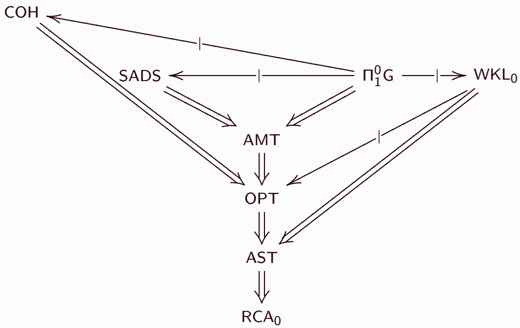
The Atomic Model Theorem and Related Model Theoretic Principles

Denis R. Hirschfeldt — University of Chicago

joint work with Richard A. Shore and Theodore A. Slaman



Computability, Reverse Mathematics and Combinatorics / Banff, December 2008

Conventions and Basic Definitions

All our theories T are countable, complete, and consistent.

All our models $\mathcal{M}$ are countable.

We work in a computable language.

Conventions and Basic Definitions

All our theories T are countable, complete, and consistent.

All our models $\mathcal{M}$ are countable.

We work in a computable language.

T is **decidable** if it is computable.

$\mathcal{M}$ is **decidable** if its elementary diagram is computable.

In reverse mathematics, we identify $\mathcal{M}$ with its elementary diagram.

Conventions and Basic Definitions II

A partial type Γ of T is a set of formulas $\{\psi_n(\vec{x})\}_{n\in\omega}$ consistent with T.

Γ is a (complete) type if it is maximal.

Γ is principal if there is a consistent φ s.t. $\forall\psi \in \Gamma\ (T + \varphi \vdash \psi)$.

$\mathcal{M}$ realizes Γ if $\exists\vec{a} \in \mathcal{M}\ \forall\psi \in \Gamma\ (\mathcal{M} \vDash \psi(\vec{a}))$. Otherwise $\mathcal{M}$ omits Γ.

The type spectrum of $\mathcal{M}$ is the set of types it realizes.

Small Models

T is atomic if every formula consistent with T can be extended to a principal type of T.

$\mathcal{M}$ is atomic if every type it realizes is principal.

$\mathcal{M}$ is prime if it can be elementarily embedded in every model of T.

Small Models

T is atomic if every formula consistent with T can be extended to a principal type of T.

$\mathcal{M}$ is atomic if every type it realizes is principal.

$\mathcal{M}$ is prime if it can be elementarily embedded in every model of T.

Thm.

- Any two atomic models of T are isomorphic.
- $\mathcal{M}$ is atomic iff $\mathcal{M}$ is prime.
- T has an atomic model iff T is atomic.

Small Models and Reverse Mathematics

Thm (HSS). The following are provable in RCA_0.

- If T has an atomic model then T is atomic.
- If $\mathcal{M}$ is prime then $\mathcal{M}$ is atomic.

The following are equivalent to ACA_0 over RCA_0.

- If $\mathcal{M}$ is atomic then $\mathcal{M}$ is prime.
- Any two atomic models of T are isomorphic.
- Every atomic T has a prime model.

The Atomic Model Theorem

Thm. T has an atomic model iff T is atomic.

$\mathsf{RCA}_0 \vdash$ If T has an atomic model then T is atomic.

The Atomic Model Theorem

Thm. T has an atomic model iff T is atomic.

$\mathsf{RCA}_0 \vdash$ If T has an atomic model then T is atomic.

AMT: If T is atomic then T has an atomic model.

The Atomic Model Theorem

Thm. T has an atomic model iff T is atomic.

$\mathsf{RCA}_0 \vdash$ If T has an atomic model then T is atomic.

AMT: If T is atomic then T has an atomic model.

It is easy to check that $\mathsf{ACA}_0 \vdash \mathsf{AMT}$.

In fact, AMT is considerably weaker than ACA_0.

Combinatorial Principles Related to RT^2_2

$\mathbf{RT^2_2}$**:** Let $f : [\mathbb{N}]^2 \to 2$. There is an infinite H s.t. f is constant on $[H]^2$.

Combinatorial Principles Related to RT^2_2

$\mathbf{RT}^2_2$: Let $f : [\mathbb{N}]^2 \to 2$. There is an infinite H s.t. f is constant on $[H]^2$.

$f : [\mathbb{N}]^2 \to 2$ is stable if $\forall m\ (\lim_n f(m,n)$ exists).

