Euler & Gauss-Bonnet for infinite graphs

- $\mathcal{R}$ a locally finite polygonal surface
- uniformly bounded edge-lengths $\frac{1}{L} \leq \ell(e) \leq L$
- $\kappa^- := \sum_{v \in V : \kappa(v) < 0} \kappa(v)$ converges
- The number of ends (t) of $G(\mathcal{R})$ is finite.

THEOREM (DeVos, BM, '07)
Under the above assumptions $\mathcal{R}$ is homeomorphic to $S - t$, a compact surface S with t points removed and

$$\boxed{\sum_{v \in V} \kappa(v) \leq 2\pi \left(\chi(S) - t \right)}$$

By adding "curvatures of the ends", this can be made into = (B. Chen & G. Chen)

$$\sum_{v}' \kappa(v) + \sum_{\varepsilon \in \mathcal{E}} \kappa(\varepsilon) = 2\pi \left(\chi(S) - t \right)$$