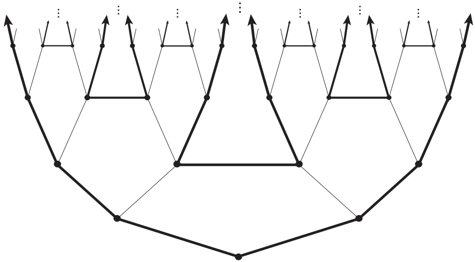
Fleischner's Theorem for Infinite Graphs

Angelos Georgakopoulos

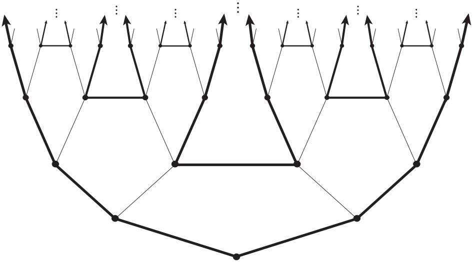
Methematisches Seminar der Universität Hamburg

Hamilton circles



The wild circle

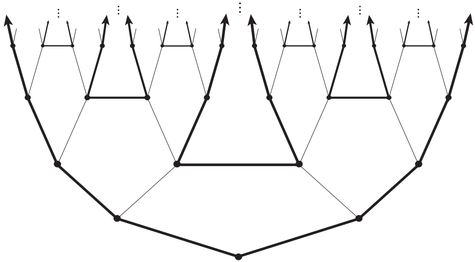
Hamilton circles



The wild circle
is a Hamilton circle:

A homeomorphic image of S^1 in $|G|$ containing all vertices

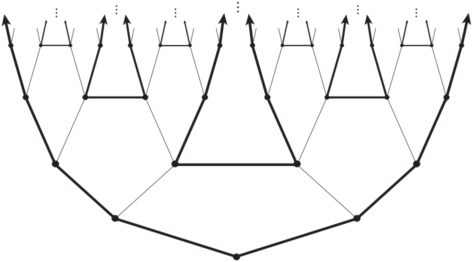
Hamilton circles



The wild circle
is a Hamilton circle:

A homeomorphic image of S^1 in $|G|$ containing all vertices
and all ends?

Hamilton circles



Hamilton circle:

A homeomorphic image of S^1 in $|G|$ containing all vertices.

Fleischner's Theorem

Theorem (Fleischner '74)

The square of a finite 2-connected graph has a Hamilton cycle.

Fleischner's Theorem

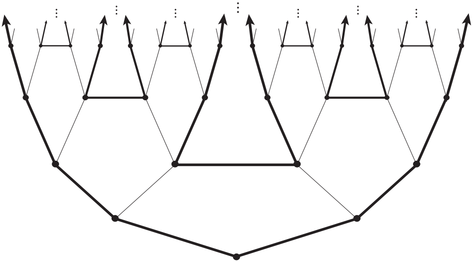
Theorem (Fleischner '74)

The square of a finite 2-connected graph has a Hamilton cycle.

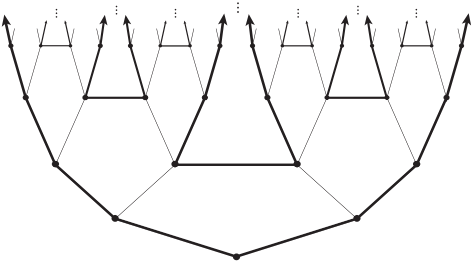
Theorem (Thomassen '78)

The square of a locally finite 2-connected 1-ended graph has a spanning double ray.

The Theorem



The Theorem



Theorem (G '06)

The square of a locally finite 2-connected graph has a Hamilton circle.

Structure of the Proof

- make G eulerian by deleting some edges and doubling some of the remaining ones

Structure of the Proof

- make G eulerian by deleting some edges and doubling some of the remaining ones
- pick an Euler tour

Structure of the Proof

- make G eulerian by deleting some edges and doubling some of the remaining ones
- pick an Euler tour
- perform "lifts" to turn the Euler tour into a Hamilton cycle

End-faithful Euler Tours

Theorem (G '06)

If a locally finite graph has an Euler tour then it also has one visiting each end exactly once.

(Euler tour: A continuous image from S^1 to $|G|$ traversing each edge exactly once.)

Structure of the Proof

- make G eulerian by deleting some edges and doubling some of the remaining ones
- pick an Euler tour
- perform "lifts" to turn the Euler tour into a Hamilton cycle

Structure of the Proof

- make G eulerian by deleting some edges and doubling some of the remaining ones
- pick an Euler tour
- perform “lifts” to turn the Euler tour into a Hamilton cycle

The hardest part is

how to avoid conficts
(i.e. make sure you don't lift any edge at both endvertices)

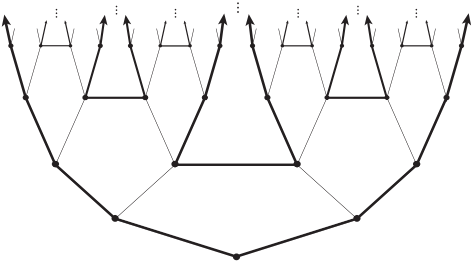
Structure of the Proof

- make G eulerian by deleting some edges and doubling some of the remaining ones
- pick an Euler tour
- perform "lifts" to turn the Euler tour into a Hamilton cycle

The hardest part is

how to avoid conficts
(i.e. make sure you don't lift any edge at both endvertices)
and
how to maintain the end-topology

The Theorem



Theorem (G '06)

The square of a locally finite 2-connected graph has a Hamilton circle.

Further reading

- AG: "Infinite hamilton cycles in squares of locally finite graphs", Preprint 2007

  http://www.math.uni-hamburg.de/home/georgakopoulos/infinitefleischner.pdf

Further reading

- AG: "Infinite hamilton cycles in squares of locally finite graphs", Preprint 2007

  http://www.math.uni-hamburg.de/home/georgakopoulos/infinitefleischner.pdf

Theorem (G '06)

If G is a locally finite connected graph then $|G^3|$ has a Hamilton circle.

Further reading

- AG: "Infinite hamilton cycles in squares of locally finite graphs", Preprint 2007

  http://www.math.uni-hamburg.de/home/georgakopoulos/infinitefleischner.pdf

Theorem (G '06)

If G is a locally finite connected graph then $|G^3|$ has a Hamilton circle.

- AG: "A short proof of Fleischner's theorem", Preprint 2007

  http://www.math.uni-hamburg.de/home/georgakopoulos/shortFleischner.pdf

Open problems

Conjecture

If G is a 4-edge-connected locally finite graph then $|L(G)|$ contains a Hamilton circle.

Open problems

Conjecture

If G is a 4-edge-connected locally finite graph then $|L(G)|$ contains a Hamilton circle.

Theorem (Jaeger)

If $F \subseteq E(G)$ contains no odd cut of G then F can be extended to an element of $\mathcal{C}(G)$.

Open problems

Conjecture

If G is a 2-connected countable graph then $|G^2|$ contains a Hamilton circle.

Open problems

Conjecture

If G is a 2-connected countable graph then $|G^2|$ contains a Hamilton circle.

Conjecture

If G is a connected countable graph then $|G^3|$ contains a Hamilton circle.


"That's all Folks!"