Alberta Number Theory Days

June 17-19, 2011

Schedule

All Talks will take place in room 159 in Max Bell building.

Saturday, June 18

09:00 *Welcoming remarks*

09:10 **Renate Scheidler**
Infrastructure of Function Fields

10:00 *Coffee*

10:30 **Nathan Ng**
Simple zeros of modular L-functions

11:30 **Masoud Kamgarpour**
Geometrization of characters of the multiplicative group over a local field

12:20 *Lunch*

14:10 **Al Weiss**
What do Artin L-functions know about Galois module structure?

15:00 *Coffee*

15:30 **Mark Bauer**
Diophantine Equations: An Excuse to Play with Continued Fractions & the Hypergeometric Method

16:30 **Paul Buckingham**
L-functions associated to multiquadratic extensions

Sunday, June 19

09:00 **Habiba Kadiri**
New explicit bounds for $\psi(x)$

10:00 *Coffee*

10:30 **Michael Jacobson**
Tabulating Class Groups of Quadratic Fields

11:30 **Tim Trudgian**
Skewes' Number

Speakers, Titles, and Abstracts

Saturday, June 18	*morning*

9:00–9:10 AM *Welcoming remarks*

9:10–9:55 AM

Renate Scheidler (University of Calgary)
Infrastructure of Function Fields

The infrastructure of a global field has been used for a variety of important applications, including computing the regulator and the class number of a global field, and even for cryptography. Originally proposed by D. Shanks in 1972 in the setting of real quadratic fields, the concept has since been generalized to number fields of higher degree and to function fields. Strictly speaking, every field extension has multiple infrastructures, one for each ideal class, each containing a certain finite subset of "small" ideals in that class. Of particular interest is the infrastructure belonging to the principal class. Geometrically, this infrastructure is a torus whose dimension is the unit rank of the field extension. It is possible to equip this torus with a binary operation that is akin to multiplication and is called a giant step. The resulting structure behaves "almost" like an Abelian monoid – and in quadratic extensions even almost like an Abelian group – failing only associativity, and just barely. In the unit rank one case, a second addition-like operation, called a baby step, imposes an ordering on the infrastructure ideals according to a natural distance which is "almost" additive under giant steps. We present the baby step giant step arithmetic framework of the infrastructure and explain what it means to "just barely" fail associativity.

10:00–10:25 AM *Coffee*

10:30–11:15 AM

Nathan Ng (University of Lethbridge)
Simple zeros of modular L-functions

An open problem concerning primitive L-functions is to show that their non-real zeros are simple. Levinson proved that 1/3 of the zeros of the Riemann zeta function are on the half-line and simple. Bauer generalized this result to Dirichlet L-functions. However, for degree two L-functions there are few results. Conrey and Ghosh showed that a particular degree two L-function has infinitely many simple zeros. In this talk, I will discuss recent joint work with Milinovich where we investigate simple zeros of modular L-functions.

11:30–12:15 PM

Masoud Kamgarpour (Univerity of British Columbia and University of Calgary)
Geometrization of characters of the multiplicative group over a local field

In geometric representation theory, one often wishes to replace functions on finite sets with sheaves on algebraic varieties. We discuss the sheaves on the multiplicative group whose trace function recovers smooth characters of F^*, where F is a local field of characteristic zero.

This is joint work with Clifton Cunningham.

14:10-14:55 PM

Al Weiss (University of Alberta)
What do Artin L-functions know about Galois module structure?

15:00–15:25 AM *Coffee*

15:30–16:15 PM

Mark Bauer (University of Calgary)
Diophantine Equations: An Excuse to Play with Continued Fractions & the Hypergeometric Method

Many Diophantine equations can be attacked using techniques from modular curves, which are basically variants of Wiles' approach to prove Fermat's Last Theorem. However, polynomial-exponential Diophantine equations do not usually yield to such techniques. For equations of this type, one is often reduced to using restricted irrationality measures.

In this talk, we will describe how to use the Hypergeometric Method to construct restricted (and unrestricted) irrationality measures. Perhaps unsurprisingly, it relies heavily on the use of continued fractions convergents and mediants as an input to the machinery. After seeing how the machinery works, it is remarkable that anything meaningful could be derived from this technique. We will show that it works more often than it has any right to work.

This is joint work with Mike Bennett.

16:30–17:15 PM

Paul Buckingham (University of Alberta)
L-functions associated to multiquadratic extensions

The Lifted Root Number Conjecture, also known as the Equivariant Tamagawa Number Conjecture, can be thought of as a refined class number formula for Artin L-functions. It predicts a subtle connection between derivatives of L-functions associated to a Galois extension of number fields (the "analytic" side of the conjecture) and the Galois structure of the unit group (the "algebraic" side). After giving an overview of the conjecture, we will provide a full description of the analytic side for multiquadratic extensions of an arbitrary base field, and hint at the class field theory involved in understanding the algebraic side in this case.

9:00–9:45 AM

Habiba Kadiri (University of Lethbridge)
New explicit bounds for $\psi(x)$

The prime number theorem establishes that the prime counting function $\psi(x)$ is asymptotic to x. Unconditionally, one can show that the error term in this estimate is of size $O\left(x \exp\left(-c_2 (log x)^{1/2}\right)\right)$, where c_2 is an effective computable constant. Explicit bounds for the error term have been established by Rosser, Rosser and Schoenfeld, and more recently by Dusart. We will generalize their work, which allows us to obtain improved explicit bounds. The proof uses smooth functions and an explicit estimate for density of zeros for the Riemann zeta function.

10:00–10:25 AM *Coffee*

10:30–11:15 AM

Michael Jacobson (University of Calgary)
Tabulating Class Groups of Quadratic Fields

Class groups of quadratic fields have been studied since the time of Gauss, and in modern times have been used in applications such as integer factorization and public-key cryptography. Tables of class groups are used to provide valuable numerical evidence in support of a number of unproven heuristics and conjectures, including those due to Cohen and Lenstra. In this talk, we discuss recent efforts to extend existing, unconditionally correct tables of both imaginary and real quadratic fields. After a summary of the state-of-the-art in the imaginary case, we will discuss recent efforts to extend tables in the real case. This includes incorporating ideas of Sutherland for computing orders of elements in a group, as well as constructing an unconditional verification algorithm using the trace formula of Maass forms based on ideas of Booker.

11:30–12:15 PM

Tim Trudgian (University of Lethbridge)
Skewes' Number

The prime number theorem asserts that $\pi(x)$, the number of primes not exceeding x, is asymptotic to $\mathrm{li}(x)$, the logarithmic integral. It is known that, for all $2 \leq x \leq 10^{14}$, the inequality $\pi(x) < \mathrm{li}(x)$ is true. That there are infinitely many x for which this inequality does not hold was proved by Littlewood in 1914. His student, Skewes, constructed the first upper bound on the eponymous number below which there is at least one x for which $\pi(x) > \mathrm{li}(x)$. This talk will survey the current method of estimating Skewes' number.