Black Holes: Theoretical, Mathematical and Computational aspects, 08w5033

Scientific Report

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1 Introduction

During the workshop there were discussed several important subjects concerning physical and mathematical aspects of the black hole theory. They include the following topics

- Higher dimensional gravity and black holes
- Geometrical properties of higher dimensional isolated black holes
- Black holes in Randall-Sundrum models
- Black-hole–black-string transitions
- Mini black creation in the high energy collider experiments
- Generation of new solutions of higher dimensional Einstein-Maxwell equations
- Viscosity bound and AdS/CFT correspondence
- Primordial black hole formation after inflation
- Black holes in tidal environments
- New exact solutions of the higher dimensional gravity

This report contains a brief summary of the problems discussed at the workshop. It is focused on the recent developments reported at the talks, and the questions which attract main attention in the discussions.

2 Introduction: Black Holes and Extra Dimensions

Spacetime in physics is modeled by a differential manifold with a non-degenerate metric with the signature $(-,+,+,+)$ on it. The metric, describing the gravitational field, is a solution of the Einstein equations or their modifications. The metric specifies an interval between two close events. It also defines local null cones which determine the causal structure of the spacetime. A black hole is a spacetime region from where no information carrying signals can escape to infinity. Existence of a black hole indicates that the global causal structure of the spacetime manifold is non-trivial. The theory of black holes in the four dimensional case has been developed and main features of the 4D black holes are well understood now. The astrophysical observations gives very strong evidences of the existence of stellar mass and massive black holes. Rapid increase of the of accuracy of the observations poses new interesting problems concerning black holes, their interaction, and behavior of the matter in their vicinity. Several talks at the workshop discuss some of these problems.

The subject of black holes is now attracting a lot of attention also in the connection with the hypothesis of the existence of extra dimensions. The idea that the spacetime can have more than four dimensions is very old. Kaluza and Klein used this idea about 80 years ago in their attempts to unify electromagnetism with gravity. The modern superstring theory is consistent (free of conformal anomalies) only if the spacetime has a fixed number (10) of dimensions. It is assumed that extra dimensions are compactified. The natural size of compactification in the string theory is of order of the Planckian scale. In recently proposed models with large extra-dimensions it is also assumed that the spacetime has more than 3 spatial dimensions. New feature is that the size of the extra dimensions (up to 0.1mm) can be much larger than the Planckian size, 10^{-33}cm. In order to escape contradictions with observations it is usually assumed that the matter and fields (except the gravitational one) are confined to a four-dimensional brane representing our world, while the gravity can propagate in the bulk. In so called ADD models [1, 2] these extra dimensions are flat. In the Randall-Sundrum type of models [3, 4] the bulk 5D spacetime is curved and it has anti-deSitter asymptotics. Black holes in the string theory and in the models with larger extra dimensions play an important role serving as probes of extra dimension. Study of higher dimensional black holes is a very important problem of the modern theoretical and mathematical physics. Several talks and a lot of discussions during the workshop were devoted to these problems.

3 Geometrical properties of higher dimensional isolated black holes

An isolated stationary black hole in 4-dimensional asymptotically flat spacetime is uniquely specified by two parameters, its mass and angular momentum. The corresponding Kerr metric possesses a number of properties, which was called by Chandrasekhar 'miraculous'. In particular the Kerr metric allows the separation of variables in the geodesic Hamilton-Jacobi equation and a massless field equations. These properties look 'miraculous' since the spacetime symmetries of the Kerr metric are not sufficient to explain them. It was shown by Carter [5] that these properties of the Kerr metric are directly connected with the existence of 'hidden symmetries' generated by the so-called Killing and Killing-Yano tensors. Separability of massless field equations (including the gravitational perturbations) in the Kerr metric plays a key role in study of properties of rotating black holes, including the proof of their stability and the calculation of Hawking radiation.

Recently higher dimensional analogues of the Kerr black holes attracted a lot of attention. The most general solution of the higher dimensional Einstein equations describing higher dimensional rotating black holes was obtained in 2006 by Chen, Lu and Pope [6]. During the past 2 years an important break-though in study of the properties of these solutions has been made(see e.g. [7] and references therein).

Namely, it was demonstrated that the most general higher dimensional black hole solution has many properties which are similar to the properties of the 4 dimensional Kerr metric. It was shown that the main role in the explanation of these properties is played by hidden symmetries. Besides the evident spacetime symmetries, connected with integrals of motion of the first order in momentum, higher-dimensional Kerr-NUT-(A)dS metrics possess also the hidden symmetries, connected with the conserved quantities of higher than first order in momentum. The main object in this study is a non-degenerate, rank 2, closed, conformal Killing-Yano tensor, which was called a principal conformal Killing-Yano (PCKY) tensor. It is possible to show that an external product of two closed conformal Killing-Yano tensors (CCKYT) is again a CCKYT, so that these objects form an algebra. Using this property it is possible to construct a tower of CCKYTs, which in its turn, generates a tower of Killing tensors. Moreover the PCKY tensor generates also a tower of the Killing vectors.

