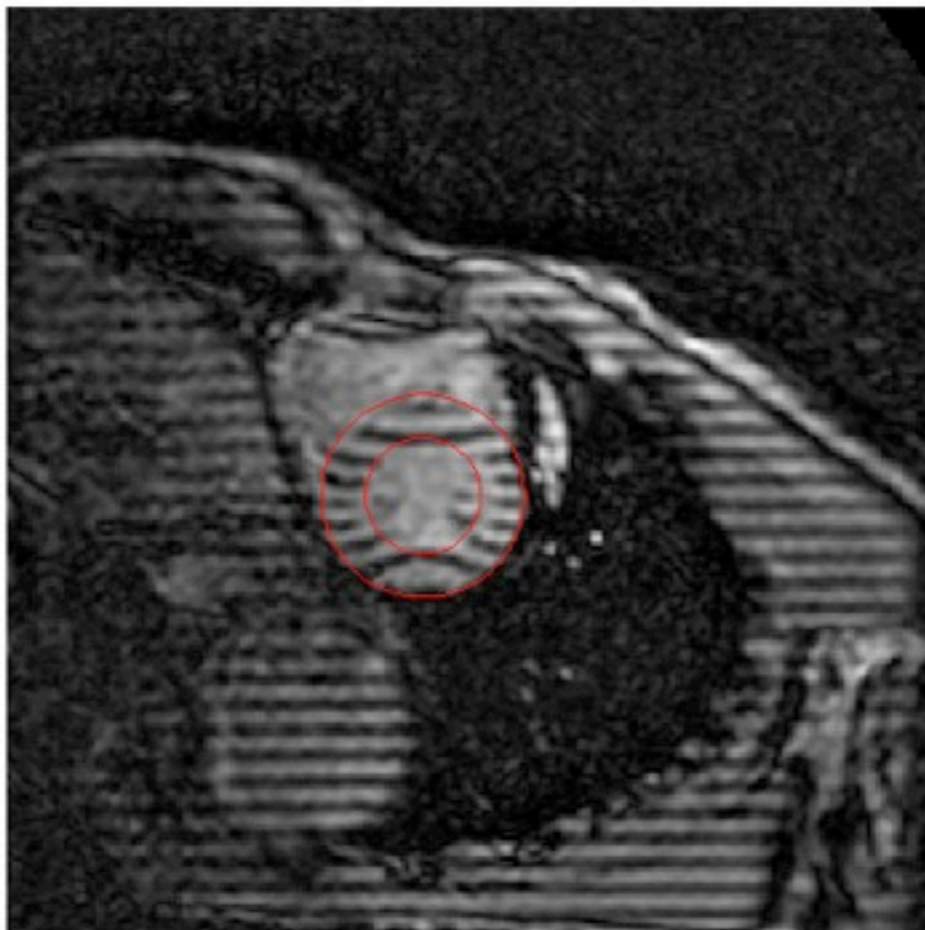
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