$\mathbf{SRT}^2_2$: Let $f : [\mathbb{N}]^2 \to 2$ be stable. There is an infinite H s.t. f is constant on $[H]^2$.

Combinatorial Principles Related to RT^2_2

RT^2_2: Let $f : [\mathbb{N}]^2 \to 2$. There is an infinite H s.t. f is constant on $[H]^2$.

$f : [\mathbb{N}]^2 \to 2$ is stable if $\forall m\ (\lim_n f(m,n)$ exists$)$.

SRT^2_2: Let $f : [\mathbb{N}]^2 \to 2$ be stable. There is an infinite H s.t. f is constant on $[H]^2$.

COH: Let $A_0, A_1, \ldots \subseteq \mathbb{N}$. There is an infinite C s.t.

$$\forall i\ (|C \cap A_i| < \infty \ \vee\ |C \cap \overline{A_i}| < \infty).$$

ADS: Every infinite linear order has an infinite ascending or descending sequence.

Combinatorial Principles Related to RT^2_2 II

ADS: Every infinite linear order has an infinite ascending or descending sequence.

A linear order is stable if every element has either finitely many predecessors or finitely many successors.

SADS: Every infinite stable linear order has an infinite ascending or descending sequence.

Combinatorial Principles Related to RT^2_2 II

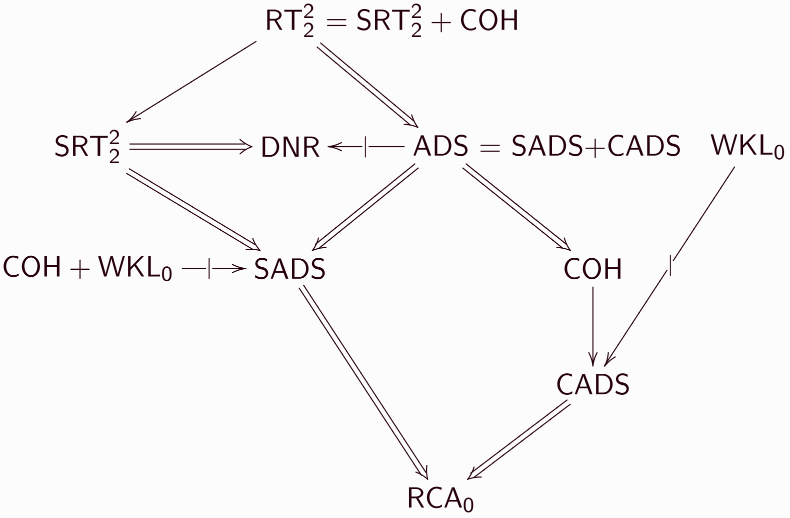
ADS: Every infinite linear order has an infinite ascending or descending sequence.

A linear order is stable if every element has either finitely many predecessors or finitely many successors.

SADS: Every infinite stable linear order has an infinite ascending or descending sequence.

CADS: Every infinite linear order has an infinite stable suborder.

Combinatorial Principles Related to RT^2_2 III



$\Longrightarrow$: not reversible $\longrightarrow$: opposite direction open

The Atomic Model Theorem Revisited

AMT: If T is atomic then T has an atomic model.

Recall that $\mathrm{ACA}_0 \vdash \mathrm{AMT}$.

The Atomic Model Theorem Revisited

AMT: If T is atomic then T has an atomic model.

Recall that $\mathsf{ACA}_0 \vdash \mathsf{AMT}$.

Thm (HSS). $\mathsf{RCA}_0 \vdash \mathsf{SADS} \to \mathsf{AMT}$.

The Atomic Model Theorem and Lowness

Thm (Csima). Every decidable atomic T has a low atomic model.

The Atomic Model Theorem and Lowness

Thm (Csima). Every decidable atomic T has a low atomic model.

By iteration, we can build an ω-model of $\mathsf{RCA}_0 + \mathsf{AMT}$ consisting entirely of low sets.

The Atomic Model Theorem and Lowness

Thm (Csima). Every decidable atomic T has a low atomic model.

By iteration, we can build an ω-model of $\mathsf{RCA}_0 + \mathsf{AMT}$ consisting entirely of low sets.

Thus AMT does not imply any principle that does not have low solutions in general, such as SRT^2_2 or CADS.

