



# Banff International Research Station

for Mathematical Innovation and Discovery

## Discontinuous Galerkin Methods for Partial Differential Equations

November 25-30, 2007

### MEALS

\*Breakfast (Buffet): 7:00–9:00 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

### MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155–159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

### SCHEDULE

#### SUNDAY, NOVEMBER 25

- |                    |   |
|--------------------|---|
| <b>16:00</b>       | Check-in begins (Front Desk - Professional Development Centre - open 24 hours)  |
| <b>17:30–19:30</b> | Buffet Dinner, Sally Borden Building  |
| <b>20:00</b>       | Informal gathering in 2nd floor lounge, Corbett Hall<br>Beverages and small assortment of snacks available on a cash honour-system. |



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## MONDAY, NOVEMBER 26

- 7:00–8:45** Breakfast
- 8:45–9:00** Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159
- 9:00–9:30** **Sherwin, Spencer J.**  
*DG methods using spectral/hp element discretisations:  
Balancing utility and efficiency*
- 9:30–10:00** **Zhang Zhimin**  
*Discontinuous Galerkin methods for singularly perturbed problems*
- 10:00–10:30** Coffee Break, 2nd floor lounge, Corbett Hall
- 10:30–11:00** **Li, Fengyan**  
*A second order DGM based fast sweeping method for eikonal equations*
- 11:00–11:30** **Cheng, Yingda**  
*Discontinuous Galerkin solver for Boltzmann-Poisson transients*
- 11:30–13:00** Lunch
- 13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
- 14:00** Group Photo; meet on the front steps of Corbett Hall
- 14:30–15:00** **Kanschat, Guido**  
*Convergent adaptive algorithms for the interior penalty method*
- 15:00–15:30** **Xu, Yan**  
*A local discontinuous Galerkin method for the Camassa-Holm equation*
- 15:30–16:00** Coffee Break, 2nd floor lounge, Corbett Hall
- 16:00–16:30** **Ayuso, Blanca**  
*Discontinuous Galerkin methods for advection-diffusion-reaction problems*
- 16:30–17:00** **Dong, Bo**  
*Optimal convergence of the original DG method for the transport-reaction equation on special meshes*
- 17:30–19:30** Dinner



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## TUESDAY, NOVEMBER 27

- 7:00–9:00** Breakfast
- 9:00–9:30** **Peraire, Jaume**  
*A discontinuous Galerkin formulation for Lagrangian dynamic analysis*
- 9:30–10:00** **Celiker, Fatih**  
*Adaptive stabilization of discontinuous Galerkin methods for nonlinear elasticity*
- 10:00–10:30** Coffee Break, 2nd floor lounge, Corbett Hall
- 10:30–11:00** **Grote, Marcus**  
*Discontinuous Galerkin methods and local time stepping for second-order wave equations*
- 11:00–11:30** **Warburton, Timothy**  
*Advances in wave propagation with the discontinuous Galerkin method*
- 11:30–13:30** Lunch
- 14:00–14:30** **Van der Vegt, Jaap**  
*Discontinuous Galerkin finite element methods for hyperbolic nonconservative partial differential equations*
- 14:30–15:00** **Rivière, Beatrice**  
*Coupling of surface and subsurface flows*
- 15:00–15:30** Coffee Break, 2nd floor lounge, Corbett Hall
- 15:30–16:00** **Brenner, Susanne**  
*Multigrid algorithms for weakly over-penalized interior penalty methods*
- 16:00–16:30** **Antonietti, Paola**  
*Preconditioning discontinuous Galerkin approximations of elliptic problems*
- 17:30–19:30** Dinner



# Banff International Research Station

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WEDNESDAY, NOVEMBER 28

- 7:00–9:00** Breakfast
- 9:00–9:30** **Zhang, Yongtao**  
*Application of a discontinuous Galerkin finite element method to reaction-diffusion systems in developmental biology*
- 9:30–10:00** **Wang, Wei**  
*The discontinuous Galerkin method for the multiscale modeling of dynamics of crystalline solids*
- 10:00–10:30** Coffee Break, 2nd floor lounge, Corbett Hall
- 10:30–11:00** **Wheeler, Mary**  
*Coupling discontinuous Galerkin and mixed finite element discretizations using mortar finite elements*
- 11:00–11:30** **Wihler, Thomas P.**  
*A-posteriori error estimation for hp-dG timestepping for parabolic PDEs*
- 11:30–13:30** Lunch  
Free Afternoon
- 17:30–19:30** Dinner



