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## TOPOLOGY

February 25 – March 2, 2007

### PROPOSED TALKS AND STATEMENT OF INTERESTS

Adem, Alejandro  
University of British Columbia

Title: Commuting Elements and Spaces of Homomorphisms

Consider the space  $\text{Hom}(Q, G)$  of homomorphisms between a discrete group  $Q$  and a Lie group  $G$ . In this talk I will describe basic properties of these spaces and how for certain discrete groups their contribution to bundle theory can be quantified using the cohomology of  $Q$ . We will also discuss the cohomology of some of these spaces, with particular attention to the case when  $Q$  is a free abelian group. A stable splitting for the space of commuting elements will be described. This is joint work with Fred Cohen.

Baird, Tom  
University of Toronto

Moduli spaces of flat connections on nonorientable surfaces

I will present some recent work studying the topology of moduli spaces of flat connections on nonorientable surfaces and describe some relationships with their counterparts for orientable surfaces. The main tool will be equivariant cohomology.

Bartels, Arthur  
Universität Münster

The Farrell-Jones Conjecture in algebraic K-theory for hyperbolic groups.

This is joint work with Wolfgang Lück and Holger Reich. We prove the Farrell-Jones Conjecture in algebraic K-theory for hyperbolic groups in the sense of Gromov. This means that the algebraic K-theory  $K_*^{\text{alg}}(RG)$  of  $RG$ , for a ring  $R$  and a hyperbolic group  $G$ , can be computed in terms of  $K_*^{\text{alg}}(RV)$ , where  $V$  varies over the family of virtually cyclic subgroups. This result has (among others) applications to Whitehead groups, the Bass conjectures and the Kaplansky conjecture.



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Behrstock, Jason  
University of Utah

Title: Dimension and rank of mapping class groups.

We will discuss recent work with Yair Minsky towards understanding the large scale geometry of the mapping class group. In particular, we'll explain how to obtain various topological properties of the asymptotic cone of the mapping class group including a computation of its dimension. An application of this analysis is an affirmative solution to Brock-Farb's Rank Conjecture which asserts that MCG has quasi-flats of dimension  $N$  if and only if it has a rank  $N$  free abelian subgroup.

I thought would be of interest to a broad audience of topologists since it contains geometric group theory, low dimensional topology, and some classical dimension theory.

Bryan, Jim  
University of British Columbia

Title: The Quantum McKay Correspondence

Abstract: Let  $G$  be a finite subgroup of  $SU(2)$  or  $SO(3)$ . The classical McKay correspondence describes the cohomology of the resolution of the orbifold  $C^2/G$  or  $C^3/G$  in terms of the representation theory of  $G$ . We give a quantum version of this. We describe the quantum cohomology (and, more generally, all the Gromov-Witten invariants) of the resolution in terms of the ADE root system associated to  $G$ .

Davis, Jim  
Indiana University

Mapping tori of self-homotopy equivalences of lens spaces (or - there are no exotic beasts in Hillman's zoo)

This is joint work with Shmuel Weinberger. We conjecture that the mapping torus of a self-homotopy equivalence of three-dimensional lens spaces is homotopy equivalent to a closed manifold. We prove this conjecture in the case where the lens space has prime order fundamental group. A feature of the proof is that it uses Gauss' Lemma on quadratic residues. This answers a question of Jonathan Hillman.



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Brent Doran  
Institute for Advanced Study, Princeton

Title: Unipotent groups, contractible varieties, and some classical questions in affine geometry

Abstract: The study of contractible topological spaces began in earnest in 1935 with J.H.C. Whitehead's construction of the Whitehead space--a counter-example to his proof of the 3-dimensional Poincaré conjecture. In the 1960s geometric topologists studied properties of contractible topological spaces in detail as a testing ground for general theory. We investigate algebraic varieties which are contractible from the standpoint of algebraic geometry, formalized using the  $A^1$ -homotopy theory of Morel and Voevodsky. The class of such varieties is surprisingly rich including many smooth examples beyond affine spaces and, in many ways, the theory is analogous to the theory developed for contractible topological spaces. Over  $C$  or  $R$ , many of these are diffeomorphic to  $C^n$  or  $R^n$ . We will discuss a general construction of such varieties using a version of geometric invariant theory for unipotent groups and show how they relate to, and provide a testing ground for, various long-standing general conjectures in algebraic geometry. A basic conclusion: many of our most sophisticated invariants miss an enormous amount of structure in algebraic geometry. Optimistic corollary: we should look to methods of topology, adapted to algebraic geometry via  $A^1$ -homotopy theory, for aid in formulating classification theorems. Joint work with Aravind Asok.