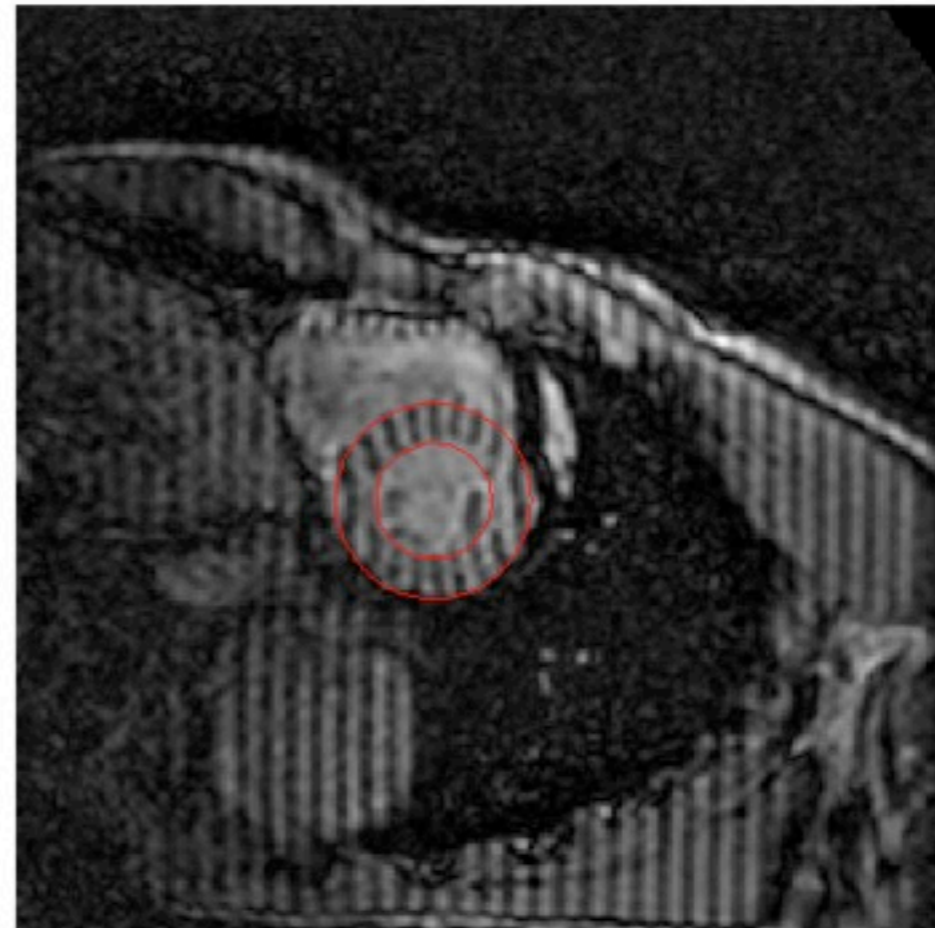
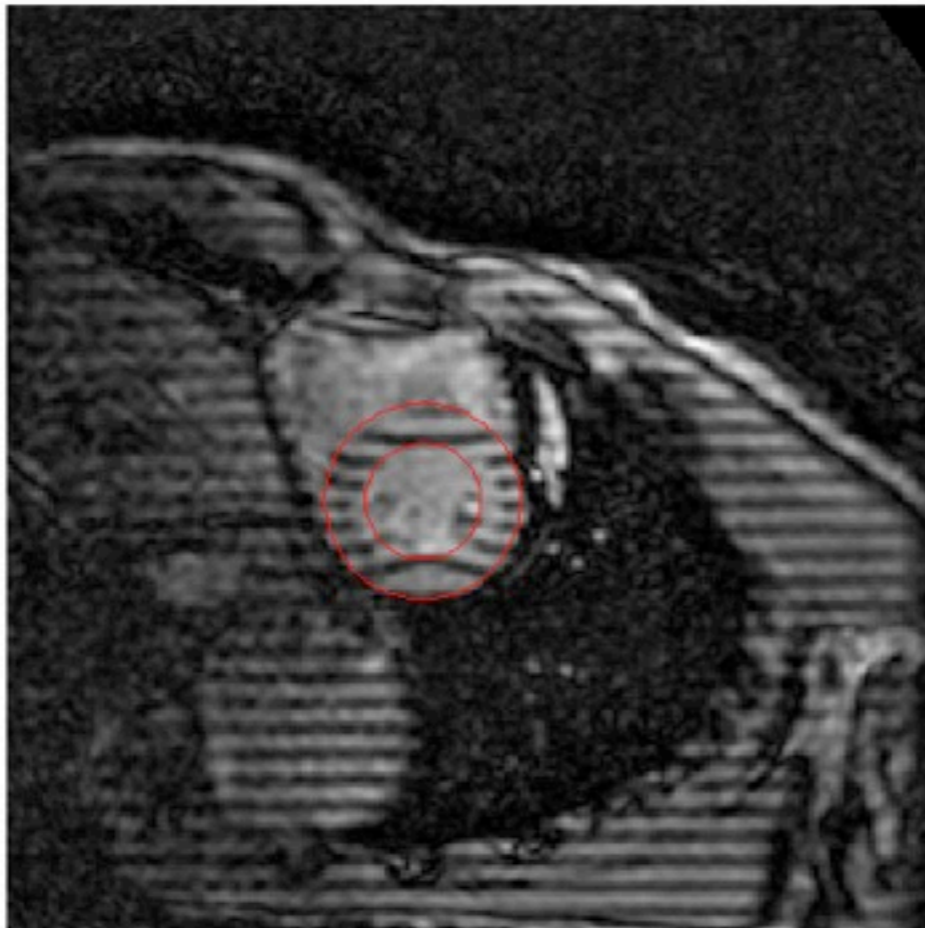
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