Targeting Rossby-Wave Trains Precursors

Modeling Using Wave Action Singular Vectors

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Singular vectors (SV) of the **tangent linear model (TLM)** are often used to define **targeting subspaces** and **flow-dependent structure functions** for **data assimilation (DA)**. It is suggested that **SVs could be shifted** due to nonlinear interaction between synoptic or planetary waves and the **Jet Stream**. Better understanding of the triggered Rossby-wave train effect on SVs position is needed.

Problematic

- Recent observing system experiments (OSE) suggest that the forecast error reduction due to targeted observations assimilation is generally positive but small, ~10-15% (Rabier *et al.*, 2008; Langland, 2005).
- Targeting observations in the context of extratropical transitions (ET) results in more important error reduction (Cardinali *et al.*, 2007).
- It has been shown that the potential vorticity (PV) gradient in the vicinity of the Jet Stream can act as waveguide for Rossby-waves (Buizza *et al.*, 1999; Schwierz *et al.*, 2004).
- Recent studies reveal that precursor signal in the atmosphere have a very small amplitude, often below the level of observation error, making them difficult to be detected (Lupu and Gauthier, 2011).

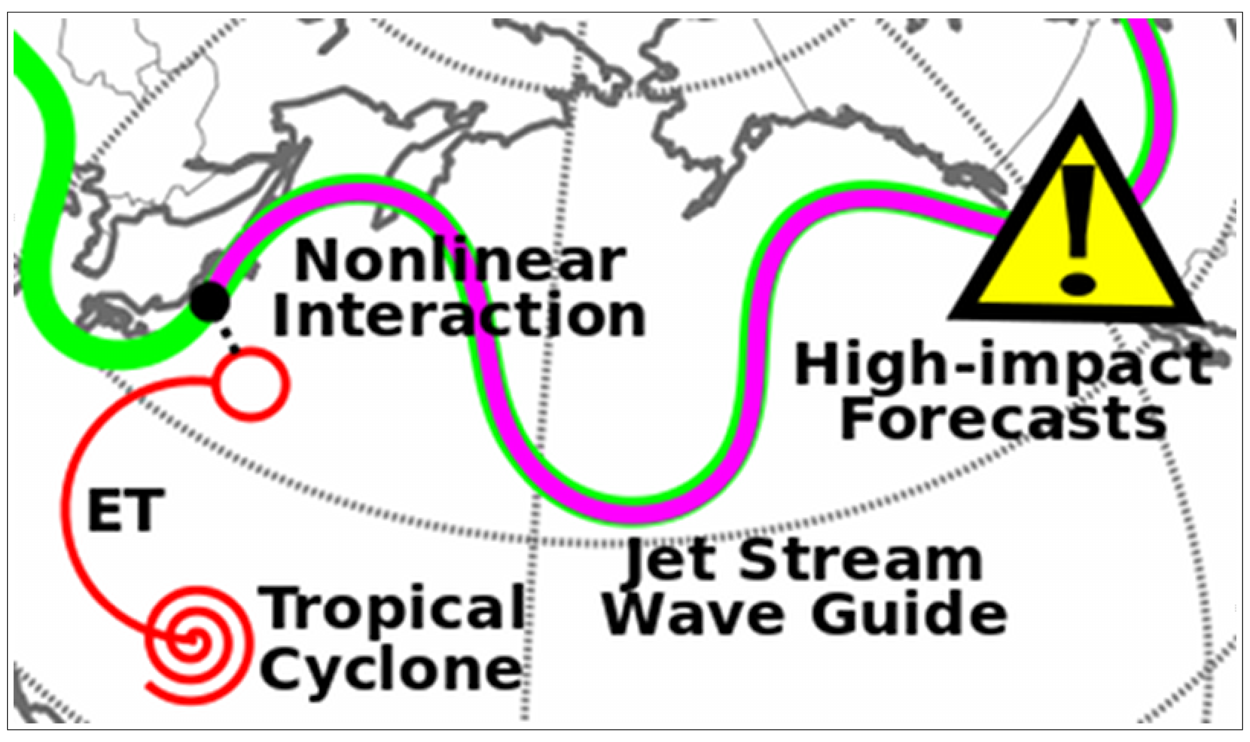

Figure 1: Wave train triggering by the interaction between a post ET perturbation and the Jet Stream and a related downstream high-impact event and flawed forecast.