# Banff International Research Station

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## THURSDAY, NOVEMBER 29

- 7:00–9:00** Breakfast
- 9:00–9:30** **Dawson, Clint**  
*Good and bad aspects of discontinuous Galerkin methods for some geoscience problems*
- 9:30–10:00** **Gopalakrishnan, Jay**  
*Hybridized DG methods*
- 10:00–10:30** Coffee Break, 2nd floor lounge, Corbett Hall
- 10:30–11:00** **Guzmán, Johnny**  
*Superconvergent discontinuous Galerkin methods for second-order elliptic problems*
- 11:00–11:30** **Ryan, Jennifer K.**  
*Local Post-processing for discontinuous Galerkin methods: Mesh, derivatives, and applications*
- 11:30–13:30** Lunch
- 14:00–14:30** **Hesthaven, Jan**  
*Nodal DG-FEM for the modeling of free surface flows using high-order Boussinesq approximations*
- 14:30–15:00** **Persson, Per-Olof**  
*Preconditioning of Newton-GMRES solvers for discontinuous Galerkin problems*
- 15:00–15:30** Coffee Break, 2nd floor lounge, Corbett Hall
- 15:30–16:00** **Chen, Yanlai**  
*An adaptive high order discontinuous Galerkin method with error control for the Hamilton-Jacobi equations*
- 16:00–16:30** **Shu, Chi-Wang**  
 *$L^2$ -Stability analysis of the central discontinuous Galerkin method and a comparison between the central and regular discontinuous Galerkin methods*
- 17:30–19:30** Dinner



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**FRIDAY, NOVEMBER 30**

**7:00–9:00** Breakfast

**11:30–13:30** Lunch

**Checkout by 12 noon.**

\*\* 5-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*



# Banff International Research Station

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## Discontinuous Galerkin Methods for Partial Differential Equations

November 25-30, 2007

### PARTICIPANTS (in alphabetic order)

**Antonietti, Paola** (University of Nottingham, paola.antonietti@nottingham.ac.uk)  
**Ayuso, Blanca** (University of Madrid, blanca.ayuso@uam.es)  
**Brenner, Susanne** (Louisiana State University, brenner@math.lsu.edu)  
**Celiker, Fatih** (Wayne State University, celiker@math.wayne.edu)  
**Chen, Yanlai** (University of Minnesota, ylchen@math.umn.edu)  
**Cheng, Yingda** (University of Texas, ycheng@math.utexas.edu)  
**Cockburn, Bernardo** (University of Minnesota, cockburn@math.umn.edu)  
**Dawson, Clint** (University of Texas, clint@ices.utexas.edu)  
**Djournna, Georges** (University of Laval, gdjournna@mat.ulavel.ca)  
**Dong, Bo** (Brown University, bdong@dam.brown.edu)  
**Gopalakrishnan, Jay** (University of Florida, jayg@math.ufl.edu)  
**Grote, Marcus** (University of Basel, Marcus.Grote@unibas.ch)  
**Guzmán, Johnny** (University of Minnesota, guzma033@umn.edu)  
**Hesthaven, Jan** (Brown University, Jan.Hesthaven@Brown.edu)  
**Kanschat, Guido** (Texas A& M University, kanschat@math.tamu.edu)  
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**Rivière, Beatrice** (University of Pittsburgh, riviere@math.pitt.edu)  
**Ryan, Jennifer K.** (Virginia Tech, jkryan@vt.edu)  
**Schötzau, Dominik** (University of British Columbia, schoetzau@math.ubc.ca)  
**Sherwin, Spencer J.** (Imperial College London, s.sherwin@imperial.ac.uk)  
**Shu, Chi-Wang** (Brown University, shu@dam.brown.edu)  
**Van der Vegt, Jaap** (University of Twente, J.J.W.vanderVegt@ewi.utwente.nl)  
**Wang, Wei** (Brown University, wwang@dam.brown.edu)  
**Warburton, Timothy** (Rice University, timwar@caam.rice.edu)  
**Wei, Xiaoxi** (University of British Columbia, weixiaoxi@math.ubc.ca)  
**Wheeler, Mary** (University of Texas, mfw@ices.utexas.edu)  
**Wihler, Thomas P.** (McGill University, wihler@math.mcgill.ca)  
**Xu, Yan** (University of Science and Technology of China, yxu@ustc.edu.cn)  
**Zhang, Yongtao** (University of Notre Dame, yzhang10@nd.edu)  
**Zhang, Zhimin** (Wayne State University, zzhang@math.wayne.edu)  
**Zhu, Liang** (University of British Columbia, zhuliang@math.ubc.ca)



# Banff International Research Station

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## Discontinuous Galerkin Methods for Partial Differential Equations

November 25-30, 2007

### ABSTRACTS

(in alphabetic order)

**Antonietti, Paola** (University of Nottingham, [paola.antonietti@nottingham.ac.uk](mailto:paola.antonietti@nottingham.ac.uk))

Title: *Preconditioning discontinuous Galerkin approximations of elliptic problems*

Abstract: In recent years, much attention has been given to domain decomposition methods for linear elliptic problems that are based on a partitioning of the domain of the physical problem. Since the subdomains can be handled independently, such methods are very attractive for coarse-grain parallel computers.