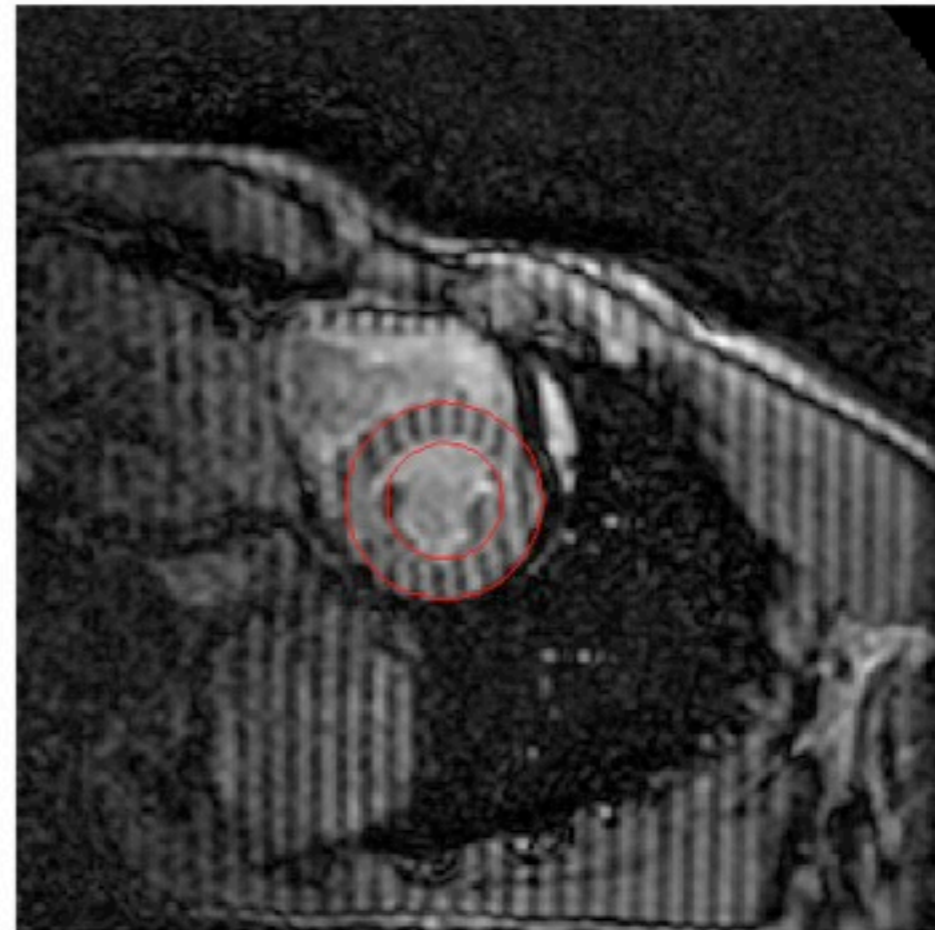
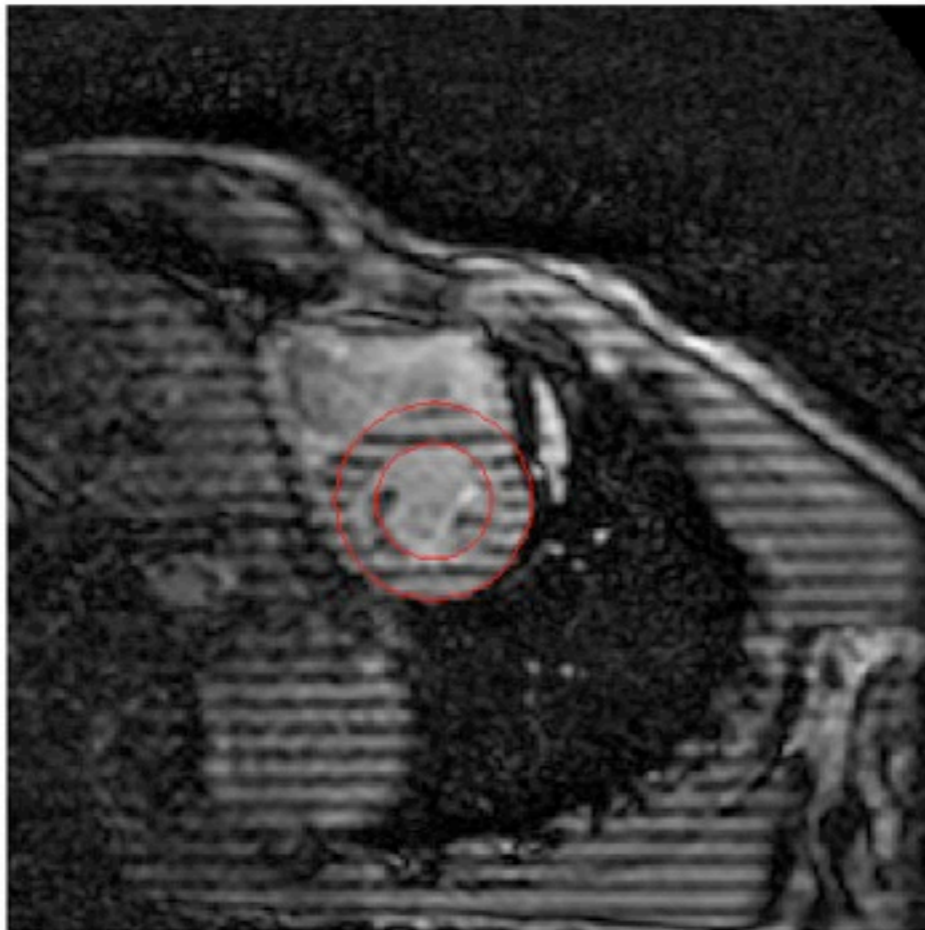
