Random Fields and Stochastic Geometry

Feb 22 - Feb 27, 2009

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any food and beverage in those areas.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00
17:30–19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall.
	Beverages and small assortment of snacks available on a cash honour system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and welcome to BIRS by BIRS Station Manager.
9:00–09:50	Taylor, *Integral geometry of random sets: the Gaussian Kinematic Formula*
9:50–10:40	Charmandy, *Saddlepoint approximations for asymptotically Gaussian random fields*
10:40–11:00	Coffee Break
11:00–11:50	Azïais, *Some applications of an implicit formula for the maximum of a Gaussian random field*
12:00–13:30	Lunch
13:30–14:30	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
15:00–15:30	Coffee
15:30–17:30	Short lectures: Nardi, Renaud, Sun, Vadlamani, Wschebor, Xiao
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:50	Schwartzman, *Inference for eigenvalues and eigenvectors of Gaussian symmetric matrices*
9:50–10:40	Marinucci, *Challenges in the analysis of cosmic microwave background radiation*
10:40–11:00	Coffee Break
11:00–11:50	Worsley (by Taylor), *The statistical analysis of fMRI data*
12:00–13:30	Lunch
13:30–15:00	Discussion sessions
15:00–15:30	Coffee
15:30–17:30	Discussion sessions
17:30–19:30	Dinner

Wednesday

7:00–9:00	Breakfast
9:00–9:50	Schneider, *Poisson hyperplanes*
9:50–10:40	Vitale, *Gaussian processes and convex bodies*
10:40–11:00	Coffee Break
11:00–11:50	Samorodnistky, *Geometric characteristics of the excursion sets over high levels of non-Gaussian infinitely divisible random fields*
11:55	Group Photo; meet on the front steps of Corbett Hall
12:00–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–9:50	Zelditch, *Large deviations for zeros of random polynomials and generalizations to Riemann surfaces*
9:50–10:40	Dennis, *Complex nodes in Gaussian random waves: Optics, quantum waves and cosmology*
10:40–11:00	Coffee Break
11:00–11:50	Wigman, *Nodal lines for random eigenfunctions of the Laplacian on the torus and the sphere*
12:00–13:30	Lunch
13:30–15:00	Short lectures Discussion sessions
15:00–15:30	Coffee
15:30–17:50	Short lectures: Hug, Khoshnevisan, Kuriki, Takemura, Teguia, Weil, Younes
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00–9:50	Buergisser, *Stochastic analysis of numerical algorithms*
9:50–10:40	Joshi, *Statistics of shape: Simple statistics on interesting spaces*
10:40–11:00	Coffee Break
11:00–11:50	Spodarev, *Efficient simulation of stable random fields*
12:00–13:30	Lunch

Checkout by 12 noon.

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TITLES OF AFTERNOON TALKS

Hug, *Steiner formula and contact distributions.*

Khoshnevisan, *TBA*

Kuriki, *Multiplicity adjustments in detecting reproductive barriers caused by loci interactions.*

Nardi, *Maxima of empirical (asymptotically Gaussian) random fields*

Renaud, *Correlated random field: the application to detection of features in electro-encephalogram (EEG) data*

Sun, *Joint behavior of average and supremum of a Gaussian random field - challenges and applications to joint control charts.*

Takemura, *Properties of Ehrhart quasi-polynomials for hyperplane arrangements with integral coefficients.*

Teguia, *Random fields and stochastic microlensing.*

Vadlamani, *Gaussian Minkowski functionals in infinite dimensional Gaussian spaces.*

Weil, *Boolean models and translative integral formulas.*

Wschebor, *Random systems of polynomial equations and Rice formula.*

Xiao, *Some fractal properties of Gaussian random fields.*

Younes, *Applications of parallel transport on Riemannian manifolds of shapes for the analysis of anatomical time series.*

TITLES AND ABSTRACTS OF MORNING TALKS

Azaïs, Jean-Marc
Some applications of an implicit formula for the maximum of a Gaussian random field

Looking, among the local maxima of a random field, to those that are act ually a global maxima, we obtain an implicit formula for the density of the maximum of a regular process on a regular set.

This formula can be used in two directions.

One is to study the regularity of the density. When the parameter is one-dimensional an induction proves that under certain conditions the distribution can very regular. When the parameter is multidimensi onal results are more limited but in both cases we obtain result that g o beyond the classical results of Tsirelson.

The other direction is to give bounds bounds to the density to obtain non-asymptotic bounds for the distribution of the maximum as well as expansion to the second order.

