

# Affine Convex Geometric Analysis

## January 12 - 16, 2009

### MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

### MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

### SCHEDULE

#### Sunday

**16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

**17:30–19:30** Buffet Dinner, Sally Borden Building

**20:00** Informal gathering in 2nd floor lounge, Corbett Hall

Beverages and small assortment of snacks available on a cash honour-system.

#### Monday

**7:00–9:00** Breakfast

**9:00–9:15** Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159

**9:15–10:05** **Rolf Schneider**

The role of the volume product in stochastic geometry

**10:05–11:00** Coffee Break, 2nd floor lounge, Corbett Hall

**11:00** Group Photo; meet on the front steps of Corbett Hall

**11:15–11:40** **Mathieu Meyer**

Once again on Mahler's problem

**11:50–13:00** Lunch

**13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

**14:00–14:50** **Nicole Tomczak-Jaegermann**

Random points uniformly distributed on an isotropic convex body

**15:00–15:30** Coffee Break, 2nd floor lounge, Corbett Hall

**15:30–15:55** **Alexander Litvak**

On vectors uniformly distributed over a convex body

**16:00–16:25** **Joseph Lehec**

A functional approach to the Blaschke-Santaló inequality

**16:30–16:55** **Matthieu Fradelizi**

The volume product of convex bodies with many symmetries

**18:00–19:30** Dinner

## Tuesday

7:00–9:00 Breakfast

9:30–10:20 **Matthias Reitzner**  
A classification of  $SL(n)$  invariant valuations

10:30–11:00 Coffee Break, 2nd floor lounge, Corbett Hall

11:00–11:25 **Fedor Petrov**  
Affine surface area and rational points on convex surfaces

11:30–11:55 **Christoph Haberl**  
Blaschke valuations

12:00–13:30 Lunch

14:00–14:50 **Peter J. Olver**  
Moving frames, differential invariants and surface geometry

15:00–15:30 Coffee Break, 2nd floor lounge, Corbett Hall

15:30–15:55 **Franz Schuster**  
Valuations and affine Sobolev inequalities

16:00–16:25 **Christian Steiner**  
The volume of the projection body and binary coding sequences

16:30– 16:55 **Carlo Nitsch**  
Affine isoperimetric inequalities and Monge-Ampère equations

18:00–19:30 Dinner

## Wednesday

7:00–9:00 Breakfast

9:30–10:20 **Franck Barthe**  
Remarks on conservative spin systems and related questions in convexity

10:30–11:00 Coffee Break, 2nd floor lounge, Corbett Hall

11:00–11:25 **Andrea Colesanti**  
Recent contributions to the Christoffel problem coming from partial differential equations

11:30–11:55 **Marina Yaskina**  
Non-symmetric convex bodies and the Fourier transform

12:00–13:30 Lunch  
Free Afternoon

17:30–19:30 Dinner

## Thursday

- 7:00–9:00** Breakfast
- 10:00–10:25** **Mark Rudelson**  
TBA
- 10:30–11:00** Coffee Break, 2nd floor lounge, Corbett Hall
- 11:00–11:25** **Mark Meckes**  
On the measure-theoretic analogue of Dvoretzky’s theorem
- 11:30–11:55** **Eugenia Saorín Gómez**  
Approaching  $K$  by its form body and kernel
- 12:00–13:30** Lunch
- 14:00–14:25** **Vlad Yaskin**  
On strict inclusions in hierarchies of convex bodies
- 14:30–14:55** **Marisa Zymonopoulou**  
New examples of non-intersection bodies
- 15:00–15:30** Coffee Break, 2nd floor lounge, Corbett Hall
- 15:30–15:55** **Olivier Guédon**  
Invertibility of matrices with iid columns
- 16:00–16:25** **Efrén Morales Amaya**  
Characterization of ellipsoids by means of parallel translated sections
- 16:30– 16:55** **Daniel Hug**  
Random polytopes: geometric and analytic aspects
- 18:00–19:30** Dinner

## Friday

- 7:00–9:00** Breakfast
- 9:30–10:30** To be determined
- 10:30–11:00** Coffee Break, 2nd floor lounge, Corbett Hall
- 11:00–11:30** To be determined
- 11:30–13:30** Lunch
- Checkout by 12 noon.**

\*\* 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

**Affine Convex Geometric Analysis**  
**January 12 - 16, 2009**

**ABSTRACTS**

**Franck Barthe**, franck.barthe@math.univ-toulouse.fr

**Remarks on conservative spin systems and related questions in convexity**

Abstract:

We provide precise estimates for the spectral gap and logarithmic Sobolev constants for canonical Gibbs measures associated to gamma distributions. This turns out to be related to the KLS conjecture for simplices of  $L_p$  balls.