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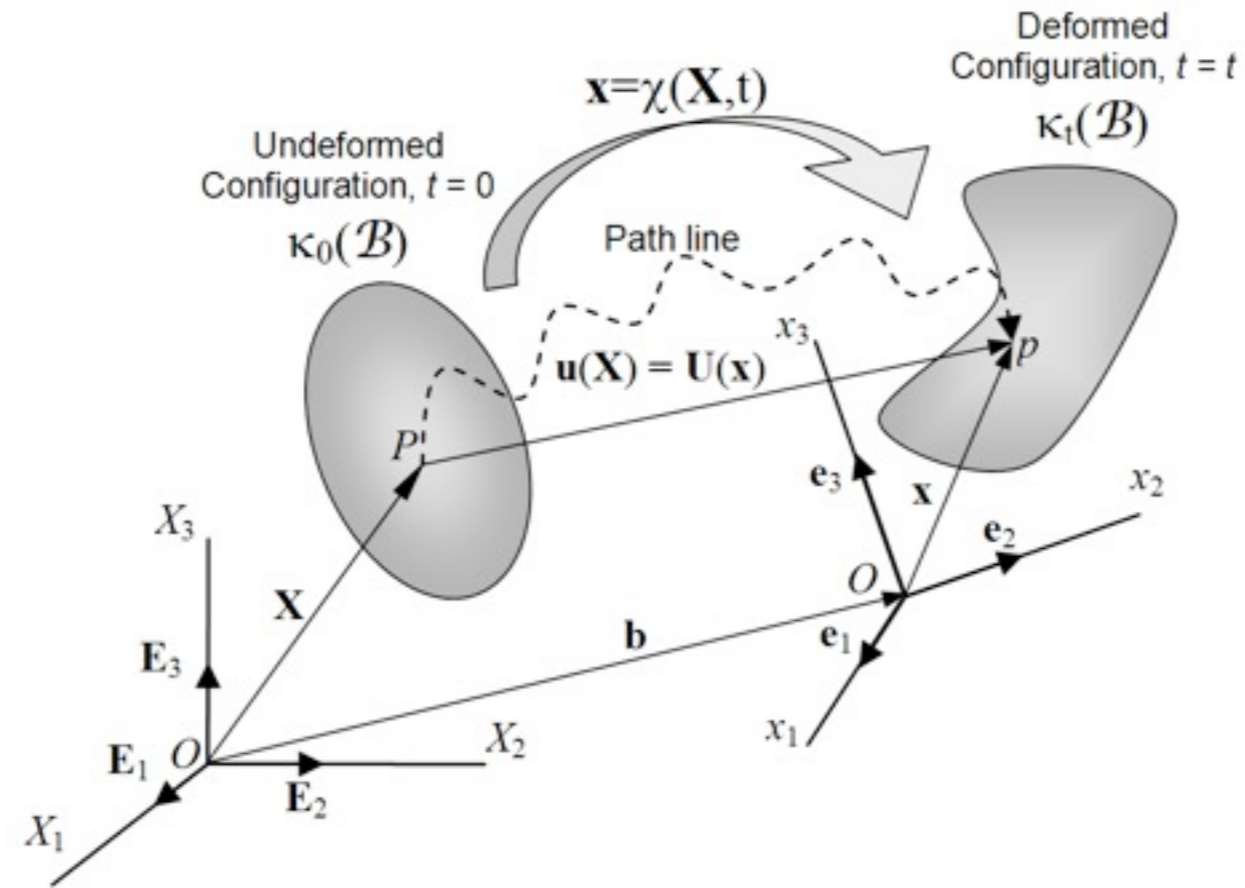