As a result, a spacetime which admits the PCKY tensor possesses the following properties: (1) Geodesics equations are completely integrable; (2) The Hamilton-Jacobi, Klein-Gordon, and Dirac equations are separable; (3) Stationary string equations are completely integrable; (4) the equations for the parallel transport of frames along geodesics in these spacetimes are integrable and can be reduced to a set of the first order ODE which allow a separation of variables. Moreover it was demonstrated that the most general solution of the Einstein equations with the cosmological constant which possesses the PCKY tensor is the Kerr-NUT-(A)dS metric. These results were presented during the workshop by scientists from the University of Alberta. Discussion of these results focused on the possibility to extend these results to the case of Einstein-Maxwell equations and a possibility of the decoupling and the variable separation in the Maxwell equations and the equations for the gravitational perturbations in the background of the spacetimes which allow the PCKY tensor. Another open problem, which was discussed during the workshop is: How the hidden symmetries become real. This question is connected with the study of the degeneracies of the PCKY tensor. Some interesting results in this direction were obtained recently.

4 Black holes in Randall-Sundrum models

The problem of existence of black holes in Randall-Sundrum models [3, 4] has been discussed in several publications, but it still attracts a lot of attention. The reason is that the obtained analytical results are based on some non-confirmed assumptions. In such a situation an important role is played by the numerical simulations. The problem can be formulated as a set of 2D non-linear elliptic equations with singular boundary conditions (regularity of a solution at the Anti-deSitter infinity). Namely the latter boundary condition creates a lot of problems in, both the analytical and numerical, approaches. In the talk by Hirotaka Yoshino there were given arguments, based on an accurate 4th order simulations, in favor of a conjecture of non-existence of the (large) black hole solutions in the Randall-Sundrum type II models. This talk generated a lot of discussions. Many arguments 'pro' and 'contra' of this conclusion were proposed. The discussion of the black hole problem in the Randall-Sundrum models continued during all 5 days of the workshop. This is certainly an interesting direction for the future study, which was singled out during the workshop.

5 Black-hole–black-string transitions

In the ADD models with large extra dimensions, the periodicity conditions are imposed on the extra dimensions. Under these conditions the black hole problem gets 'new dimensions'. Namely, besides 'standard' black holes with the spherical topology of the horizon S^{4+k} (k is the number of extra dimensions) there exist solutions of the Einstein equations, called black strings, with the topology of the horizon $S^2 \times T^k$. Thus there exist 2 different 'phases'. One can expect that when the dimensionless ratio $\alpha = r_0/L$ of the black hole size, r_0, to the size L of extra

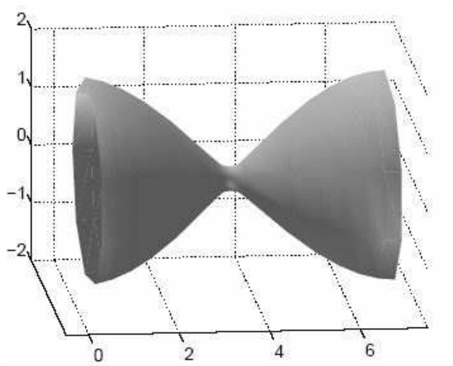

Figure 1: Non-uniform string (From Kol's talk at HDG, Bremen,2008)

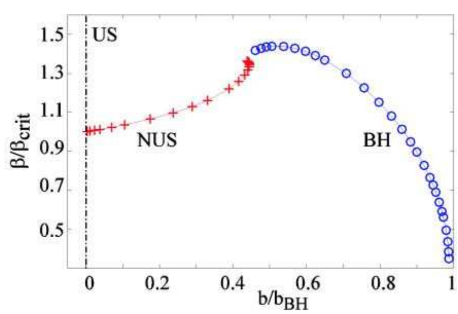

Figure 2: Phase diagram for a BH-BS system (From Kudo-Wiseman, 2004)

dimensions, changes there is a phase transition between black hole and black string phases. During this phase transition the Euclidean topology of the solution changes. Such transitions have been investigated for 15 years, after Gregory and LaFlamme [8] demonstrated the existence of classical instability in the solutions of the Einstein equations with compactified extra dimensions, which develops at the special critical values of the control (order) parameter α. Figure 2 schematically shows a non-uniform string which arises when the order parameter approaches its critical value. Figure 2 show the the tension parameter as a function of the order parameter.

During the workshop there were presented new results concerning such 'caged' black holes for the case of charged and rotation higher dimensional black holes (mainly in the talk by Kunz [9]). The discussions of this subject during workshop mainly focused on the open problem on the nature and detailed description of the BH-BS phase transitions.