Buergisser, Peter
Stochastic analysis of numerical algorithms

Iterative numerical algorithms are often easy to state, but difficult to analyze on random input data. Important examples include the simplex method, interior-point methods of linear and convex optimization, as well as homotopy methods for solving systems of polynomial equations. The probabilistic analysis of such algorithms leads to fascinating problems of stochastic geometry, that can be systematically studied with tools from integral geometry. A key concept here is the notion of condition number, which has a clear geometric meaning. Besides in algorithmics, the above work has also triggered new investigations on the zero sets of systems of random polynomials and has led to developments of independent mathematical interest, e.g. on the curvatures and Euler characteristic of random (semi-) algebraic sets.

Chamandy, Nick

Saddlepoint approximations for asymptotically Gaussian random fields, with applications

Local increases in the 'activation' of a random field are detected (conservatively) by thresholding an image of test statistics at a level u chosen to control the tail probability or p-value of its supremum. This probability is commonly approximated by the expected Euler characteristic (EC) of the excursion set above u, a functional which has been computed for a number of processes, mainly those derived from (multivariate) Gaussian fields. I will argue that the Gaussian assumption is untenable in many applications, and present a correction to the expected EC formula involving a saddlepoint approximation.

For asymptotically Gaussian fields of a particular type, and when the threshold is allowed to grow with the sample size, this 'tilted' expression has a smaller relative asymptotic error than the Gaussian one. Practical implications will be discussed, and inference under the Gaussian and tilted regimes will be contrasted using real data and simulation. Finally, I will allude to multivariate extensions of the tilted theory. The applications considered originate from brain mapping, astronomy, cognitive psychology and web search eye-tracking.

Dennis, Mark

Complex nodes in Gaussian random waves: Optics, quantum waves and cosmology

Gaussian random fields are a natural model of randomly interfering wave fields in physics, and thus are an important tool in, for example, statistical optics, quantum chaos, and cosmology. Geometric and topological aspects of these random fields – such as the configuration of their nodal sets – provides a handle to understand the morphology of these complicated physical fields. I will describe analytic and numerical calculations of the distribution and correlations of nodes in Gaussian random functions modelling wave fields from the following three physical examples:

1: Quantum chaos: The complex quantum waves in a 2-dimensional chaotic enclosure, modelled by a gaussian random solution of the Helmholtz equation, the nodal points in this case are vortices of the probability current flow.

2: Cosmology: The coherent state representation of the cosmic microwave temperature fluctuations. The random fields reduce to random polynomials with roots on the Riemann sphere correspond to multipole vectors.

3: Statistical optics: Optical speckle patterns occur as the mottled pattern in laser light reflected from a rough surface, and are modelled by complex gaussian random waves propagating in three-dimensional space. The nodes are a dense tangle of lines in three dimensions, which has fractal and scaling properties revealed by numerical calculations.

Joshi, Sarang

Statistics of shape: Simple statistics on interesting spaces.

A primary goal of Computational Anatomy is the statistical analysis of anatomical variability. A natural question that arises is how dose one define the image of an "Average Anatomy". Such an "average" must represent the intrinsic geometric anatomical variability present. Large Deformation Diffeomorphic transformations have been shown to accommodate the geometric variability but performing statistics of diffeomorphic transformations remains a challenge. In this lecture I will further extend this notion of averaging for studying change of anatomy on average from a cross sectional study of growth.

Regression analysis is a powerful tool for the study of changes in a dependent variable as a function of an independent regressor variable, and in particular it is applicable to the study of anatomical growth and shape change. When the underlying process can be modeled by parameters in a Euclidean space, classical regression techniques are applicable and have been studied extensively. However, recent work suggests that attempts to describe anatomical shapes using *flat Euclidean spaces* undermines our ability to represent natural biological variability. In this lecture I will develop a method for regression analysis of general, manifold-valued data. Furthermore I will extend the notion of robust estimation to manifold valued data. The median is a classic robust estimator of centrality for data. In this lecture I will formulate the geometric median of data on a Riemannian manifold as the minimizer of the sum of geodesic distances to the data points. I will exemplify the robustness of the estimation technique by applying the procedure to various manifolds commonly used in the analysis of medical images. Using this approach, we also present a robust brain atlas estimation technique based on the geometric median in the space of deformable images.

Marinucci, Domenico

Statistics and probability challenges in the analysis of cosmic microwave background radiation

Cosmic Microwave Background radiation (hereafter CMB) can be viewed as a relic radiation providing a snapshot of the Universe approximately 13.7 billion years ago. Although CMB was detected in the mid sixties, it was only very recently that the first full-sky maps of observations were produced, leading to the 2006 Nobel prize for Physics to Smoot and Mather. In the last few years, the analysis of CMB data from satellite missions has attracted an enormous amount of attention from theoretical and experimental physicists; indeed CMB is considered a goldmine of information on a number of fundamental issues in Theoretical Physics and Cosmology, such as the large scale structure of the Universe, models for the Big Bang dynamics, the existence and nature of dark matter and dark energy, and several others.