The Atomic Model Theorem and Nonlow$_2$ness

A degree $\mathbf{d}$ is atomic bounding if every decidable atomic T has a $\mathbf{d}$-decidable atomic model.

Thm (Csima, Hirschfeldt, Knight, and Soare). A Δ_2^0 degree is atomic bounding iff it is nonlow$_2$.

The Atomic Model Theorem and Nonlow$_2$ness

A degree $\mathbf{d}$ is atomic bounding if every decidable atomic T has a $\mathbf{d}$-decidable atomic model.

Thm (Csima, Hirschfeldt, Knight, and Soare). A Δ^0_2 degree is atomic bounding iff it is nonlow$_2$.

Thus $\mathsf{WKL}_0 \nvdash \mathsf{AMT}$.

The Atomic Model Theorem and Nonlow$_2$ness

A degree $\mathbf{d}$ is atomic bounding if every decidable atomic T has a $\mathbf{d}$-decidable atomic model.

Thm (Csima, Hirschfeldt, Knight, and Soare). A Δ^0_2 degree is atomic bounding iff it is nonlow$_2$.

Thus $\mathsf{WKL}_0 \nvdash \mathsf{AMT}$.

The previous theorems can be combined and iterated to produce an ω-model of $\mathsf{RCA}_0 + \mathsf{AMT}$ whose elements are all computable in a given nonlow$_2$ Δ^0_2 degree.

The Atomic Model Theorem and Nonlow$_2$ness

A degree $\mathbf{d}$ is atomic bounding if every decidable atomic T has a $\mathbf{d}$-decidable atomic model.

Thm (Csima, Hirschfeldt, Knight, and Soare). A Δ^0_2 degree is atomic bounding iff it is nonlow$_2$.

Thus $\mathrm{WKL}_0 \nvdash \mathrm{AMT}$.

The previous theorems can be combined and iterated to produce an ω-model of $\mathrm{RCA}_0 + \mathrm{AMT}$ whose elements are all computable in a given nonlow$_2$ Δ^0_2 degree.

Thus $\mathrm{RCA}_0 + \mathrm{AMT} \nvdash \mathrm{WKL}_0$.

Restricted Π^1_2 Conservativity

Many principles such as WKL, RT^2_2, ADS, etc. can be put into the form

$$\forall A\ (\Theta(A) \rightarrow \exists B\ \Phi(A, B)),$$

where Θ is arithmetic and Φ is Σ^0_3.

Restricted Π^1_2 Conservativity

Many principles such as WKL, RT^2_2, ADS, etc. can be put into the form

$$\forall A\ (\Theta(A) \rightarrow \exists B\ \Phi(A, B)),$$

where Θ is arithmetic and Φ is Σ^0_3.

P is r-Π^1_2 conservative over Q if every sentence of the above form provable from $P + Q$ is provable from Q.

Restricted Π^1_2 Conservativity

Many principles such as WKL, RT^2_2, ADS, etc. can be put into the form

$$\forall A\ (\Theta(A) \rightarrow \exists B\ \Phi(A, B)),$$

where Θ is arithmetic and Φ is Σ^0_3.

P is r-Π^1_2 conservative over Q if every sentence of the above form provable from $P + Q$ is provable from Q.

r-Π^1_2 conservativity implies Π^1_1 conservativity.

Restricted Π^1_2 Conservativity

Many principles such as WKL, RT^2_2, ADS, etc. can be put into the form

$$\forall A\ (\Theta(A) \to \exists B\ \Phi(A, B)),$$

where Θ is arithmetic and Φ is Σ^0_3.

P is r-Π^1_2 conservative over Q if every sentence of the above form provable from $P + Q$ is provable from Q.

r-Π^1_2 conservativity implies Π^1_1 conservativity.

Thm (Hirschfeldt and Shore). COH is r-Π^1_2 conservative over RCA_0.

So RCA_0 + COH cannot prove statements like ADS or even SADS.

Restricted Π^1_2 Conservativity of AMT

Thm (HSS). AMT is r-Π^1_2 conservative over RCA_0.

So $\mathsf{RCA}_0 + \mathsf{AMT}$ cannot prove statements like ADS or even SADS.

Restricted Π^1_2 Conservativity of AMT

Thm (HSS). AMT is r-Π^1_2 conservative over RCA_0.

So $\mathsf{RCA}_0 + \mathsf{AMT}$ cannot prove statements like ADS or even SADS.