In this talk we shall present in a unified framework some non-overlapping additive and multiplicative Schwarz domain decomposition methods for the solution of the algebraic linear systems of equations arising from discontinuous Galerkin approximations of elliptic problems. In particular, two-level methods for both symmetric and non-symmetric DG schemes will be introduced and some interesting features, which have no analog in the conforming case, will be discussed. For symmetric DG discretizations, optimal convergence estimates will be presented, and we will show that the proposed Schwarz methods can be successfully accelerated with suitable Krylov iterative solvers. A discussion on the issue of preconditioning non-symmetric DG approximations of second order elliptic equations will be included. Numerical experiments to validate our theory and to illustrate the performance and robustness of the proposed two-level methods will be shown. This work has been carried out in collaboration with Blanca Ayuso (Departamento de Matemáticas, Universidad Autónoma de Madrid, Spain).

**Ayuso, Blanca** (University of Madrid, [blanca.ayuso@uam.es](mailto:blanca.ayuso@uam.es))

Title: *Discontinuous Galerkin methods for advection-diffusion-reaction problems*

Abstract: We apply the weighted-residual approach recently introduced by Brezzi et. al, to derive discontinuous Galerkin formulations for advection-diffusion-reaction problems. We devise the basic ingredients to ensure stability and optimal error estimates in suitable norms, and propose two new methods. The talk is based on joint work with L. Donatella Marini from the University of Pavia.

**Brenner, Susanne** (Louisiana State University, [brenner@math.lsu.edu](mailto:brenner@math.lsu.edu))

Title: *Multigrid algorithms for weakly over-penalized interior penalty methods*

Abstract: In this talk we will introduce two weakly over-penalized interior penalty methods that are stable for any penalty parameter and are quasi-optimal in both the energy norm and the L2 norm. We will also discuss multigrid algorithms for these interior penalty methods. Both theoretical and numerical results will be presented.

**Celiker, Fatih** (Wayne State University, [celiker@math.wayne.edu](mailto:celiker@math.wayne.edu))

Title: *Adaptive stabilization of discontinuous Galerkin methods for nonlinear elasticity*

Abstract: We introduce a novel approach to stabilizing discontinuous Galerkin methods in nonlinear elasticity problems. The new stabilization strategy possesses the distinguishing feature of allowing the size of the stabilization term to vary throughout the mesh, and to automatically adjust the local level of stabilization according to the solution sought. This stabilization strategy is hence adaptive. The proposed

scheme computationally efficient and remains stable for a fairly lengthy quasistatic loading path. This is demonstrated with two and three dimensional numerical examples. We further propose a slight modification of this approach for which we are able to prove theoretical estimates for the minimal values of the stabilization parameters defining the method.

**Chen, Yanlai** (University of Minnesota, ylchen@math.umn.edu)

Title: *An adaptive high order discontinuous Galerkin method with error control for the Hamilton-Jacobi equations*

Abstract: We propose and study an adaptive version of the discontinuous Galerkin method for Hamilton-Jacobi equations. It works as follows. Given the tolerance and the degree of the polynomial of the approximate solution, the adaptive algorithm finds a mesh on which the approximate solution has an  $L^\infty$ -distance to the viscosity solution no bigger than the prescribed tolerance. The algorithm uses three main tools. The first is an iterative solver combining the explicit Runge-Kutta Discontinuous Galerkin method and the implicit Newton's method that enables us to solve the Hamilton-Jacobi equations efficiently. The second is a new a posteriori error estimate based on the approximate resolution of an approximate problem for the actual error. The third is a method that allows us to find a new mesh as a function of the old mesh and the ratio of the a posteriori error estimate to the tolerance. We display extensive numerical evidence that indicates that, for any given polynomial degree, the method achieves its goal with optimal complexity independently of the tolerance. This is done in the framework of one-dimensional steady-state model problems with periodic boundary conditions.