From the mathematical point of view, CMB can be viewed as the realization of a mean-square continuous and isotropic random field on the sphere. The aim of this talk is to survey several probability and statistical issues which arise in connection with CMB; most of these problems are still open for research. We shall focus in particular on the characterizations of Fourier coefficients under isotropy and their relationships with group representation theory; on high resolution asymptotics for Gaussian and Gaussian subordinated random fields; on the construction of wavelets and tight frames systems for spherical random fields; and on probability and statistical issues relating to tensor, rather than scalar-valued, random fields. The latter issue is gaining particular importance, in connection with so-called CMB polarization data.

These lines of research have been developed in joint works with P.Baldi, D.Geller, G.Kerkyacharian, Xiaohong Lan, G. Peccati and D.Picard.

Samorodnitsky, Gennady

Geometric characteristics of the excursion sets over higl levels of non-Gaussian infinitely divisible random fields

We consider smooth infinitely disivible random fields of the type $\big(X(t),\, t \in [-1,1]^d\big)$, with regularly varying Lévy measure. For such random fields we are interested in the geometric characteristics of the excursion sets

$$A_u = \big\{t \in [-1,1]^d : \, X(t) > u\big\}$$

over high level u.

For a large class of random fields we compute the asymmtotic (as $u \to \infty$), conditional on A_u being non-empty) joint distribution of the of the numbers of critical points over the level u of all types. This allows us, for example, to obtain the asymptotic conditional distribution of the Euler characteristic of the excursion set.

In a significant departure from the Gaussian situation, the excursion set over a high level for smooth random fields we are considering, can have complicated geometry. In the Gaussian case the excursion set, unless it is empty, is nearly certain to be "a ball-like" and have its Euler characteristic equal to one. In contrast, the Euler characteristic of the excursion sets in our model can have a highly non-degenerate conditional distribution.

Joint work with R. Adler and J. Taylor.

Schneider, Rolf

Poisson hyperplanes

Beginning with the early work of R. Miles and G. Matheron, Poisson processes in the space of hyperplanes in d-dimensional Euclidean space have been a prominent subject of stochastic geometry. After a brief introduction to this topic, we describe some results that have been obtained in recent years. These concern the tessellations generated by Poisson hyperplane processes and, in particular in the homogeneous case, their 'average cells' and k-dimensional faces. Various types of average faces can be considered, distinguished by different weightings, and formally defined with the help of suitable Palm distributions.

In a first class of problems, we establish sharp bounds for the expected vertex numbers of weighted typical cells. The extremal cases characterize particularly simple hyperplane processes, like isotropic ones or those with only d directions of the hyperplanes.

A second class of problems deals with the phenomenon that average cells of large size (with different interpretations) may approximate certain definite shapes, with high probability. There are also results of this type for Voronoi and Delaunay mosaics.

Schwartzman, Armin

Inference for eigenvalues and eigenvectors of Gaussian symmetric matrices

This work presents maximum likelihood estimators (MLEs) and log-likelihood ratio (LLR) tests for the eigenvalues and eigenvectors of Gaussian random symmetric matrices of arbitrary dimension, where the observations are independent repeated samples from one or two populations. These inference problems are relevant in the analysis of Diffusion Tensor Imaging data, where the observations are 3-by-3 symmetric positive definite matrices. The parameter sets involved in the inference problems for eigenvalues and eigenvectors are subsets of Euclidean space that are either affine subspaces, embedded submanifolds that are invariant under orthogonal transformations or polyhedral convex cones. We show that for a class of sets that includes the ones considered here, the MLEs of the mean parameter do not depend on the covariance parameters if and only if the covariance structure is orthogonally invariant. Closed-form expressions for the MLEs and the associated LLRs are derived for this covariance structure.

Spodarev, Evgeny

Efficient simulation of stable random fields

In this talk, an overview of the simulation methods for Gaussian, alpha- and max–stable random fields will be given. New efficient algorithms for the simulation of alpha-stable random fields will be discussed together with their error bounds and performance issues.

Joint work with Wolfgang Karcher and Hans-Peter Scheffler.

Taylor, Jonathan

Integral geometry of random sets: the Gaussian Kinematic Formula

In various scientific fields from astrophysics to neuroimaging, researchers observe entire images or functions rather than single observations. The integral geometric properties, notably the Euler characteristic of the level/excursion sets of these functions, typically modelled as Gaussian random fields, have found some interesting applications in these domains. In this talk, I will describe some of the integral geometric properties of these random sets, particularly their Lipschitz-Killing curvature measures. I will focus on describing the results for a class of non-Gaussian random fields (built up of Gaussians) which highlights the relation between their Lipschitz-Killing curvature measures and the classical Kinematic Fundamental Formulae of integral geometry.