This result is tight, in that AMT is itself of the form

$$\forall A\ (\Theta(A) \to \exists B\ \Phi(A, B))$$

with Φ being Π^0_3.

Restricted Π^1_2 Conservativity of AMT

Thm (HSS). AMT is r-Π^1_2 conservative over RCA_0.

So RCA_0 + AMT cannot prove statements like ADS or even SADS.

This result is tight, in that AMT is itself of the form

$$\forall A\ (\Theta(A) \to \exists B\ \Phi(A, B))$$

with Φ being Π^0_3.

The r-Π^1_2 conservativity of COH and AMT come from their connection with forcing notions.

Restricted Π^1_2 Conservativity and Forcing: COH

Thm (Cholak, Jockusch, and Slaman). Let $\mathcal{N} \vDash \mathsf{RCA}_0$ be countable. Let G be Mathias 1-generic over $\mathcal{N}$.

- Then every sequence in $\mathcal{N}$ has a cohesive set in $\mathcal{N}[G]$.

Restricted Π^1_2 Conservativity and Forcing: COH

Thm (Cholak, Jockusch, and Slaman). Let $\mathcal{N} \vDash \mathsf{RCA}_0$ be countable. Let G be Mathias 1-generic over $\mathcal{N}$.

- Then every sequence in $\mathcal{N}$ has a cohesive set in $\mathcal{N}[G]$.

Thm (Hirschfeldt and Shore). Let $\mathcal{N} \vDash \mathsf{RCA}_0$ be countable. Let G be Mathias 2-generic over $\mathcal{N}$. Let $\Phi(A, B)$ be Σ^0_3 and $A \in \mathcal{N}$ be s.t. $\forall B \in \mathcal{N}\ (\mathcal{N} \vDash \neg\Phi(A, B))$.

- Then $\forall B \in \mathcal{N}[G]\ (\mathcal{N}[G] \vDash \neg\Phi(A, B))$.

Restricted Π^1_2 Conservativity and Forcing: COH

Thm (Cholak, Jockusch, and Slaman). Let $\mathcal{N} \vDash \mathsf{RCA}_0$ be countable. Let G be Mathias 1-generic over $\mathcal{N}$.

- Then every sequence in $\mathcal{N}$ has a cohesive set in $\mathcal{N}[G]$.

Thm (Hirschfeldt and Shore). Let $\mathcal{N} \vDash \mathsf{RCA}_0$ be countable. Let G be Mathias 2-generic over $\mathcal{N}$. Let $\Phi(A, B)$ be Σ^0_3 and $A \in \mathcal{N}$ be s.t. $\forall B \in \mathcal{N}$ $(\mathcal{N} \vDash \neg\Phi(A, B))$.

- Then $\forall B \in \mathcal{N}[G]$ $(\mathcal{N}[G] \vDash \neg\Phi(A, B))$.

So by iterating the CJS result, we get the r-Π^1_2 conservativity of COH.

Restricted Π^1_2 Conservativity and Forcing: AMT

Thm (HSS). Let $\mathcal{N} \vDash \mathsf{RCA}_0$ be countable and let G be Cohen 2-generic over $\mathcal{N}$.

- Then every atomic T in $\mathcal{N}$ has an atomic model in $\mathcal{N}[G]$.
- Let $\Phi(A, B)$ be Σ^0_3 and $A \in \mathcal{N}$ be s.t. $\forall B \in \mathcal{N}\ (\mathcal{N} \vDash \neg\Phi(A, B))$.

  Then $\forall B \in \mathcal{N}[G]\ (\mathcal{N}[G] \vDash \neg\Phi(A, B))$.

Restricted Π^1_2 Conservativity and Forcing: AMT

Thm (HSS). Let $\mathcal{N} \vDash \mathsf{RCA}_0$ be countable and let G be Cohen 2-generic over $\mathcal{N}$.

- Then every atomic T in $\mathcal{N}$ has an atomic model in $\mathcal{N}[G]$.
- Let $\Phi(A, B)$ be Σ^0_3 and $A \in \mathcal{N}$ be s.t. $\forall B \in \mathcal{N}\ (\mathcal{N} \vDash \neg\Phi(A, B))$.

  Then $\forall B \in \mathcal{N}[G]\ (\mathcal{N}[G] \vDash \neg\Phi(A, B))$.