**Cheng, Yingda** (University of Texas, ycheng@math.utexas.edu)

Title: *Discontinuous Galerkin solver for Boltzmann-Poisson transients*

Abstract: We present results of a discontinuous Galerkin scheme applied to deterministic computations of the transients for the Boltzmann-Poisson system describing electron transport in semiconductor devices. The collisional term models optical-phonon interactions which become dominant under strong energetic conditions corresponding to nano-scale active regions under applied bias. The proposed numerical technique is a finite element method using discontinuous piecewise polynomials as basis functions on unstructured meshes. It is applied to simulate hot electron transport in bulk silicon, in a silicon  $n^+ - n - n^+$  diode and in a double gated 12nm MOSFET. Additionally, the obtained results are compared to those of a high order WENO scheme simulation.

**Dawson, Clint** (University of Texas, clint@ices.utexas.edu)

Title: *Good and bad aspects of discontinuous Galerkin methods for some geoscience problems*

Abstract: We will discuss the development and implementation of DG methods for two applications arising in geosciences. The first application is the shallow water equations. These equations are a hyperbolic/parabolic system of equations which is well-suited for approximation by the Runge-Kutta DG methods of Cockburn and Shu. We will describe recent successes and lessons learned in the application of hp-adaptive DG methods for shallow water under very complex flow scenarios. The second application is to flow in porous media, in particular flow in the vadose zone described by Richards' equation. Richards' equation is a highly nonlinear parabolic equation and requires implicit time discretization, which leads to the solution of large nonlinear and linear systems of equations. For these problems, DG methods have been observed to give solutions which are comparable to standard finite element methods, but are much more computationally intensive. Several numerical examples will be presented to illustrate our experiences in this area.

**Dong, Bo** (Brown University, bdong@dam.brown.edu)

Title: *Optimal convergence of the original DG method for the transport-reaction equation on special meshes*

Abstract: We show that the approximation given by the original discontinuous Galerkin method for the transport-reaction equation in  $d$  space dimensions is optimal provided the meshes are suitably chosen: the  $L^2$ -norm of the error is of order  $k + 1$  when the method uses polynomials of degree  $k$ . These meshes are not

necessarily conforming and do not satisfy any uniformity condition; they are only required to be made of simplexes each of which has a unique *outflow* face. We also find a new, element-by-element postprocessing of the derivative in the direction of the flow which superconverges with order  $k + 1$ .

**Gopalakrishnan, Jay** (University of Florida, jayg@math.ufl.edu)

Title: *Hybridized DG methods*

Abstract: Discontinuous Galerkin (DG) methods have often been criticized for having too many unknowns. However, we show that DG methods can be made competitive with mixed and continuous Galerkin (CG) methods via hybridization. We achieve this by extending hybridization techniques already developed for mixed methods into a unified framework that allows hybridization of a variety of methods. This framework facilitates the discovery of new methods as well as new connections between DG, mixed and CG methods. Moreover, the unified framework allows easy coupling of these different methods, even across non-matching mesh interfaces.

**Grote, Marcus** (University of Basel, Marcus.Grote@unibas.ch)

Title: *Discontinuous Galerkin methods and local time stepping for second-order wave equations*

Abstract: The accurate and reliable simulation of wave phenomena is of fundamental importance in a wide range of engineering applications such as fiber optics, wireless communication, radar and sonar technology, and non-invasive testing. To address the wide range of difficulties involved, we consider symmetric interior penalty discontinuous Galerkin (IP-DG) methods, which easily handle elements of various types and shapes, irregular non-matching grids, and even locally varying polynomial order. Moreover, in contrast to standard (conforming) finite element methods, IP-DG methods yield an essentially diagonal mass matrix; hence, when coupled with explicit time integration, the overall numerical scheme remains truly explicit in time. To circumvent the severe stability (CFL) condition imposed on the time step by the smallest elements in the mesh, we propose local time-stepping schemes, which allow arbitrarily small time steps where small elements in the mesh are located. When combined with the symmetric IP-DG discretization, the resulting fully discrete scheme is explicit and exactly conserves a discrete energy. Starting from the standard second order “leap-frog” scheme, time integrators of arbitrary order of accuracy are derived. Numerical experiments illustrate the usefulness of these methods and validate the theory.