**Andrea Colesanti**, colesant@math.unifi.it

**Recent contributions to the Christoffel problem coming from partial differential equations**

Abstract:

In a recent paper Guan and Ma gave a rather simple sufficient condition on a function  $f$ , defined on the unit sphere  $\mathbf{S}^n$  of  $\mathbf{R}^n$ , to be the density of the  $k$ -th area measure of a convex body of  $\mathbf{R}^n$ . In the particular case  $k = 1$  this leads to a sufficient condition to solve the Minkowski problem. The aim of this talk is to give a brief description of this result and of the technique used for its proof.

**Matthieu Fradelizi**, matthieu.fradelizi@univ-mlv.fr

**The volume product of convex bodies with many symmetries**

Abstract:

We present a joint work with Franck Barthe. We give lower bounds on the volume product of convex bodies with many symmetries.

**Olivier Guédon**, guedon@math.jussieu.fr

**Invertibility of matrices with iid columns**

(joint work with R. Adamczak, A. Litvak, A. Pajor and N. Tomczak-Jaegermann)

Abstract:

Recent work on the subject will be presented.

**Christoph Haberl**, haberl@geometrie.tuwien.ac.at

### **Blaschke valuations**

Abstract:

In recent years, Monika Ludwig classified convex and star body valued valuations which are compatible with the general linear group. She obtained characterizations of the projection, intersection, and centroid body operator. I present a corresponding classification for valuations with respect to Blaschke addition. It turns out that the only non-trivial such valuation is the curvature image operator on symmetric bodies.

**Daniel Hug**, daniel.hug@math.uka.de

### **Random polytopes: geometric and analytic aspects**

(joint work with Károly J. Böröczky, Ferenc Fodor, Lars Hoffmann)

Abstract:

The theory of random polytopes has developed substantially in the last decade. Various classical results concerning the convergence of mean values have been generalized, for instance to sharp estimates of higher moments, thus leading to limit theorems. Moreover, several results which were only available in the plane and under strong smoothness and curvature assumptions are understood much better now. This progress is due to new techniques coming from probability and also to new and improved geometric arguments. In the talk, we mainly describe progress on the geometric side. Here quite naturally affine invariant functionals arise. In particular, geometric duality arguments in combination with measure geometric (curvature) transformations are applied for the study of random polyhedral sets.

**Joseph Lehec**, joseph.lehec@univ-mlv.fr

### **A functional approach to the Blaschke-Santaló inequality**

Abstract:

We reprove various functional Santaló inequalities, due to Keith Ball, to Artstein, Klartag and Milman and to Fradelizi and Meyer. Unlike the original ones, our proofs are direct, in the sense that they do not rely on the classical Blaschke-Santaló inequality.

**Alexander Litvak**, alexandr@math.ualberta.ca

### **On vectors uniformly distributed over a convex body**

(joint work with R. Adamczak, A. Pajor, N. Tomczak-Jaegermann)

Abstract:

We discuss recent results on the topic.

Mark Meckes, mwm2@case.edu

### **On the measure-theoretic analogue of Dvoretzky's theorem**

Abstract:

Recently Klartag showed that any convex body has high-dimensional marginals (projections of its uniform measure) which are nearly Gaussian. This result can be interpreted as a measure-theoretic analogue of Dvoretzky's theorem, and as in Dvoretzky's theorem the existence of nice projections is proved nonconstructively.

In contrast to this and to the situation for Dvoretzky's theorem, we will show that for many convex bodies it is possible to identify specific nearly Gaussian marginals.