Error Modeling on the Tangent Space $\mathcal{TE}$

Let $\mathbf{L}$ be the propagator of the TLM for a time interval t. It's an application between two metric spaces associated with the metrics $\mathbf{G}_0$ and $\mathbf{G}_t$:

$$\mathbf{L}\ :\left\{\begin{array}{ccc}(\mathcal{TE},\mathbf{G}_0) & \longmapsto & (\mathcal{TE},\mathbf{G}_t)\\ \delta\mathbf{x}_0 & \longmapsto & \delta\mathbf{x}_t\end{array}\right. \qquad (1)$$

The objective of **targeting techniques** is to find which additional observations to assimilate to maximize forecast improvement. Singular value decomposition of $\mathbf{L}$ produces these amplifying error structures:

$$\|\delta\mathbf{x}_t\|^2 \overset{\text{def}}{=} \langle \mathbf{L}\delta\mathbf{x}_0\,|\mathbf{G}_t|\,\mathbf{L}\delta\mathbf{x}_0\rangle \overset{\text{def}}{=} \left\langle \mathbf{L}^*_{(\mathbf{G}_0,\mathbf{G}_t)}\mathbf{L}\delta\mathbf{x}_0\,|\mathbf{G}_0|\,\delta\mathbf{x}_0\right\rangle \qquad (2)$$

Eigenvectors of $\mathbf{L}^*_{(\mathbf{G}_0,\mathbf{G}_t)}\mathbf{L}$ are SVs of the TLM, they spawn the **unstable subspace**. In a predictability problem, $\mathbf{G}_0$ is constrained to be the inverse of the analysis covariances matrix. $\mathbf{G}_t$ characterizes the dynamical process of interest (Lacarra and Talagrand, 1988).

Wave Action

- Nonlinear interaction between the Jet Stream and synoptic perturbations can generate fast propagating wave trains.
- As perturbations, they evolve in the tangent space and could be responsible for an important fraction of the forecast error.

Metrics characterizing wave propagation error

A **wave action (WA)**, $\mathcal{A}$, is a **globally conserved** scalar on $\mathcal{TE}$:

$$\frac{\partial\alpha}{\partial t}+\boldsymbol{\nabla}\cdot(\mathbf{c}_g\alpha)=0,\qquad \mathcal{A}\equiv\iiint_{\mathcal{V}}\alpha\;dx\;dy\;d\eta \qquad (3)$$

For a given WA, we define an associated metric, $\mathbf{A}$, and obtain SVs of $\mathbf{L}^*_{(\mathbf{G}_0,\mathbf{A})}\mathbf{L}$. Under certain assumptions, **pseudoenergy** is a WA and we define the associated metric:

$$\langle\delta\mathbf{x}\,|\mathbf{A}_E|\,\delta\mathbf{y}\rangle \equiv \iiint_{\mathcal{V}}\left(E-U\frac{q_{\mathbf{x}}q_{\mathbf{y}}}{2\partial_y Q}\right)dx\;dy\;d\eta \qquad (4)$$

where $\delta\mathbf{x}$ and $\delta\mathbf{y}$ are perturbation state vectors, E, the perturbative energy, U the reference state zonal wind speed, Q the reference state PV and $q_{\mathbf{x}}$, its anomaly. $\mathcal{V}$ is the volume of the atmosphere: x, y and η are zonal, meridional and normalized pressure coordinates respectively.