Other current research interests: non-reductive geometric invariant theory and applications; moduli problems, especially at the moment moduli of bundles on curves and sheaves on surfaces;  $A^1$ -homotopy, motives, and motivic cohomology; intersection cohomology of compactifications of locally symmetric spaces. There are some interesting interrelations among these topics.

Ebert, Johannes  
Muenster

Title: Spin structures on surface bundles

A spin structure on a surface bundle  $\pi: E \rightarrow B$  with connected compact oriented fiber  $F$  is a spin structure on the vertical tangent bundle  $T_{\text{v}} E$ . We address the question of necessary and sufficient conditions on the existence of a spin structure. First of all, there must exist a spin structure  $\sigma$  on  $F$  which is invariant under the image of the monodromy homomorphism  $\pi_1(B) \rightarrow \pi_0(\text{Diff}(F))$ . But this is not sufficient. For any spin structure  $\sigma$  on  $F$ , there exists a class in  $H^2(B; \text{Diff}(F))$



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$(F; \sigma); \mathbb{Z}/2$  which is an obstruction to the existence of a spin structure on a surface bundle whose monodromy fixes  $\sigma$ . We show that this obstruction class is nonzero for any spin structure on any surface.

As necessary conditions for the existence of spin structures, we have divisibility relations for the Mumford classes  $\kappa_n (\pi) \in H^{2n}(B; \mathbb{Z})$ . We show that the previously known divisibility relations without the assumption of a spin structure is strengthened by the factor  $2^{n+1}$ . For even  $n$ , we show that this relation is optimal in the stable range, i.e. if the genus  $g$  of  $F$  is large compared to  $n$ .

Galatius, Soren  
Stanford University

Title: The homotopy type of the cobordism category.

Abstract: The  $d$ -dimensional cobordism category  $C_d$  has closed  $(d-1)$ -dimensional manifolds as objects and compact  $d$ -dimensional cobordisms as morphisms. Thom's theorem determines  $\pi_0$  of the classifying space  $BC_d$ . I will discuss joint work with Madsen, Tillmann and Weiss, in which we determine the homotopy type of  $BC_d$ . As a corollary we get a new proof of Madsen-Weiss' theorem.

Ghiggini, Paolo  
Université du Québec à Montréal

Title: Contact structures, Heegaard Floer homology, and fibred knots

Recently I proposed a strategy to prove that knot Floer homology detects fibred knots using taut foliations and contact structures. This strategy was implemented by myself in the particular case of genus-one knots, and by Yi Ni in the general case.

In the talk I will outline the strategy, and give some hints about the proof in the case of genus-one fibred knots. I will also point out the difficulties that Yi Ni had to overcome in order to arrive to a complete proof.

Grodal, Jesper  
Chicago/Copenhagen

Title: Local-to-global principles for classifying spaces

Abstract: In this talk I will show how one can sometimes "uncomplete" the  $p$ -completed classifying space of a finite group, to obtain the original (non-completed) classifying space, and hence the original finite group. This "uncompletion" process is closely related to well-known local-to-global



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questions in group theory, such as the classification of finite simple groups. The approach goes via the theory of  $p$ -local finite groups, more precisely a certain fundamental group. This talk is a report on joint work with Bob Oliver.

Hanke, Bernhard  
University of Munich  
Enlargeability, coarse geometry and the Baum-Connes map

Enlargeability was introduced by Gromov and Lawson as an obstruction to the existence of positive scalar curvature metrics on closed spin manifolds  $M$ .