**Guzmán, Johnny** (University of Minnesota, guzma033@umn.edu)

Title: *Superconvergent discontinuous Galerkin methods for second-order elliptic problems*

Abstract: We identify discontinuous Galerkin methods for second-order elliptic problems having superconvergence properties similar to those of the Raviart-Thomas and the Brezzi-Douglas-Marini mixed methods. These methods use polynomials of degree  $k$  for both the potential as well as the flux. We show that the approximate flux converges with the optimal order of  $k + 1$ , and that the approximate potential and its numerical trace superconverge, to suitably chosen projections of the potential, with order  $k + 2$ . We also apply element-by-element postprocessings of the approximate solution to obtain new approximations of the flux and the potential. The new approximate flux is proven to have normal components continuous across inter-element boundaries, to converge with order  $k + 1$ , and to have a divergence converging in with order  $k + 1$ . The new approximate potential is proven to converge with order  $k + 2$ . This is joint work with Bernardo Cockburn and Haiying Wang.

**Hesthaven, Jan** (Brown University, Jan.Hesthaven@Brown.edu)

Title: *Nodal DG-FEM for the modeling of free surface flows using high-order Boussinesq approximations*

Abstract: We shall discuss the modeling of free surface flows and fluid-structure interactions using high-order Boussinesq approximations. These sets of equations are characterized by being purely dispersive and strongly non-linear, with additional complications introduced by high-order spatial derivatives.

We shall discuss the key elements of the formulation and some of the properties of the Boussinesq system. These properties shall be used to argue why DG-FEM may be a suitable approach for the solution of these equations. We shall continue to develop the basic elements required for solving this system,

discussing a number of subtleties and addressing practical concerns of performance and efficient solvers. The computational approach will be extensively validated with both benchmark tests and experimental data. This is work done in collaboration with A.P. Engsig-Karup (DTU, Denmark), P. Madsen (DTU, Denmark), H. Bingham (DTU, Denmark) and T. Warburton (Rice).

**Kanschat, Guido** (Texas A& M University, kanschat@math.tamu.edu)

Title: *Convergent adaptive algorithms for the interior penalty method*

Abstract: We discuss an adaptive strategy producing a sequence of meshes with a guaranteed error reduction in each step. Mesh refinement is based on a bulk criterion employing an energy norm a posteriori error estimate. The convergence proof relies on a modified discrete local efficiency. Numerical experiments show the feasibility of our approach.

**Li, Fengyan** (Rensselaer Polytechnic Institute, lif@rpi.edu)

Title: *A second order DGM based fast sweeping method for eikonal equations*

Abstract: The original fast sweeping method for solving static Hamilton-Jacobi equations is in the finite difference framework. By incorporating the causality of the equation into the discretization and combining the Gauss-Seidel iteration with alternating sweeping orderings, the method defines an efficient solver for these nonlinear equations with linear computational complexity. That is, the number of iterations needed for the error to be reduced to certain threshold is independent of the number of unknowns. On the other hand, this finite difference based fast sweeping method is only first order accurate. In this talk, I will discuss our recent progress in developing a second order fast sweeping method based on the discontinuous Galerkin (DG) discretization for an important family of static Hamilton-Jacobi equations - Eikonal equation. The main new components in our algorithm are the properly chosen numerical Hamiltonian in the DG formulation and the procedure to consistently enforce the causality. Numerical examples will be presented to demonstrate the performance of the proposed algorithm.

**Peraire, Jaume** (MIT, peraire@MIT.EDU)

Title: *A discontinuous Galerkin formulation for Lagrangian dynamic analysis*

Abstract: Dynamic analyses in solid mechanics are typically carried out by integrating the second order system expressing the balance of momentum. In this work, we follow a different approach. The nonlinear governing equations are first cast as a system of first order conservation laws for momentum (and energy, if required by the constitutive law). In addition, a first order time evolution equation is also written for the deformation gradient tensor,  $\mathbf{F}$ . This results in a coupled system of 6 first order equations in 2 dimensions and 12 first order equations in 3 dimensions. The resulting system is found to possess involutions for the form  $\nabla \times \mathbf{F} = \mathbf{0}$ , which must be accounted for in the solution process in order to preserve stability. We develop an approximate Riemman solver and discretize the resulting equations using a high order discontinuous Galerkin method and integrate in time using an explicit Runge-Kutta algorithm. The method preserves momentum exactly and numerical results show excellent energy conservation properties for long time integrations.

It is well known that for non-dissipative materials the solutions may develop shock waves which need to be handled numerically by the explicit addition of artificial viscosity. Shock capturing techniques developed in the context of non-linear conservation laws are then readily applicable. Unlike other formulations, the method is purely Lagrangian and therefore is formulated in the reference configuration.