So by iteration, we get the r-Π^1_2 conservativity of AMT.

Restricted Π^1_2 Conservativity and Forcing: AMT

Thm (HSS). Let $\mathcal{N} \models \mathsf{RCA}_0$ be countable and let G be Cohen 2-generic over $\mathcal{N}$.

- Then every atomic T in $\mathcal{N}$ has an atomic model in $\mathcal{N}[G]$.
- Let $\Phi(A, B)$ be Σ^0_3 and $A \in \mathcal{N}$ be s.t. $\forall B \in \mathcal{N}\ (\mathcal{N} \models \neg\Phi(A, B))$.

  Then $\forall B \in \mathcal{N}[G]\ (\mathcal{N}[G] \models \neg\Phi(A, B))$.

So by iteration, we get the r-Π^1_2 conservativity of AMT.

We can combine the two kinds of forcing to obtain r-Π^1_2 conservativity of COH $+$ AMT.

AMT and Genericity

Cohen 2-genericity is more than we need to prove AMT.

$\mathbf{\Pi^0_1}$G: Let $(D_i)_{i\in\omega}$ be uniformly Π^0_1 dense subsets of $2^{<\omega}$. There is a G s.t. $\forall i\ \exists n\ (G \upharpoonright n \in D_i)$.

AMT and Genericity

Cohen 2-genericity is more than we need to prove AMT.

Π^0_1G: Let $(D_i)_{i \in \omega}$ be uniformly Π^0_1 dense subsets of $2^{<\omega}$. There is a G s.t. $\forall i \; \exists n \; (G \upharpoonright n \in D_i)$.

It is easy to see that $\mathsf{RCA}_0 \vdash \Pi^0_1\mathsf{G} \rightarrow \mathsf{AMT}$.

AMT and Genericity

Cohen 2-genericity is more than we need to prove AMT.

Π^0_1G: Let $(D_i)_{i\in\omega}$ be uniformly Π^0_1 dense subsets of $2^{<\omega}$. There is a G s.t. $\forall i\ \exists n\ (G \upharpoonright n \in D_i)$.

It is easy to see that $\mathsf{RCA}_0 \vdash \Pi^0_1\mathsf{G} \rightarrow \mathsf{AMT}$.

Thm (Conidis). $\mathsf{RCA}_0 + \mathsf{I}\Sigma_2 \vdash \mathsf{AMT} \rightarrow \Pi^0_1\mathsf{G}$.

AMT and Genericity

Cohen 2-genericity is more than we need to prove AMT.

Π^0_1G: Let $(D_i)_{i\in\omega}$ be uniformly Π^0_1 dense subsets of $2^{<\omega}$. There is a G s.t. $\forall i \; \exists n \; (G \upharpoonright n \in D_i)$.

It is easy to see that $\mathsf{RCA}_0 \vdash \Pi^0_1\mathsf{G} \to \mathsf{AMT}$.

Thm (Conidis). $\mathsf{RCA}_0 + \mathsf{I}\Sigma_2 \vdash \mathsf{AMT} \to \Pi^0_1\mathsf{G}$.

The use of $\mathsf{I}\Sigma_2$ cannot be dispensed with.

AMT and Genericity: Further Conservativity Results

Thm(HSS). AMT is Π^1_1 conservative over $\mathsf{RCA}_0 + \mathsf{B}\Sigma_2$.

AMT and Genericity: Further Conservativity Results

Thm(HSS). AMT is Π^1_1 conservative over $\mathsf{RCA}_0 + \mathsf{B}\Sigma_2$.

Thm(HSS). $\mathsf{RCA}_0 \vdash \Pi^0_1\mathsf{G} + \mathsf{B}\Sigma_2 \rightarrow \mathsf{I}\Sigma_2$.

Thus AMT does not imply $\Pi^0_1\mathsf{G}$ over RCA_0 (or even $\mathsf{RCA}_0 + \mathsf{B}\Sigma_2$).

AMT and Genericity: Further Conservativity Results

Thm(HSS). AMT is Π^1_1 conservative over $\mathsf{RCA}_0 + \mathsf{B}\Sigma_2$.

Thm(HSS). $\mathsf{RCA}_0 \vdash \Pi^0_1\mathsf{G} + \mathsf{B}\Sigma_2 \rightarrow \mathsf{I}\Sigma_2$.