Mathieu Meyer, mathieu.meyer@univ-mlv.fr

### **Once again on Mahler's problem**

Abstract:

TBA

Efrén Morales Amaya, efren@cimat.mx

### **Characterization of Ellipsoids by Means of Parallel Translated Sections**

(joint work with David Larman and Luis Montejano)

Abstract:

Let  $K$  be a convex body and let  $p_0 \in \text{int } K$ . Suppose that in every direction we can choose continuously a section of  $K$  which is a translated copy of the corresponding parallel section of  $K$  through  $p_0$ . Our main result essentially claims that if all these pairs of sections are different, almost everywhere, then  $K$  is an ellipsoid

Carlo Nitsch, c.nitsch@unina.it

### **Affine isoperimetric inequalities and Monge-Ampère equations**

Abstract:

We show that as well as the classical isoperimetric inequalities, affine isoperimetric inequalities can be used to obtain a priori estimates for solutions to elliptic problems. In particular we show that the eigenvalue of the Dirichlet problem for the Monge-Ampère operator, when computed on convex domains with fixed measure, is maximal on ellipsoids. This result is established by exploiting the affine invariant structure of such operator using either Blaschke-Santaló or Petty inequalities.

**Peter J. Olver**, olver@ima.umn.edu

## Moving Frames, Differential Invariants and Surface Geometry

Abstract:

I will begin with a brief discussion of the classical theory of differential invariants, followed by a survey, for the non-expert, of the new, equivariant approach to the method of moving frames that has been extensively developed with various collaborators over the past decade. The moving frame approach can be applied to arbitrary group actions, and algorithmically reveals the full structure of the associated algebra of differential invariants, pinpointing generating differential invariants as well as their differential syzygies. I will then present some surprising new results on differential invariant algebras in classical surface geometries, including equi-affine surfaces. I will close by discussing applications to equivalence problems, symmetry detection, image processing, and, time permitting, invariant surface flows.

**Fedor Petrov**, fedyapetrov@gmail.com

## Affine surface area and rational points on convex surfaces

Abstract:

Let  $\Phi \subset \mathbb{R}^d$  be a convex body. We study the asymptotic estimates on the number of integer points, lying on  $n\partial\Phi$  for large  $n$ , the number of integer points in a small neighborhood of  $n\partial\Phi$ , the number of convex polytopes with integer vertices, almost inscribed in  $n\Phi$  etc. It appears that the estimate involve affine surface area of  $\Phi$  and possible further results concerning the behavior of lattice polytopes are essentially equivalent to establishing that some values, defined via covering of the boundary by small convex sets, are proportional to affine area of the boundary.

**Matthias Reitzner**, matthias.reitzner@tuwien.ac.at

## Classification of $SL(n)$ invariant Valuations

(joint work with Monika Ludwig)

Abstract:

Let  $\mathcal{K}^n$  denote the space of convex bodies in  $\mathbb{R}^n$ . A functional  $\Phi : \mathcal{K}^n \rightarrow \mathbb{R}$  that satisfies the inclusion-exclusion relation

$$\Phi(K) + \Phi(L) = \Phi(K \cup L) + \Phi(K \cap L)$$

whenever  $K, L, K \cup L, K \cap L \in \mathcal{K}^n$ , is called a (real-valued) *valuation*. A valuation is  $SL(n)$  invariant, if  $\Phi(AK) = \Phi(K)$  for every  $A \in SL(n)$  holds.

A classification of upper semicontinuous and  $SL(n)$  invariant valuations is established. As a consequence, characterizations of centro-affine and  $L_p$  affine surface areas are obtained.

**E. Saorín Gómez**, esaorin@um.es

### Approaching $K$ by its form body and kernel

(joint work with M. A. Hernández Cifre)

Abstract:

For two convex bodies  $K, E \subset \mathbb{R}^n$  with interior points, the *inner parallel body of  $K$  (relative to  $E$ ) at distance  $\lambda$* , for  $0 \leq \lambda \leq r(K; E)$ , is the set

$$K_{-\lambda} = \{x \in \mathbb{R}^n : x + \lambda E \subseteq K\},$$

where  $r := r(K; E) = \max\{r > 0 : \exists x \in \mathbb{R}^n \text{ with } x + rE \subseteq K\}$  denotes the *relative inradius* of  $K$ . The *kernel* of  $K$ , i.e., the set of its incenters, is  $K_{-r}$ .