Wave Propagation Representativeness

Let $\boldsymbol{\mu}$ be an initial SV, and $\boldsymbol{\mu}(t)$ its evolution with the nonlinear model M:

$$\boldsymbol{\mu}(t)\equiv M(t;\ \mathbf{x}_0+\boldsymbol{\mu})-M(t;\ \mathbf{x}_0) \qquad (5)$$

As discussed by Mahidjiba *et al.* (2007), we define the moving support Ω of the propagating wave packet. **Since WA is globally conserved** and if $\boldsymbol{\mu}(t)$ is such that WA is conserved on Ω, then we conclude the SV represents well the wave propagation. Writing $\mathbf{T}_\Omega$ the projector on Ω, $R_{\mathbf{A}}$ characterizes the local conservation of WA in the wave reference frame:

$$R_{\mathbf{A}}(t;\ \boldsymbol{\mu})\equiv\sqrt{\frac{\left\langle\boldsymbol{\mu}(t)\left|\mathbf{T}^T_{\Omega_t}\mathbf{A}\mathbf{T}_{\Omega_t}\right|\boldsymbol{\mu}(t)\right\rangle}{\left\langle\boldsymbol{\mu}\left|\mathbf{T}^T_{\Omega_0}\mathbf{A}\mathbf{T}_{\Omega_0}\right|\boldsymbol{\mu}\right\rangle}}=\frac{\|\mathbf{T}_{\Omega_t}\boldsymbol{\mu}(t)\|_{\mathbf{A}}}{\|\mathbf{T}_{\Omega_0}\boldsymbol{\mu}\|_{\mathbf{A}}} \qquad (6)$$

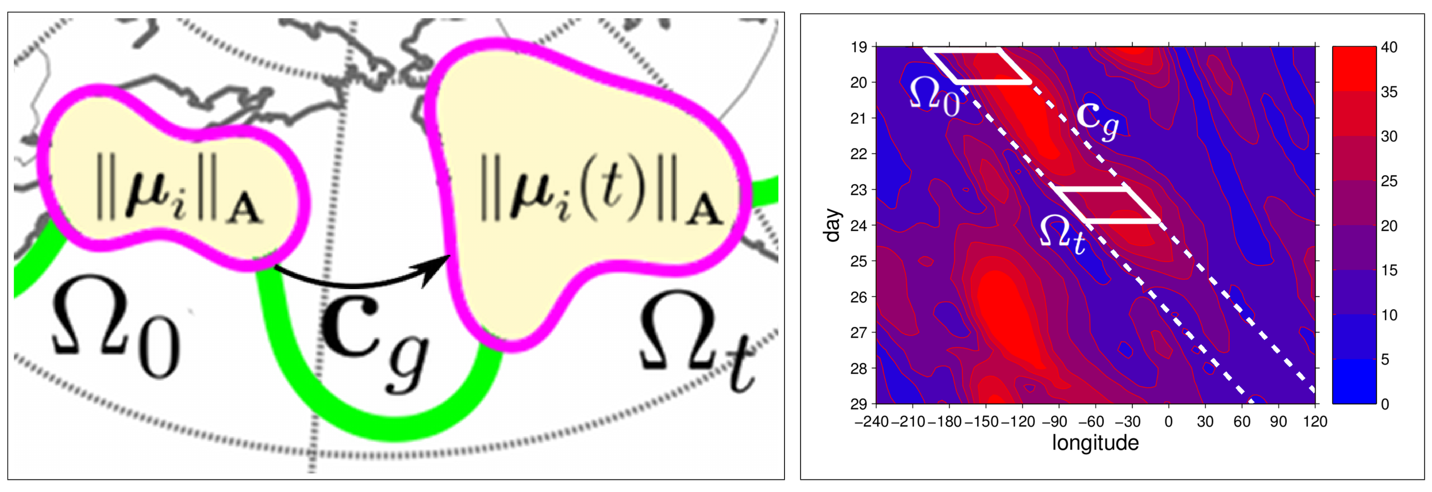

Figure 2: On the left: representation of WA density integrated on the support of a propagating Rossby-wave packet at initial and final time. On the right: definition of Ω from an Hovmöller diagram of meridional wind anomaly amplitude (ms^{-1}); from Mahidjiba *et al.* (2007).