We also consider an alternative formulation in which the deformation gradient tensor is eliminated from the equations at the expense of introducing second order spatial derivatives. These are treated in the discretization using the CDG method. This formulation has a significantly reduced number of unknowns (4 in 2D and 6 in 3D) but seems to be less adequate to deal with strong discontinuities.

This is work done in collaboration with P.-O. Persson (MIT) and J. Bonet (Swansea).

**Persson, Per-Olof** (MIT, persson@MIT.EDU)

Title: *Preconditioning of Newton-GMRES solvers for discontinuous Galerkin problems*

Abstract: A fundamental problem with Discontinuous Galerkin methods is their high computational and storage cost. This is partly because they require more degrees of freedom than other methods, but mainly due to the wide nodal stencils which result in very large Jacobian matrices in implicit solvers. In this work we address the high cost of solving the corresponding linear systems of equations and propose a new preconditioner for implicit solution of stationary or time dependent Discontinuous Galerkin problems. The viscous terms are discretized using the Compact Discontinuous Galerkin (CDG) method, but the results should be representative for other schemes as well. We consider several existing preconditioners such as block-Jacobi and Gauss-Seidel combined with multi-level schemes which have been developed and tested for specific applications. While our results are consistent with the claims reported, we find that these preconditioners lack robustness when used in more challenging situations involving low Mach numbers, stretched grids or high Reynolds number turbulent flows. We propose a preconditioner based on a coarse scale correction with post-smoothing based on a block incomplete LU factorization with zero fill-in (ILU0) of the Jacobian matrix. Our block-Minimum Discarded Fill algorithm numbers the elements such that the error in the ILU0 factorization is minimized in a greedy fashion, a step which turns out to be critical for convection dominated problems. While little can be said in the way of theoretical results, the proposed preconditioner is shown to perform remarkably well for a broad range of representative test problems. These include compressible flows ranging from very low Reynolds numbers to fully turbulent flows using the Reynolds Averaged Navier Stokes equations discretized on highly stretched grids. This is joint work with J. Peraire (MIT).

**Rivière, Beatrice** (University of Pittsburgh, riviere@math.pitt.edu)

Title: *Coupling of surface and subsurface flows*

Abstract: The study of surface/subsurface interaction is important in the environmental problem of groundwater contamination. A mathematical weak formulation of the coupled Darcy flow with Navier-Stokes flow is presented. A numerical approach coupling discontinuous Galerkin and continuous finite element methods is analyzed. Extensions to time-dependent problem are given.

**Ryan, Jennifer K.** (Virginia Tech, jkryan@vt.edu)

Title: *Local Post-processing for discontinuous Galerkin methods: Mesh, derivatives, and applications*

In this presentation an overview of aspects of post-processing for discontinuous Galerkin (DG) methods will be given. Specifically, we will examine extensions to non-uniform meshes, derivative calculation, and applications to streamlines. Improving the accuracy in the solution and its' derivatives is important for many scientific applications in such areas as fluid mechanics and chemistry. The specific technique that we will examine was shown to improve the order of accuracy from  $k+1$  of the DG approximation to  $2k+1$  for the post-processed solution over a uniform mesh where  $k$  is the highest degree polynomial used in the approximation. We will explore two techniques for extending the applications to smoothly varying and non-uniform mesh structures. Additionally, we will focus on extension of the existing kernel to improve the accuracy in the derivatives of the numerical solution. Lastly, we will present an application that uses one-dimensional filtering to aid in calculation of a streamline, regardless of the dimensionality of the solution.

**Sherwin, Spencer J.** (Imperial College London, s.sherwin@imperial.ac.uk)

Title: *DG Methods using spectral/hp element discretisations: Balancing utility and efficiency*

Abstract: Given the growing interest in discontinuous Galerkin (DG) methods from both the academic and university sectors, it is of interest to ask why and when it is useful to apply the DG formulation in the context of high-order spectral/hp element discretisations.

The spectral/hp element discretisation methodology originated from the confluence of two family of high order finite element methods: p-type finite elements and spectral element methods. Mimicking the p-type finite element techniques, spectral/hp elements employ hierarchical polynomial functions as

trial and test functions as opposed to nodal Lagrange polynomials that are traditionally used in spectral element methods. A potential draw back of hierarchical expansions is that they lead to non-diagonal mass matrix systems which can be costly to invert in hyperbolic problem discretised explicitly in time. The discontinuous Galerkin method provided a significant advantage for this problem over the standard continuous Galerkin formulation particularly when using upwind fluxes which advantageously penalize the dispersive high frequencies.

