

# COMPLEX MONGE-AMPÈRE EQUATION

18-23 October 2009

## MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

## MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

## SCHEDULE

### Sunday

- 16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)  
17:30–19:30 Buffet Dinner, Sally Borden Building  
20:00 Informal gathering in 2nd floor lounge, Corbett Hall  
Beverages and small assortment of snacks available on a cash honour-system.

### Monday

- 8:45–9:00 Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159  
**9:00–10:00** Shing-Tung Yau (Harvard University)  
*Canonical metrics and Monge-Ampère equations*  
10:05 Group Photo; meet on the front steps of Corbett Hall  
10:10–10:30 Coffee Break, 2nd floor lounge, Corbett Hall  
**10:30–11:30** Jacob Sturm (Rutgers University)  
*Regularity of geodesic rays*  
13:00–14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall  
**14:00–15:00** Vestislav Apostolov (Université du Québec à Montréal)  
*Extremal Kähler metrics on projective bundles over a curve*  
15:00–15:30 Coffee Break, 2nd floor lounge, Corbett Hall  
**15:30–16:30** Song-Ying Li (University of California, Irvine)  
*On the complex Monge-Ampère operator and its application on pseudo-Hermitian manifolds*

## Tuesday

- 9:00–10:00** Zhiqin Lu (University of California, Irvine)  
*Remarks on hypersurface K-stability*
- 10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall
- 10:30–11:30** Jean-Pierre Demailly (Université de Grenoble I)  
*Asymptotic cohomology and holomorphic Morse inequalities*
- 14:00–15:00** Gabriele LaNave (Yeshiva University)  
*TBA*
- 15:00–15:30 Coffee Break, 2nd floor lounge, Corbett Hall
- 15:30–16:30** Qun Li (Ohio State University)  
*Complex Monge-Ampère equations and totally real submanifolds*

## Wednesday

- 9:00–10:00** Sławomir Dinew (Jagiellonian University)  
*Hölder continuity of solutions of Monge-Ampère equations with right hand side in  $L^p$*
- 10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall
- 10:30–11:30** Jian Song (Rutgers University)  
*The Kähler-Ricci flow through singularities*
- Free Afternoon

## Thursday

- 9:00–10:00** Jixiang Fu (Fudan University)  
*On balanced metrics*
- 10:00–10:30 Coffee Break, 2nd floor lounge, Corbett Hall
- 10:30–11:30** Robert Berman (Chalmers University)  
*Complex Monge-Ampère equations and balanced metrics*
- 14:00–15:00** Valentino Tosatti (Columbia University)  
*Complex Monge-Ampère equations on symplectic and hermitian manifolds*
- 15:00–15:30 Coffee Break, 2nd floor lounge, Corbett Hall
- 15:30–16:30** Gábor Székelyhidi (Columbia University)  
*On convergence of the Kähler-Ricci flow*
- 16:40–17:40** Bo Guan (Ohio State University)  
*Some special Dirichlet problems for the complex Monge-Ampère equation*

## Friday

**Checkout by 12 noon.**

\*\* 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

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### ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Vestislav Apostolov** (Université du Québec à Montréal)

Title: *Extremal Kähler metrics on projective bundles over a curve*

Abstract: I will discuss the existence problem of extremal Kähler metrics (in the sense of Calabi) on the total space of a holomorphic projective bundle  $P(E)$  over a compact complex curve. The problem is not solved in full generality even in the case of a projective plane bundle over  $\mathbb{C}P^1$ . However, I will show that sufficiently “small” Kähler classes admit extremal Kähler metrics if and only if the underlying vector bundle  $E$  can be decomposed as a sum of stable factors. This result can be viewed as a “Hitchin-Kobayashi correspondence” for projective bundles over a curve, but in the context of the search for extremal Kähler metrics. The talk will be based on a recent work with D. Calderbak, P. Gauduchon and C. Tonnesen-Friedman.

Speaker: **Robert Berman** (Chalmers University)

Title: *Complex Monge-Ampère equations and balanced metrics*

Abstract: In a very recent joint work with Sebastien Boucksom, Vincent Guedj and Ahmed Zeriahi a new variational approach to solving complex Monge-Ampère equations in big cohomology classes was developed. Applications to existence and convergence of “balanced metrics” were also investigated. In this talk, I will in particular focus on these latter applications. It was Donaldson who first proposed to use suitably defined balanced metrics as canonical approximations of 1) solutions of the inhomogenous Monge-Ampère equation for a metric on an ample line bundle  $L$  over a complex manifold  $X$ , given a measure on  $X$  2) Kähler-Einstein metrics, when  $L$  is the (anti-) canonical line bundle. In the case of Kähler-Einstein metrics on Fano manifolds our proofs also require some recent estimates of Berndtsson on one hand and D.H. Phong, Jian Song, Jacob Sturm and Ben Weinkove on the other.