Precursors observability

In order to use SVs to detect instabilities and target new observations, we must ascertain that **precursors** exist in the atmosphere. As a measure, we use the **observability coefficient** defined by Lupu and Gauthier (2011) as the normalized model-equivalent of $\boldsymbol{\mu}$ projection on the innovation $\mathbf{d}$:

$$\rho(\boldsymbol{\mu})\equiv\frac{\langle\boldsymbol{\nu}\,|\mathbf{R}^{-1}|\,\mathbf{d}\rangle}{\|\boldsymbol{\nu}\|_{\mathbf{R}^{-1}}\|\mathbf{d}\|_{\mathbf{R}^{-1}}},\qquad \boldsymbol{\nu}=H(\mathbf{x}_0+\boldsymbol{\mu})-H(\mathbf{x}_0) \qquad (7)$$

where H is the nonlinear observation operator and $\mathbf{R}$ the corresponding covariances matrix.

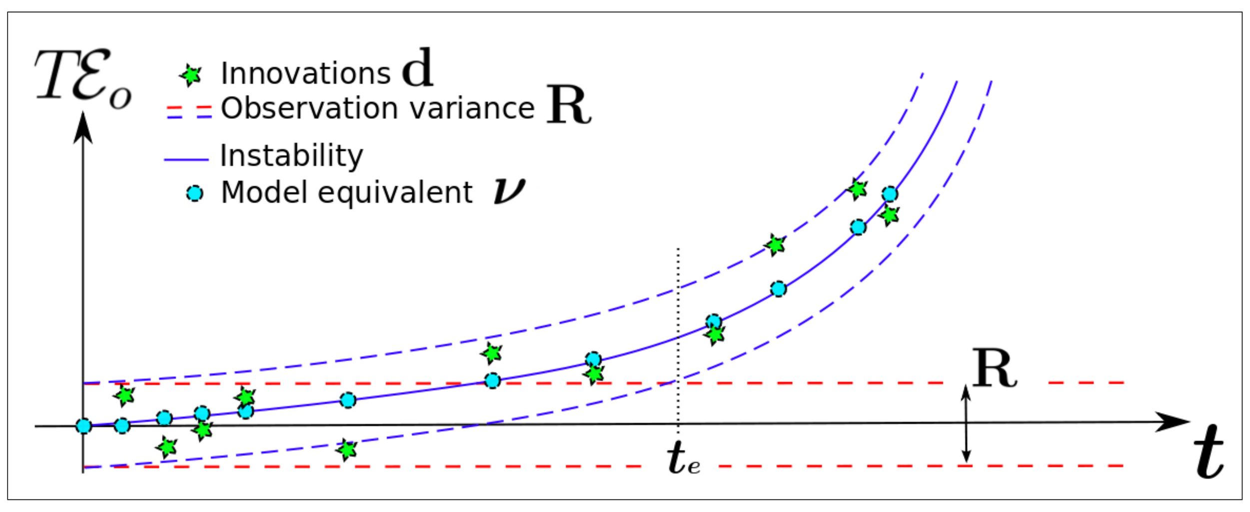

Figure 3: Illustration on the observation tangent space of the emergence time t_e necessary for a precursor to be observable.

Project and Methodology

Objectives

- Study WA energetics and precursor dynamics using SVs;
- Study the impact of model resolution on PV gradients and WA SVs;
- Contribute in explaining the weak efficiency of pseudoinverses to explain forecast error;
- Study precursors observability as they evolve.

Methodology

We intend to perturb analyses of low score, past ET events in the presence of Rossby-wave train, with different families of SVs (different metrics and TLM resolutions) and evaluate representativeness of pseudoinverses, similarity indexes, power spectra, amplification profiles and forecast error reduction.

Secondly, we'll investigate the observability of SVs as they evolve. When the amplitude of the precursor emerge from the noise level, $\partial_t\rho(t)$ should become positive definite and we will look for this **emergence time**.

Numerical Laboratory

- The Global Environmental Multiscale (GEM) Model of Environment Canada will be used for nonlinear and TLM integrations and assimilation components;
- SVs will be calculated with the ARPACK implementation of the Arnoldi iteration.

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