When we consider, however, parabolic or elliptic problems, the advantages of the DG formulation are not so apparent. For instance, the DG formulation is arguable more complicated to implement since it encapsulates many (but not all) of the continuous Galerkin components. DG formulations typically lead to a greater number of degrees of freedom compared to classical Galerkin formulations, and do not necessarily provide significant improvement in accuracy or compactness, at least for problems with constant coefficients. However, the ability to adapt a DG formulation and the absence of a direct stiffness (or global assembly) process can lead to greater flexibility/adaptivity. In addition, if the solution is required to exist in a discontinuous space then there may not be many obvious alternative. In summary if one already has a continuous Galerkin implementation the benefits of converting to a DG formulation are questionable for many standard problems, especially when using low order polynomial spatial approximations.

In this presentation we will explore the above competing arguments. Finally, we will also outline some of the potential benefits of the recently proposed LDG-Hybrid approach where the issue of additional degrees of freedom have been addressed and which may offer additional advantages in massively parallel implementations. This is joint work with Robert M. Kirby (University of Utah).

**Shu, Chi-Wang** (Brown University, shu@dam.brown.edu)

Title:  *$L^2$ -Stability analysis of the central discontinuous Galerkin method and a comparison between the central and regular discontinuous Galerkin methods*

Abstract: We give stability analysis and error estimates for the recently introduced central discontinuous Galerkin method when applied to linear hyperbolic equations. A comparison between the central discontinuous Galerkin method and the regular discontinuous Galerkin method in this context is also made. Numerical experiments are provided to validate the quantitative conclusions from the analysis. This is joint work with Yingjie Liu, Eitan Tadmor and Mengping Zhang.

**Van der Vegt, Jaap** (University of Twente, J.J.W.vanderVegt@ewi.utwente.nl)

Title: *Discontinuous Galerkin finite element methods for hyperbolic nonconservative partial differential equations*

Abstract: In this presentation a new space-time discontinuous Galerkin finite element (DGFEM) formulation will be discussed for partial differential equations containing nonconservative products, such as occur in dispersed multiphase flow equations. Standard DGFEM formulations cannot be applied to nonconservative partial differential equations. We therefore introduce the theory of weak solutions for nonconservative products into the DGFEM formulation leading to the new question how to define the path connecting left and right states across a discontinuity. The effect of different paths on the numerical solution is investigated and found to be small. We also introduce a new numerical flux that is able to deal with nonconservative products. The scheme is applied to two different systems of partial differential equations. First, we consider the shallow water equations, where topography leads to nonconservative products. Second, a simplification of a depth-averaged two-phase flow model is discussed. This model contains more intrinsic nonconservative products. This is joint work Sander Rhebergen and Onno Bokhove.

**Wang, Wei** (Brown University, wwang@dam.brown.edu)

Title: *The discontinuous Galerkin method for the multiscale modeling of dynamics of crystalline solids*

Abstract: We present a multiscale model for numerical simulation of dynamics of crystalline solids. The method couples nonlinear elastodynamics as the continuum description and molecular dynamics as another component at the atomic scale. The governing equations on the macroscale are solved by the discontinuous

Galerkin method, which is built up with an appropriate local curl-free space to produce coherent displacement field. The constitutive data are based on the underlying atomistic model: it is either calibrated prior to the computation or obtained from molecular dynamics as the computation proceeds. The decision to use either the former or the latter is made locally for each cell based on suitable criteria. This is joint work with Xiantao Li (Penn State) and Chi-Wang Shu (Brown University).

**Warburton, Timothy** (Rice University, timwar@caam.rice.edu)

Title: *Advances in wave propagation with the discontinuous Galerkin method*

Abstract: A range of important features relating to the practical application of discontinuous Galerkin (DG) method for wave propagation will be discussed.

Recent investigations of the spectral properties of the discrete discontinuous Galerkin operators have revealed important connections with their continuous Galerkin counter parts. Theoretical and numerical results will be shown which demonstrate the correct asymptotic behavior of these methods and controls spurious solutions under mild assumptions.

Given the suitability of DG for solving Maxwell's equations and their ability to propagate waves over long distance, it is natural to seek effective boundary treatments for artificial radiation boundary conditions. A new family of far field boundary conditions will be introduced which gracefully transmit propagating and evanescent components out of the domain. These conditions are specifically formulated with DG discretizations in mind, however they are also relevant for a range of numerical methods.

There is an Achilles heel to high order discontinuous Galerkin methods when applied to conservation laws. The methods are typically constructed with polynomial field representations and unfortunately these suffer from excess maximum gradients near the edges of elements. I will describe a simple filtering process that allows us to reduce these anomalous gradients and provably yield a dramatic increase in the maximum allowable time step.