Rosenberg introduced another "universal" index theoretic obstruction living in the  $K$ -theory of the reduced or maximal  $C^*$ -algebra of the fundamental group  $\pi_1(M)$ . We report on recent work of Kotschick, Roe, Schick and myself proving nonvanishing of this index obstruction for enlargeable manifolds. Our approach is independent from injectivity of the Baum-Connes assembly map.

The discussions of the reduced and maximal  $C^*$ -algebra use quite different methods: The former one has a strong coarse theoretic flavour, whereas the latter one rests on the construction of a flat Hilbert space bundle (as twisting bundle for the Dirac operator on  $M$ ) out of a sequence of asymptotically flat bundles.

This construction can also be used to prove injectivity of the restriction of the Baum-Connes assembly map (with values in the  $K$ -theory of the maximal  $C^*$ -algebra) to  $K$ -homology classes dual to classes of cohomological degree  $2$ . This verifies the strong Novikov conjecture for these classes and implies a result of Mathai and Connes-Gromov Moscovici on the invariance of higher signatures associated to cohomology classes of degree  $2$ .

Hausmann, Jean-Claude  
L'Université de Genève

The topology and geometry of Polygon spaces.

The study of polygon spaces in  $\mathbb{R}^d$  started two decades ago with the thesis of K. Walker (for  $d=2$ ). They occur in connection with statistical shape theory and robotic. For  $d=3$ , they became also a chapter of Hamiltonian geometry, as a rich source of examples, closely related to toric manifolds. This talk will be a survey of these various aspects of polygon spaces, their classification and recent results.



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Hedden, Matthew  
Massachusetts Institute of Technology

Title: On knot Floer homology and algebraic curves

Abstract: It is well known that each torus knot arises as the intersection of an algebraic curve in  $\mathbb{C}^2$  with isolated singularity at the origin with the standard three-dimensional sphere. Indeed, the class of knots which arise in this way from algebraic curves with an isolated singularity is well understood. However, by deforming the sphere or relaxing the restriction on the curve's singular locus a much wider class of knots and links is obtained. This talk will discuss the question of which knots arise from algebraic curves in the above sense, focusing our attention on some results indicating connections with the Ozsvath-Szabo Floer homology invariants. More precisely, Ozsvath and Szabo introduced an invariant, denoted  $\tau(K)$ , to knots in the three-sphere (this invariant was independently discovered by Rasmussen). We first show that  $\tau(K)$  provides an obstruction to knots arising from complex curves in the above sense. Restricting attention to fibered knots, we then prove the more surprising theorem that  $\tau(K)$  detects when a fibered knot arises from a complex curve with a certain genus constraint. Coupled with work of Ozsvath and Szabo and recent independent work of Ghiggini, Ni, and Juhasz, an immediate corollary is that any knot which admits a lens space surgery can be realized as the intersection of a complex curve with the three-sphere.

Ji, Lizhen  
University of Michigan

Title: Large scale geometry and topology of subgroups of Lie groups and mapping class groups

For a discrete group, a natural problem concerns different versions of the Novikov conjecture in surgery theory, algebraic K-theory and  $C^*$ -algebras. The original Novikov conjecture on homotopy invariance of higher signatures is equivalent to the rational Novikov conjecture in surgery theory, and the integral Novikov conjecture in surgery theory implies the stable Borel conjecture.

One approach to the Novikov conjecture uses the asymptotic dimension of the group endowed with a word metric, and another approach uses suitable compactifications of cofinite universal spaces for proper actions.



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We will study the validity of the integral Novikov conjecture and the existence of cofinite universal space for proper actions for the following closely related classes of groups:

1. Arithmetic groups such as  $SL(n, \mathbb{Z})$ , and more generally lattice subgroups of Lie groups
2. S-arithmetic subgroups of semisimple algebraic groups such as  $SL(n, \mathbb{Z}[1/p])$ , which are usually not discrete subgroups of Lie groups.
3. Finitely generated subgroups of  $GL(n, \mathbb{Q})$ .
4. Mapping class groups  $\text{Mod}_{\{g, n\}}$  of surfaces of genus  $g$  with  $n$  punctures.