Thus AMT does not imply $\Pi^0_1\mathsf{G}$ over RCA_0 (or even $\mathsf{RCA}_0 + \mathsf{B}\Sigma_2$).

Thm(HSS). AMT is Π^1_1 conservative over $\mathsf{RCA}_0 + \mathsf{I}\Sigma_2$.

Omitting Partial Types

Thm (Millar). The following hold in RCA_0.

Let A be a set of complete types of T.
There is a model of T omitting all nonprincipal types in A.

Let B be a set of nonprincipal partial types of T.
There is a model of T omitting all partial types in B.

Omitting Partial Types

Thm (Millar). The following hold in RCA_0.

Let A be a set of complete types of T.
There is a model of T omitting all nonprincipal types in A.

Let B be a set of nonprincipal partial types of T.
There is a model of T omitting all partial types in B.

OPT: Let S be a set of partial types of T.
There is a model of T omitting all nonprincipal types in S.

Omitting Partial Types and Hyperimmunity

OPT: Let S be a set of partial types of T.
There is a model of T omitting all nonprincipal types in S.

Omitting Partial Types and Hyperimmunity

OPT: Let S be a set of partial types of T.
There is a model of T omitting all nonprincipal types in S.

HYP: For every X there is a function not dominated by any X-computable function.

Omitting Partial Types and Hyperimmunity

OPT: Let S be a set of partial types of T.
There is a model of T omitting all nonprincipal types in S.

HYP: For every X there is a function not dominated by any X-computable function.

Thm (HSS). $\mathsf{RCA}_0 \vdash \mathsf{OPT} \leftrightarrow \mathsf{HYP}$.

A Weak Form of AMT

Partial types Γ and Δ of T are equivalent if they imply the same formulas over T.

$(\Delta_n)_{n \in \omega}$ is a subenumeration of the partial types of T if for every partial type Γ of T there is an n s.t. Γ and Δ_n are equivalent.

A Weak Form of AMT

Partial types Γ and Δ of T are equivalent if they imply the same formulas over T.

$(\Delta_n)_{n\in\omega}$ is a subenumeration of the partial types of T if for every partial type Γ of T there is an n s.t. Γ and Δ_n are equivalent.

AST: If T is atomic and its partial types have a subenumeration, then T has an atomic model.

A Weak Form of AMT

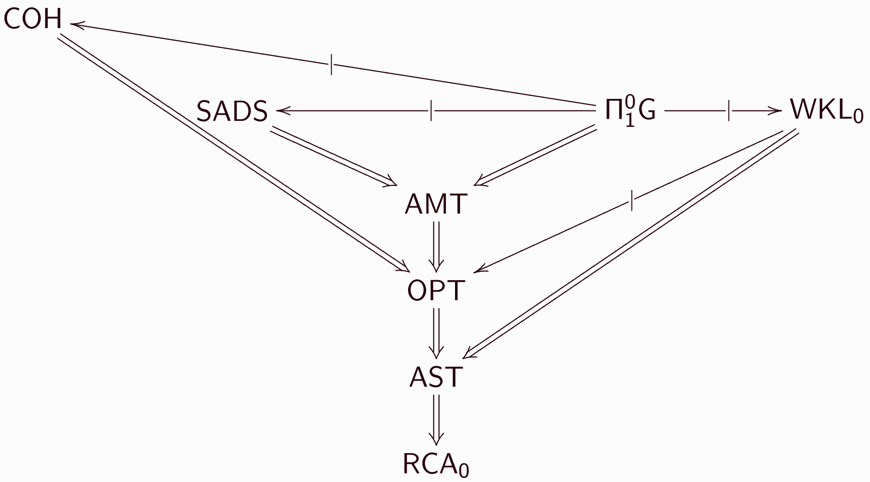
Partial types Γ and Δ of T are equivalent if they imply the same formulas over T.

$(\Delta_n)_{n\in\omega}$ is a subenumeration of the partial types of T if for every partial type Γ of T there is an n s.t. Γ and Δ_n are equivalent.

AST: If T is atomic and its partial types have a subenumeration, then T has an atomic model.

Thm (HSS). $\mathsf{RCA}_0 \vdash \mathsf{AST} \leftrightarrow \forall X\, \exists Y\, (Y \not\leq_{\mathbf{T}} X)$.