Finally, I will discuss progress in using a posteriori error estimates for mesh adaptivity and demonstrate guaranteed error reduction on refinement for some model (static) problems. These results indicate a need for very fine local refinement of meshes to accurately capture solution singularities. I will show a very simple approach for local time stepping with discontinuous Galerkin methods in order to practically use such meshes in time-domain computations.

**Wheeler, Mary** (University of Texas, mfw@ices.utexas.edu)

Title: *Coupling discontinuous Galerkin and mixed finite element discretizations using mortar elements*

Abstract: Discontinuous Galerkin (DG) and mixed finite element (MFE) methods are two popular methods that possess local mass conservation. In this paper we investigate DG-DG and DG-MFE domain decomposition couplings using mortar finite elements to impose weak continuity of fluxes and pressures on the interface. The subdomain grids need not match and the mortar grid may be much coarser, giving a two-scale method. Convergence results in terms of the fine subdomain scale  $h$  and the coarse mortar scale  $H$  are established for both types of couplings. In addition, a non-overlapping parallel domain decomposition algorithm is developed, which reduces the coupled system to an interface mortar problem. The properties of the interface operator are analyzed. Computational results are presented.

**Wihler, Thomas P.** (McGill University, wihler@math.mcgill.ca)

Title: *A-posteriori error estimation for hp-dG timestepping for parabolic PDEs*

Abstract: We will present an hp-version a posteriori error analysis for the time discretization of parabolic problems by the discontinuous Galerkin (dG) time-stepping. The resulting error estimators are fully explicit with respect to the local time steps and approximation orders. Their performance within an hp-adaptive time stepping procedure shall be illustrated with a series of numerical experiments. This talk comprises joint work with Dominik Schötzau (University of British Columbia).

**Xu, Yan** (University of Science and Technology of China, yxu@ustc.edu.cn)

Title: *A local discontinuous Galerkin method for the Camassa-Holm equation*

Abstract: In this paper, we develop, analyze and test a local discontinuous Galerkin (LDG) method for solving the Camassa-Holm equation which contains nonlinear high order derivatives. The LDG method has the flexibility for arbitrary  $h$  and  $p$  adaptivity. We prove the  $L^2$  stability for general solutions and give a detailed error estimate for smooth solutions, and provide numerical simulation results for different types of solutions of the nonlinear Camassa-Holm equation to illustrate the accuracy and capability of the LDG method. This is joint work with Chi-Wang Shu (Brown University).

**Zhang, Yongtao** (University of Notre Dame, yzhang10@nd.edu)

Title: *Application of a discontinuous Galerkin finite element method to reaction-diffusion systems in developmental biology*

Abstract: Nonlinear reaction-diffusion systems which arise from mathematical modeling in developmental biology are usually highly stiff in both diffusion and reaction terms, and they are built on high dimensional complex geometrical domains due to the complex shape of embryos. Computational challenges come from both the temporal direction and the spatial direction. The stiffness of these reaction-diffusion equations demands efficient temporal numerical discretization. Complex geometrical domain can be handled by finite element spatial discretization. Based on the recent work by Cheng and Shu, we combined the DG scheme with Strang's operator splitting and the Crank-Nicholson discretization to solve the nonlinear reaction-diffusion systems, and overcome the difficulties from high stiffness of the system and the complexity of the geometrical domain. Directly solving a coupled nonlinear system like the standard implicit schemes was avoided. We applied the method to various reaction-diffusion models in developmental biology, including a system from the skeletal pattern formation in developing chick limb, to show the accuracy and efficiency of the method. This is a joint work with Jianfeng Zhu, Mark Alber and Stuart A. Newman.

**Zhang, Zhimin** (Wayne State University, zzhang@math.wayne.edu)

Title: *Discontinuous Galerkin methods for singularly perturbed problems*

Abstract: We study DG methods for convection diffusion equations. When the diffusion coefficient  $\epsilon$  is small, the underlying equation is singularly perturbed. Compared with continuous Galerkin methods, i.e., the traditional finite element methods, DG methods have the advantage to avoid non-physical oscillations. Our main concern is the convergence independent of the singular perturbation parameter  $\epsilon$ . Furthermore, we shall apply recovery techniques to the DG methods by identifying some superconvergence points and using them to reconstruct local solutions in achieving higher order convergence on the whole domain or some sub-domains. Both the one-dimensional and two-dimensional model problems are discussed. Numerical examples will be presented.

This is joint work with Huiqing Zhu (Wayne State).