Symmetric spaces, Bruhat-Tits buildings, Teichmüller spaces and their compactifications will be used together with basic tools such as the reduction theory for arithmetic subgroups. They will also bring out similarity of these objects.

Juan-Pineda, Daniel  
Universidad Nacional Autonoma de Mexico

Kreck, Matthias  
University of Heidelberg

Equivariant (co)homology and Poincaré duality

Equivariant (co)homology defined via the Borel construction does not fulfill Poincaré duality. To motivate what I am working on consider a closed oriented free smooth  $m$ -dimensional  $G$ -manifold ( $G$  a compact Lie group of dimension  $d$ ). Then the equivariant homology of  $M$  is the homology  $H_k(M/G)$  of  $M/G$ . By ordinary Poincaré duality this is isomorphic to  $H^{m-d-k}(M/G)$ .

Question: Is there an equivariant multiplicative cohomology theory  $h^r_G(M)$  such that if  $M$  is as above a closed free  $G$  manifold, then  $h^r_G(M) = H^r(M/G)$ ? If yes we then consider the shifted equivariant homology  $h_k^G(M) = H_{k+d}^G(M)$ . Poincaré duality for the theory  $h$  then holds for closed free  $G$ -manifolds and one can ask if this is the case for arbitrary closed  $G$ -manifolds.

I have constructed such a theory  $h^k_G(M)$  as bordism classes of free  $G$  stratifolds of dimension  $m - k$  together with a proper equivariant map to  $M$ . The corresponding homology theory  $h_k^G(M)$  of bordism classes of free compact  $G$ -stratifolds with an equivariant map to  $M$  is canonically isomorphic to  $H_{k+d}^G(M)$ . Thus we obtain a geometric description of equivariant homology. Presently I'm investigating this new cohomology theory further and construct corresponding Bredon type equivariant (co)homology theories.

Lueck, Wolfgang  
Universitat Munster



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Topological rigidity for non-aspherical manifolds (with M. Kreck)

The Borel Conjecture predicts that closed aspherical manifolds are topological rigid. We want to investigate when a non-aspherical oriented connected closed manifold  $M$  is topological rigid in the following sense. If  $f: N \rightarrow M$  is an orientation preserving homotopy equivalence with a closed oriented manifold as target, then there is an orientation preserving homeomorphism  $h: N \rightarrow M$  such that  $h$  and  $f$  induce up to conjugation the same maps on the fundamental groups. We call such manifolds Borel manifolds. We give partial answers to this questions for  $S^k \times S^d$ , for sphere bundles over aspherical closed manifolds of dimension less or equal to 3 and for 3-manifolds with torsionfree fundamental groups. We show that this rigidity is inherited under connected sums in dimensions greater or equal to 5. We also classify manifolds of dimension 5 or 6 whose fundamental group is the one of a surface and whose second homotopy group is trivial.

Equivariant Chern characters

We first recall Dolds rational computation of a generalized homology theory in terms of

singular homology. Essentially Dold shows that the Atiyah-Hirzebruch spectral sequence rationally collapses.

The aim of this talk is to generalize it to the equivariant setting.

We introduce the notion of an equivariant homology theory. We explain how under certain assumptions

on the coefficients such as a Mackey structure it can be computed in terms of Bredon homology.

This has many applications in connection with the Farrell-Jones Conjecture, and the Baum-Connes Conjecture and leads

to a rational computation of the topological  $K$ -theory of  $BG$  for a discrete group  $G$  which has a finite model for its classifying space of proper  $G$ -actions.

Lurie, Jacob

Harvard University

Equivariant Cohomology Theories and Algebraic Groups

I will sketch a construction which produces an equivariant cohomology theory starting with an algebraic group (in a suitable setting). If time permits, I will explain how this construction can be used to produce equivariant elliptic cohomology.

List of interests: Interactions between homotopy theory and algebraic geometry, elliptic cohomology, geometric representation theory.