Speaker: **Jean-Pierre Demailly** (Université de Grenoble I)

Title: *Asymptotic cohomology and holomorphic Morse inequalities*

Abstract: The concept of volume of a line bundle or of a pseudoeffective class can be generalized to introduce an asymptotic cohomology function  $\hat{h}^q(\alpha)$  defined on the space of real  $(1, 1)$  classes of a compact Kähler manifold, in such a way that the alternating sum yields the intersection product  $\alpha^n$ . This is strongly related to the (still conjectural) holomorphic Morse inequalities for transcendental classes and, in the case of the volume, is related to Monge-Ampère operators via an upper envelope construction.

Speaker: **Sławomir Dinew** (Jagiellonian University)

Title: *Hölder continuity of solutions of Monge-Ampère equations with right hand side in  $L^p$*

Abstract: We will discuss some recent developments regarding regularity of weak solutions of the complex Monge-Ampère equation in the setting of compact Kähler manifolds in the case when the measure has an  $L^p$  density. We will show how to prove Hölder continuity via pluripotential methods and will discuss the problem of optimal Hölder exponents.

Speaker: **Jixiang Fu** (Fudan University)

Title: *On balanced metrics*

Abstract: In this talk, I will give the definition and properties of the complex manifold with balanced metric. I will also describe some examples of such manifolds.

Speaker: **Qun Li** (Ohio State University)

Title: *Complex Monge-Ampère equations and totally real submanifolds*

Abstract: We study the Dirichlet problem of complex Monge-Ampère equations in Hermitian manifolds. As an application, we will also discuss the homogeneous complex Monge-Ampère equation with prescribed singularity on a totally real submanifold.

Speaker: **Song-Ying Li** (University of California, Irvine)

Title: *On the complex Monge-Ampère operator and its application on pseudo-Hermitian manifolds*

Abstract: In this talk, I will present my recent works related to the degenerated complex Monge-Ampère operators and their applications to the rigidity theorems on several complex variables and on pseudo-Hermitian manifolds.

Speaker: **Zhiqin Lu** (University of California, Irvine)

Title: *Remarks on hypersurface K-stability*

Abstract: Is a smooth hypersurface K-stable? In this talk, we study the K-stability on a hypersurface and its relation to the Chow-Mumford stability. Let  $M$  be a smooth hypersurface of  $\mathbb{C}\mathbb{P}^n$ ; we prove that for any test configuration induced by a generic one-parameter subgroup of  $Aut(\mathbb{C}\mathbb{P}^n)$ , the leading term of the Mabuchi energy is always a positive number times  $\log |t|$ . Our result shows that there should be a deep relation between the Chow-Mumford stability and the K-stability. This is joint work with Duong Phong.

Speaker: **Jian Song** (Rutgers University)

Title: *The Kähler-Ricci flow through singularities*

Abstract: We prove the existence and uniqueness of the weak Kähler-Ricci flow on projective varieties with log terminal singularities. It is also shown that the weak Kähler-Ricci flow can be uniquely continued through divisorial contractions and flips if they exist. We then propose an analytic version of the Minimal Model Program with Ricci flow.

Speaker: **Jacob Sturm** (Rutgers University)

Title: *Regularity of geodesic rays*

Abstract: Let  $L$  be a holomorphic line bundle over a Kähler manifold  $X$ , let  $T$  be a test configuration for the pair  $(X, L)$  and let  $H$  be the space of Kähler metrics in the class  $c_1(L)$ . We show that the geodesic rays in  $H$  attached to the test configuration  $T$  are  $C^{1,1}$ .

Speaker: **Gábor Székelyhidi** (Columbia University)

Title: *On convergence of the Kähler-Ricci flow*

Abstract: I will discuss joint work with Ovidiu Munteanu studying the convergence of the Kähler-Ricci flow on a Fano manifold under some stability conditions.

Speaker: **Valentino Tosatti** (Columbia University)

Title: *Complex Monge-Ampère equations on symplectic and hermitian manifolds*

Abstract: We discuss two natural generalizations of the complex Monge-Ampère equation to symplectic and hermitian manifolds. We will explain to which extent the classical theory on Kähler manifolds extends to these two cases, and give some applications. This is joint work with B. Weinkove and with S.-T. Yau.