6 Mini black creation in the high energy collider experiments

One of the main features of the popular now models with large extra dimensions is a prediction that gravity becomes strong at small distances. This conclusion implied that for the particle collision with the energy of the order of TeV the gravitational channel would be as important as the electroweak channel of the interaction. Under these conditions two qualitatively new effects are possible: (1) bulk emission of the gravitons, and (2) mini black hole production. These effects have been widely discussed in the connection with the expected new data at the Large Hadronic Collider (LHC) (see e.g. [10] and references therein).

At the workshop there was given a detailed review of the possible expected experimental consequences of the models with large extra dimensions for high energy particle collision at LHC. The talks cover the detailed description of the (higher dimensional) Hawking radiation produced by TeV scale black holes. Another problem, discussed during the workshop, is a numerical simulation of collision of two ultra-relativistic particles with a formation of a larger black hole. In these calculations the initial small black holes are used to model colliding particles. For this approach, the numerical problem becomes similar to the problem of black hole merge which has been recently under study in connection to the astrophysical applications and the LIGO project. Using similar developed tools new results were obtained for the collision of the ultrarelativistic black holes up to the gamma-factor of order of 2. In particular, the cross section of the larger black hole formation was calculated. Unfortunately, this gamma factor is much smaller than the realistic gamma factor in the collider experiments, which for the energy 7TeV is about 7,500. Discussions during the workshop were focused on the possibility to increase the value of the gamma factor, for example by combining the numerical and analytical approaches.

7 Generation of new solutions of higher dimensional Einstein-Maxwell equations

Solving the higher dimensional Einstein equations is a challenge because of their complexity. Nevertheless recently new physically interesting solutions were obtained by applying a method the solution generating transformations. By using the methods similar to the Belinsky-Zakharov back-scattering approach in 4D gravity, black hole solutions with the topology different from S^n were obtained (black rings, black saturns, etc.). At the workshop there was discussed a new solution generating technique in the higher dimensional gravity (Robert Mann [11]). This method allows one to obtain a charged version of the black-ring type solutions. These results looks very promising and generate a lot of discussion during the workshop.

8 Viscosity bound and AdS/CFT correspondence

The anti-de Sitter (AdS) – conformal field theory (CFT) correspondence has yielded striking insights into the dynamics of strongly coupled gauge fields [12]. Recently using this conjecture it was established a universal restrictions on the ratio of the viscosity and entropy density for all gauge fields in the strong coupling limit. The nature of this universality is now widely discussed. This subject was also under discussion of the workshop. Main results reported at the workshop (by Robert Myers) is the demonstration of the modifications of this inequality for a class of the 4D CFTs with the Gauss-Bonnet 5D bulk theory.

9 Primordial black hole formation after inflation

The subject of cosmological primordial black holes has a long history. Such black holes might be produced at the early stages of the evolution of the universe from the initial inhomogeneities of the matter. The analysis performed by Carr [13, 14] demonstrated that corresponding gravitational perturbations must be quite large, so that in the radiation dominated universe the probability of creation of PBHs is very small. During the workshop there was a discussion (generated by the talk of Misao Sasaki) of different models of behavior of the matter after the inflation in which the effect of the PBH production would be important. This subject is very important since possible observation of such primordial black holes might give an important direct information about the properties of the universe at the stage just after the inflation.

10 Black holes in tidal environments

Black holes interacting with surrounding objects are distorted. In particular, the black hole deformation is of interest for the problem of a black-hole–black-hole or black-hole–neutron-star coalescence. In the astrophysical set-up this distortion is small. A natural approach to describe the distortion is to use a perturbation analysis. Two problems make such an analysis complicated: non-linearity of gravity and gauge invariance reflecting the general covariance of the problem. During the workshop there was discussed a new approach to this problem developed by Eric Poisson [15] and his collaborators. A part of this discussion focused on the following problem: Is it possible to extract the information concerning the black hole distortion from the recently performed numerical calculations of the black-hole–black-hole merger.

11 New exact solutions of the higher dimensional gravity

There exists an interesting class of solutions of the higher dimensional vacuum Einstein equations which possesses the property that its all polynomial type invariants constructed from the Riemann tensor and its covariant derivatives vanish. In such spaces local quantum effects of the vacuum polarization vanish. This class of solutions, for example, includes physically interesting solutions, called gyratons, which describe the gravitational field created by the spinning radiation pulse. These solutions, originally obtained in the asymptotically flat spacetime, were later generalized to the spacetime with AdS asymptotics. These solutions have all the polynomial curvature invariants identical to the similar invariants in the AdS spacetime. Possible further generalizations describing the gyraton motion in a more general constant curvature spacetimes were discussed at the workshop (Andrey Zelnikov). An interesting problem is to describe the most general solutions of these type.

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