Vitale, Rick

Gaussian processes and convex bodies

As discussed in the Objectives of the Workshop, probability and geometry have had a long and fruitful interaction. Here we survey some connections between two specific areas. Among the topics will be GB/GC classification and Ito-Nisio singularities from a geometric viewpoint, Gaussian representation of intrinsic volumes, Gaussian samples and random projection of regular simplices, the Wills functional in a Gaussian context, deviation inequalities, and metrization of convex bodies.

Worsley, Keith

The statistical analysis of fMRI data

(While Keith Worsley will not be able to attend the workshop due to illness, we felt that this talk, which was meant to describe some of the flavor of the applications of random fields in neuroimaging, was too important to be left out of the program. A talk similar to Keith's proposed talk (though not giving justice to his most recent work, SurfStat) will be delivered by Jonathan Taylor from Keith's slides.)

We propose a method for the statistical analysis of fMRI data that seeks a compromise between efficiency, generality, validity, simplicity and execution speed. The main differences between this analysis and previous ones are: a simple bias reduction and regularization for voxel-wise autoregressive model parameters; the combination of effects and their estimated standard deviations across different scans/sessions/subjects via a hierarchical random effects analysis using the EM algorithm; overcoming the problem of a small number of scans/session/subjects using a regularized variance ratio to increase the degrees of freedom.

Wigman, Igor

Nodal lines for random eigenfunctions of the Laplacian on the torus and the sphere.

We are interested in the length of nodal lines for eigenfunctions of the Laplacian corresponding to large eigenvalues. In case of the torus or the sphere, the eigenspaces are highly degenerate, so that we may endow the eigenspaces with the Gaussian probability measure. We study the distribution of the length of nodal lines of random eigenfunction in the corresponding ensemble.

First, using a standard technique, we compute an exact expression for the expected value of the length. Our main result concerns the variance.

This work is joint with Zeev Rudnick. Time permitting, I will also mention recent results with John Toth and Andrew Granville.

Zelditch, Steve

Large deviations for zeros of random polynomials and generalizations to Riemann surfaces

We define inner products on the space P_N of polynomials of degree N using a weight $e^{-N\phi}$ and a positive measure ν on the Riemann sphere. The inner product induces a Gaussian measure on P_N and on the possible configurations of N zeros. We encode the configurations by the empirical measure of zeros. As $N \to \infty$, these measures tend almost surely to the weighted equilibrium measure for (ϕ, ν). Moreover, they satisfy a large deviations principle with a good rate functional. The result generalizes to any compact Riemann surface.

Joint work with Ofer Zeitouni.

List of Participants

Adler, Robert	Technion - Israel Institute of Technology
Anderes, Ethan	University of California, Berkeley
Azais, Jean-Marc	Toulouse
Bartz, Kevin	Harvard University
Buergisser, Peter	Univerity of Paderborn
Carbonell, Felix	McGill University
Chamandy, Nicholas	Google
Dennis, Mark	University of Bristol
Farzan, Rohani	McGill University
Feldman, Raya	University of California at Santa Barbara
Ghosh, Souvik	Cornell University
Hug, Daniel	Universitat Duisburg-Essen
Jakobson, Dmitry	McGill University
Joshi, Sarang	University of Utah
Khoshnevisan, Davar	University of Utah
Kou, Samuel	Harvard
Kuriki, Satoshi	Institute of Statistical Mathematics
Marinucci, Domenico	University of Rome
Mazza, Christian	Universite de Fribourg
Nardi, Yuval	Carnegie Mellon University
Ninomiya, Yoshiyuki	Kyushu University
Renaud, Olivier	Université de Genève
Rootzen, Holger	Chalmers Institute of Technology
Samorodnitsky, Gennady	Cornell University
Schneider, Rolf	University of Freiburg
Schwartzman, Armin	Harvard School of Public Health
Siegmund, David	Stanford University
Spodarev, Evgeny	Ulm University
Sun, Jiayang	Case Western University
Takemura, Akimichi	University of Tokyo
Taylor, Jonathan	Stanford University
Teguia, Alberto.	Duke University
Tecuapetla, Gomez	Cornell University
Vadlamani, Sreekar	Stanford University
Vitale, Rick	University of Connecticut
Weil, Wolfgang	Universitaet Karlsruhe
Wigman, Igor	CRM, Universite de Montreal
Wilson, Richard	University of Queensland
Wschebor, Mario	Universidad de la Republica Uruguay
Xiao, Yimin	Michigan State University
Younes, Laurent	John Hopkins University
Zelditch, Steve	John Hopkins University