Heart Function

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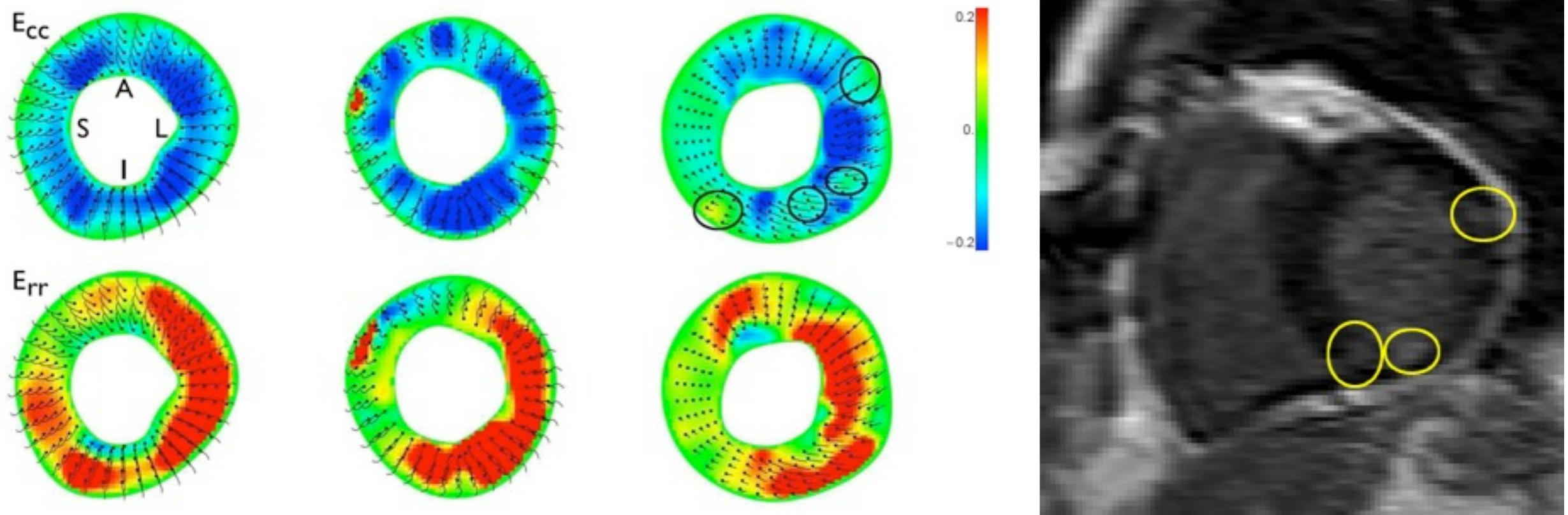
Methodology



- velocity gradient $\mathbf{L} = \nabla \mathbf{v} \rightarrow$ deformation tensor $\mathbf{F} \rightarrow$ strain tensor \mathbf{E}
- $d\mathbf{F}/dt = \mathbf{L}\mathbf{F}$: exact solution via non-commutative multiplicative calculus

$$\mathbf{F}(t) = \prod_0^t \exp(\mathbf{L}(s)ds)$$

Work in Progress



Hans van Assen

- strain tensor atlas of “normals” + “deviations” → pathology
- strain tensor evolution for quantitative stem cell therapy assessment

The End



TUe Technische Universiteit
Eindhoven
University of Technology

**Van beeld
naar beslissing**

Where innovation starts