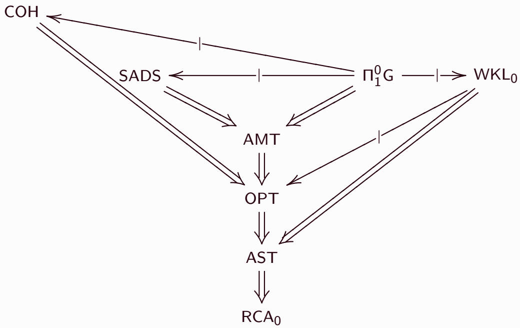
The Picture



$\Longrightarrow$: not reversible $\qquad$ $\longrightarrow$: opposite direction open

Open Questions

Completing the Picture



Does COH (or CADS) imply AMT over RCA_0?

Does CADS imply OPT over RCA_0?

Is AMT r-Π^1_2 conservative over $B\Sigma_2$?

The Homogeneous Model Theorem

$\mathcal{M}$ is homogeneous if for $\vec{a}, \vec{b} \in \mathcal{M}$ of the same type, $(\mathcal{M}, \vec{a}) \cong (\mathcal{M}, \vec{b})$.

Goncharov gave closure conditions on a set of types S of T necessary and sufficient for S to be the type spectrum of a homogeneous model of T.

The Homogeneous Model Theorem

$\mathcal{M}$ is homogeneous if for $\vec{a}, \vec{b} \in \mathcal{M}$ of the same type, $(\mathcal{M}, \vec{a}) \cong (\mathcal{M}, \vec{b})$.

Goncharov gave closure conditions on a set of types S of T necessary and sufficient for S to be the type spectrum of a homogeneous model of T.

- Closure under permutations of variables.
- Closure under subtypes.
- Closure under unions of types on disjoint sets of variables.
- Closure under type / type amalgamation.
- Closure under type / formula amalgamation.

The Homogeneous Model Theorem

$\mathcal{M}$ is homogeneous if for $\vec{a}, \vec{b} \in \mathcal{M}$ of the same type, $(\mathcal{M}, \vec{a}) \cong (\mathcal{M}, \vec{b})$.

Goncharov gave closure conditions on a set of types S of T necessary and sufficient for S to be the type spectrum of a homogeneous model of T.

- Closure under permutations of variables.
- Closure under subtypes.
- Closure under unions of types on disjoint sets of variables.
- Closure under type / type amalgamation.
- Closure under type / formula amalgamation.

HMT: If S satisfies the Goncharov conditions, then there is a homogeneous model of T with type spectrum S.

The Homogeneous Model Theorem and AMT

Computability theoretic results suggest that HMT behaves like AMT.

For example:

Thm (Lange). TFAE for a Δ^0_2 degree $\mathbf{d}$.

For every computable S satisfying the Goncharov conditions, there is a $\mathbf{d}$-decidable homogeneous model of T with type spectrum S.

$\mathbf{d}$ is nonlow$_2$.

The Homogeneous Model Theorem and AMT

Computability theoretic results suggest that HMT behaves like AMT.

For example:

Thm (Lange). TFAE for a Δ^0_2 degree $\mathbf{d}$.

For every computable S satisfying the Goncharov conditions, there is a $\mathbf{d}$-decidable homogeneous model of T with type spectrum S.

$\mathbf{d}$ is nonlow$_2$.

Lange has shown that AMT implies HMT computability-theoretically.

The Homogeneous Model Theorem and AMT

Computability theoretic results suggest that HMT behaves like AMT.

For example:

Thm (Lange). TFAE for a Δ^0_2 degree $\mathbf{d}$.

For every computable S satisfying the Goncharov conditions, there is a $\mathbf{d}$-decidable homogeneous model of T with type spectrum S.

$\mathbf{d}$ is nonlow$_2$.

Lange has shown that AMT implies HMT computability-theoretically.

Open Question: Are HMT and AMT equivalent over RCA_0?

References

- C.J. Conidis, Classifying model-theoretic properties, JSL 73 (2008) 885–905.
- D.R. Hirschfeldt and R.A. Shore, Combinatorial Principles Weaker than Ramsey's Theorem for Pairs, JSL 72 (2007) 171–206.
- D.R. Hirschfeldt, R.A. Shore, and T.A. Slaman, The Atomic Model Theorem and Type Omitting, to appear in TAMS.
- K. Lange, The Degree Spectra of Homogeneous Models, JSL 73 (2008), 1009–1028.