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Morel, Fabien  
Ludwig-Maximilians-Universität Munich

Title: Towards a surgical approach to the classification of smooth projective varieties over a field

In this talk we will sketch a new approach to the study of smooth projective  $A^1$ -connected varieties over a field inspired by the classical surgery approach in differential topology. This approach relies on recent progresses in the  $A^1$ -homotopy theory of smooth varieties. We will explain basic facts concerning the  $A^1$ -fundamental group and will illustrate the slogan that it should play a major role in this approach, as in classical differential topology.

Morgan, John  
Columbia

Title: Overview of Perelman's proof of the Poincaré Conjecture and the Geometrization Conjecture.

Starting with the Ricci flow introduced by Hamilton, Perelman showed how to control the finite-time singularities in 3-dimensional flows and consequently extend such a flow to a Ricci with surgery defined for all positive time. The surgeries analytically necessary to deal with the finite-time singularities in fact perform the topological operation of connected sum decomposition necessary in order to simplify 3-manifolds into prime pieces. With the existence result for Ricci flow with surgery defined for all positive times, and a complete understanding of the topological change at the surgery times, to prove the geometrization conjecture for a compact 3-manifold it suffices to prove it for any of the 3-manifolds that appear in the Ricci flow with surgery at any later time. The proof of the Poincaré Conjecture is completed by showing that if the initial 3-manifold is a homotopy sphere then after some finite time the 3-manifold that appears in the resulting Ricci flow with surgery is empty (and hence satisfies the Geometrization Conjecture).

To prove the general geometrization conjecture requires studying the limits as time goes to infinity in a general 3-dimensional Ricci flow with surgery. Here is where the incompressible tori appear and according the pieces that



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result from cutting the manifold open along these tori are either hyperbolic or are collapsed. Perelman then states a result showing that the collapsed pieces are graph manifolds. This allows one to prove the full geometrization conjecture.

In these lectures we will give an overview of the ideas and techniques that go into these arguments and give an evaluation of the current state of confidence that these arguments are complete and correct.

Olbermann, Martin  
University of Heidelberg

Title: Conjugations on six-manifolds

When we are trying to find simply-connected asymmetric manifolds, i.e. manifolds not admitting any non-trivial finite group action (for example among spin 6-manifolds), V. Puppe's method shows that in some cases, the only possible action would have to be a "conjugation". Conjugation spaces are spaces with involution such that the fixed point set of the involution has  $H_2$ -cohomology isomorphic to the  $H_2$ -cohomology of the space itself, with the little difference that all degrees are divided by two (e.g.  $\mathbb{C}P^n$  with the complex conjugation). One also requires that a certain conjugation equation is fulfilled.

I will apply a new characterization of conjugation spaces to realize conjugation 6-manifolds. The main result is that for every closed oriented 3-manifold  $M$  there exists a simply connected spin conjugation 6-manifold with fixed point set  $M$ .

Pedersen, Erik  
SUNY Binghamton

A few years ago T. Bauer, N. Kitchloo, D. Notbohm and I proved that if  $X$  is a loop space and the homology of  $X$  is finitely generated as an abelian group then  $X$  is homotopy equivalent to a compact, smooth, parallelisable manifold. It is likely this result holds without assuming that  $X$  is a loop space only assuming  $X$  is an H-space. This is not even known in the simply connected case because of our very poor understanding of the Arf invariant. So one of the things I am thinking about is how to get a better understanding of the Arf invariant. Notice this is different from trying to prove the so-called Arf invariant problem.

Ranicki, Andrew  
University of Edinburgh



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## Survey of codimension one splitting

Much of high-dimensional manifold topology depends on codimension 1 splitting techniques, using algebraic K- and L-theory to decide if a homotopy equivalence of manifolds can be split along a codimension 1 submanifold. The talk will survey the obstruction theory involved, and some of the applications.