A vector  $u \in \mathbb{S}^{n-1}$  is a *0-extreme normal vector* of  $K$  if we cannot write it as the sum of 2 linearly independent normal vectors at one and the same boundary point of  $K$ . The set of 0-extreme normal vectors of  $K$  is denoted by  $\mathcal{U}_0(K)$ . Then, the (relative) *form body* of  $K$  with respect to  $E$ , denoted by  $K^*$ , is defined as

$$K^* = \bigcap_{u \in \mathcal{U}_0(K)} \{x : \langle x, u \rangle \leq h(E, u)\}.$$

The trivial inclusion  $K_{-\lambda} + \lambda E \subseteq K$  was improved in 1978 by Sangwine-Yager, who proved that  $K_{-\lambda} + \lambda K^* \subseteq K$  for every  $\lambda \in [0, r]$ . We characterize the equality case by proving that, for any  $\lambda \in [0, r]$ ,  $K = K_{-\lambda} + \lambda K^*$  if and only if  $K$  is a so-called tangential body of  $K_{-r} + rE$  satisfying an additional condition on 0-extreme vectors; we will see that the control of the 0-extreme vectors of the bodies involved in the inequality is crucial. We also use this result in order to get optimal bounds on the quermassintegrals of the inner parallel body  $K_{-\lambda}$  in terms of magnitudes of the original body and its form body  $K^*$ .

**Rolf Schneider**, rolf.schneider@math.uni-freiburg.de

### The role of the volume product in stochastic geometry

Abstract:

TBA

**Franz Schuster**, schuster@geometrie.tuwien.ac.at

### Valuations and Affine Sobolev Inequalities

(joint work with Christoph Haberl)

Abstract:

In this talk, I will present a new sharp affine  $L_p$  Sobolev inequality for functions on  $\mathbb{R}^n$ . This inequality strengthens and directly implies the previously known sharp affine  $L_p$  Sobolev inequality of Lutwak, Yang, and Zhang which in turn is stronger than the classical  $L_p$  Sobolev inequality of Aubin and Talenti. The geometry behind our new inequality is an  $L_p$  affine isoperimetric inequality which is based on recent advances in valuation theory by Ludwig.

**Christian Steineder**, christian.steineder@tuwien.ac.at

### **The volume of the projection body and binary coding sequences**

Abstract:

Let  $P \subset [0, 1)^d$  be a convex polytope.  $P$  can be interpreted as a subset of the  $d$ -dimensional torus group,  $\mathbb{T}^d = (\mathbb{R}/\mathbb{Z})^d$  in a natural way. Let  $T : \mathbb{T}^d \rightarrow \mathbb{T}^d$ ,  $T(x) = x + \vec{\alpha}$  be the standard ergodic group translation on  $\mathbb{T}^d$  related to the classical sequence  $n\vec{\alpha} \pmod{1}$ .  $T$  and  $P$  yield the binary coding sequence  $\mathbf{h} \in \{0, 1\}^{\mathbb{Z}}$ ,  $h_k = 1$  iff  $T^k(0) \in P$ ,  $k \in \mathbb{Z}$ . It turns out that the volume of the projection body of  $P$  can directly be computed from the coding sequence  $\mathbf{h}$ .

**Nicole Tomczak-Jaegermann**, nicole@ellpspace.math.ualberta.ca

### **Random points uniformly distributed on an isotropic convex body**

Abstract:

We review the recent progress in the area.

**Vlad Yaskin**, vlad.yaskin@math.ualberta.ca

### **On strict inclusions in hierarchies of convex bodies**

Abstract:

Let  $\mathcal{I}_k$  be the class of convex  $k$ -intersection bodies in  $\mathbb{R}^n$  (in the sense of Koldobsky) and  $\mathcal{I}_k^m$  be the class of convex bodies all of whose  $m$ -dimensional central sections are  $k$ -intersection bodies. We show that 1)  $\mathcal{I}_k^m \not\subset \mathcal{I}_k^{m+1}$ ,  $k + 3 \leq m < n$ , and 2)  $\mathcal{I}_l \not\subset \mathcal{I}_k$ ,  $1 \leq k < l < n - 3$ .

**Marina Yaskina**, yaskinam@gmail.com

### **Non-symmetric convex bodies and the Fourier transform**

Abstract:

We derive formulas for the Fourier transform of homogeneous functions which are not necessarily even. We apply these formulas to the problem of unique determination of convex bodies. We also develop a new approach to Christoffel's problem.

**Marisa Zymonopoulou**, marisa@cwru.edu

### **New examples of non-intersection bodies**

Abstract:

TBA