## The geometric Hopf invariant

This is a joint project with Michael Crabb. The geometric Hopf invariant of a stable map  $F: \Sigma^k X \rightarrow \Sigma^k Y$  is a  $\mathbb{Z}_2$ -equivariant map  $h_{\{R^k\}(F)}$  which counts the double points of  $F$ . The homotopy class of  $h_{\{R^k\}(F)}$  is the primary obstruction to  $F$  being homotopic to the  $k$ -fold suspension  $\Sigma^k F_0$  of an unstable map  $F_0: X \rightarrow Y$ . The geometric Hopf invariant has applications to double points of immersions of manifolds, and to surgery obstruction theory, including the non-simply connected cases.

Rosenthal, David

St. Johns University

Title: On the K-theory of groups with finite asymptotic dimension

Abstract: In this work it is proved that the assembly maps in algebraic K- and L-theory with respect to the family of finite subgroups is injective for groups with finite asymptotic dimension that admit a finite model for the classifying space for proper actions. The result also applies to certain groups that admit only a finite dimensional model for this space. In particular, it applies to discrete subgroups of virtually connected Lie groups. This is joint work with Arthur Bartels.

Sauer, Roman

University of Chicago

On and around proportionality of the simplicial volume of finite volume manifolds.

Abstract: The simplicial volume of compact and non-compact manifolds can behave quite differently. We give a criterion saying in which cases the proportionality principle for Riemannian finite volume manifolds holds. In contrast to that, the well-known proportionality theorem for closed manifolds holds in general. Furthermore, we explain some related results about the relation between the locally finite and the relative simplicial volume and the relation to  $L^2$ -Betti numbers. This joint work with Clara Loeh



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Schommer-Pries, Chris  
Berkeley

Stern, Ronald  
University of California, Irvine

Together with Ron Fintushel we have created techniques that demonstrate how to change smooth structures on a given smooth 4-manifold and how to create infinitely many distinct smooth structures. Our current goal is to show that any two smooth 4-manifolds are homeomorphic iff they are obtained by a sequence of these operations.

Symington, Margaret  
Mercer University  
Title: Applications of toric geometry to more general manifolds.

Abstract: This talk will be an advertisement for the use of techniques motivated by toric geometry in dimension four to study the topology four-manifolds. Toric four-manifolds are quite tame, consisting exclusively of  $S^2 \times S^2$  and blowups of  $CP^2$ . However, if one is willing to consider a toric structure on only part of the manifold, one can exploit the "local toric structure" to prove that a smooth surgery (rational blowdowns) preserves symplectic structures.

More recently, David Gay and I relaxed the symplectic condition on a toric manifold to characterize "toric near-symplectic manifolds". Doing so provides both examples and tools to understand and calculate (in terms of graphs in moment map images) emerging Gromov-Witten type invariants due to Taubes.

Taylor, Laurence  
Notre Dame

Homology with local coefficients:

Farrell and Hsiang noticed that the action of conjugation on Wall groups implies that the geometric surgery groups defined in Wall Chapter 9 do not have the naturality Wall claims for them. They fixed the problem.

The observation here is that the definition of geometric Wall groups involves homology with local coefficients and these also lack Wall's claimed naturality.

One would hope that a geometric bordism theory involving non-orientable manifolds would enjoy the same naturality as that enjoyed by homology with local  $\mathbb{Z}$  coefficients. A setting for this naturality entirely in terms of local  $\mathbb{Z}$  coefficients is presented in this paper.



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Applying this theory to the example of non-orientable Wall groups restores much of the elegance of Wall's original approach.

Even manifolds:

A  $4k$ -dimensional, oriented manifold is even if the intersection form on the integral homology has all squares even. There is a condition on the tangent bundle which is equivalent to even and following Lashof we can study the resulting structures on bundles. Several corollaries will be given including computing the resulting bordism groups in terms of more classical ones. The 4-manifold case is especially interesting.

Pin structures on surfaces

This note records some results about  $\text{pin}^\pm$ - structures on surfaces that probably should have been included in Kirby-Taylor. The action of the symplectic group is described using quadratic enhancements. The quadratic enhancement vanishes on the Lagrangian determined on a boundary is proved as well as a bit more.

The quadratic enhancement on a dual to  $w_2$  in an oriented 4-manifold vanishes on the image of  $H^1$  is proved.

Unlu, Ozgun

McMaster University

Free actions of extraspecial  $p$ -groups on products of spheres

Abstract : Let  $p$  be an odd regular prime, we show that the extraspecial  $p$ -group of order  $p^3$  and exponent  $p$  acts freely and smoothly on two equidimensional spheres. We also discuss the problem for  $p$ -groups of larger order and give some partial results. (Joint work with Ian Hambleton.)

Vogtmann, Karen

Cornell University

Outer Spaces of Right-angled Artin groups

Right-angled Artin groups form a bridge between free groups and free abelian groups, and hence their outer automorphism groups can be thought of as interpolating between  $\text{Out}(F_n)$  and  $\text{GL}(n, \mathbb{Z})$ . The group  $\text{Out}(F_n)$  is the group of symmetries of Outer space, a space of actions of  $F_n$  on trees, and  $\text{GL}(n, \mathbb{Z})$  is a group of symmetries of the homogeneous space  $\text{GL}(n, \mathbb{R})/\text{O}(n, \mathbb{R})$ , which can be described as a space of actions of  $\mathbb{Z}^n$  on  $\mathbb{R}^n$ . We define an outer space for the outer automorphism group of a right-angled Artin groups  $G$ , in the case when the associated graph is connected and has no triangles, as a space of actions of  $G$  on appropriate objects. We prove that this space is finite-dimensional and contractible, that the action is proper, and we give upper and lower bounds on the virtual cohomological dimension of



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the outer automorphism group.

Wahl, Nathalie  
University of Copenhagen

Stabilizing mapping class groups of 3-manifolds (joint work with Allen Hatcher)

Let  $M$  be a compact, connected 3-manifold with a fixed boundary sphere  $d_0M$ . For each prime manifold  $P$ , we consider the mapping class group of the manifold  $M_n^P$  obtained from  $M$  by taking a connected sum with  $n$  copies of  $P$ . We prove that the  $i$ th homology of this mapping class group is independent of  $n$  in the range  $n > 2i + 1$ . Our theorem moreover applies to certain subgroups of the mapping class group and include, as special cases, homological stability for the automorphism groups of free groups and of other free products, for the symmetric groups and for wreath products with symmetric groups.

Williams, Bruce  
University of Notre Dame

A Parametrized Signature Theorem with Converse  
(Joint work with Michael Weiss)

Suppose  $X^n$  is an oriented  $n$ -dim Poincare complex. If  $4|n$ , the signature of  $X$ ,  $\sigma(X) \in \mathbf{Z}$  is defined using symmetric structure on  $H_{\frac{n}{2}}(X)$ . If  $X$  is a manifold, then Hirzebruch showed  $\sigma(X)$  has a "local description" in terms of Pontrjagin classes. This follows from the index theorem applied to the signature operator.

By using the symmetric structure on  $C(\tilde{X})$ , the cellular chain complex of the universal cover of  $X$ , Ranicki defined the (visible) symmetric signature of  $X$ ,  $\sigma_V(X)$  which is a refinement of  $\sigma(X)$ . He proved that when  $n > 4$ ,  $X$  is homotopy equivalent to a topological manifold if and only if  $\sigma_V(X)$  has a local description in terms of a symmetric L-theory fundamental class for  $X$ .

If  $p: E \rightarrow B$  is a fibration with fibers  $n$ -dim Poincare complexes, then  $p$  has a parametrized (visible) symmetric signature,  $\sigma_V(p)$ . If  $p$  is a topological fiber bundle with closed  $n$ -dim fibers, then  $\sigma_V(p)$  satisfies a certain fiberwise index theorem.

In this talk I'll describe a further refinement  $\sigma_{VA}(p)$  of  $\sigma_V(p)$ . We again get a family index theorem, but we also get a



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converse when  $\dim B < n/3$ ,  $B$  is path connected, and  $p^{-1}(b)$  is homotopy equivalent to a smooth manifold for some  $b \in B$ . Then the fibration  $p$  satisfies our signature family index theorem if and only if  $p$  is fiber homotopy equivalent to a fiber bundle with fibers closed  $n$ -dim manifolds.