



**Banff International Research Station**

for Mathematical Innovation and Discovery

# **Operator methods in fractal analysis, wavelets and dynamical systems (06w5027)**

**December 2 – December 7, 2006**

**Organizers:**

Ola Bratteli (University of Oslo, Norway)

bratteli@math.uio.no

Palle Jorgensen (The University of Iowa, USA)

jorgen@math.uiowa.edu

David Kribs (University of Guelph, Canada)

dkribs@uoguelph.ca

Gestur Olafsson (Louisiana State University, USA)

olafsson@lsu.edu

Sergei Silvestrov (Lund University, Sweden)

sergei.silvestrov@math.lth.se

## MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by bridge on 2<sup>nd</sup> floor of Corbett Hall). Hours: 6 am–12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.

## MEALS

\*Breakfast (Buffet): 7:00 – 9:00 am, Donald Cameron Hall, Sunday – Thursday

\*Lunch (Buffet): 11:30 am – 1:30 pm, Donald Cameron Hall, Sunday – Thursday

\*Dinner (Buffet): 5:30 – 7:30 pm, Donald Cameron Hall, Saturday – Wednesday

Coffee Breaks: As per daily schedule, 2<sup>nd</sup> floor lounge, Corbett Hall

\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

## \*\*\*\*\*PRELIMINARY SCHEDULE\*\*\*\*\*

### Saturday, December 2

- 16:00 Check-in begins (Front Desk – Professional Development Centre - open 24 hours)  
Lecture rooms available after 16:00 (if desired)
- 17:30-19:30 Buffet Dinner, Donald Cameron Hall
- 20:00 Informal gathering in 2<sup>nd</sup> floor lounge, Corbett Hall (if desired)  
Beverages and small assortment of snacks available on a cash honour-system.

### Sunday, December 3

7:00-8:45 Breakfast

Chair: Gestur Olafsson

8:45-9:00 Introduction and Welcome to BIRS by BIRS Station Manager, Max Bell 159

9:00-9:40 Palle Jorgensen (The University of Iowa, USA)  
Wavelets on Fractals.

9:45-10:25 Dorin Dutkay (Rutgers University, USA)  
Covariant representations, scaling functions and affine fractals

10:25-10:40 Coffee Break, 2<sup>nd</sup> floor lounge, Corbett Hall

10:40-11:20 David R. Larson (Texas A&M University, USA)  
Frames and Operator Theory

11:30-13:30 Lunch

Chair: Palle Jorgensen

13:30-14:10 Iain Raeburn (University of Newcastle, Australia)  
Direct limits, the Cuntz relations and wavelets.

14:15-14:55 Nadia S. Larsen (University of Oslo, Norway)  
Projective multi-resolution analyses arising from direct limits of Hilbert modules

14:55-15:15 Coffee Break, 2<sup>nd</sup> floor lounge, Corbett Hall

15:15-15:55 Kjetil Røysland (University of Oslo, Norway)  
Transition operators on bundle maps.

16:00-16:40 Jonas D'Andrea (University of Colorado, USA)  
Fractal wavelets of Dutkay-Jorgensen Type for the Sierpinski gasket space

16:45-17:25 Mary Beth Ruskai (Tufts University, USA)  
The relation between frames and POVM's in quantum information theory

17:30-19:30 Dinner  
Informal discussions

## Monday, December 4

7:00-9:00 Breakfast

Chair: David Kribs

9:00-9:40 Kenneth R Davidson (University of Waterloo, Canada)  
Operator algebras for multivariable dynamics I

9:45-10:25 Elias G. Katsoulis (East Carolina University, USA)  
Operator algebras for multivariable dynamics II

10:25-10:40 Coffee Break, 2<sup>nd</sup> floor lounge, Corbett Hall

10:40-11:20 Anilesh Mohari (N. Bose Centre for Basic Sciences – India)  
Translation invariant pure state in quantum spin chain and Kolmogorov property

11:30-13:30 Lunch

Chair: Ken Davidson

13:00-14:00 Guided Tour of The Banff Centre; meet in the 2<sup>nd</sup> floor lounge, Corbett Hall

14:00-14:15 Group Photo; meet on the front steps of Corbett Hall

14:15-14:55 Sergei Silvestrov (Lund University, Sweden)  
C\*-crossed Products and Shift Spaces

15:00-15:40 Søren Eilers (University of Copenhagen, Denmark)  
Classification of C\*-algebras associated to irreducible shift spaces

15:40-16:00 Coffee Break, 2<sup>nd</sup> floor lounge, Corbett Hall

16:00-16:40 Takeshi Katsura (Hokkaido University, Japan)  
Cuntz-Krieger algebras and factor maps between topological graphs

16:45-17:25 Berndt Brenken (University of Calgary, Canada)  
Topological quivers as multiplicity free relations

17:30-19:30 Dinner  
Informal discussions

## Tuesday, December 5

7:00-9:00 Breakfast

Chair: Michael Lamoureux

9:00-9:40 N Christopher Phillips (University of Oregon, USA)  
Crossed products of the irrational rotation algebras  
by the "standard" actions of  $\mathbb{Z}/2\mathbb{Z}$ ,  $\mathbb{Z}/3\mathbb{Z}$ ,  $\mathbb{Z}/4\mathbb{Z}$ ,  $\mathbb{Z}/6\mathbb{Z}$  are AF.

9:45-10:35 Florin Boca (University of Illinois, USA)  
C\*-algebras and continued fractions

10:35-10:50 Coffee Break, 2<sup>nd</sup> floor lounge, Corbett Hall

10:50-11:30 Cristina Ivan (University of Hannover, Germany)  
Spectral triples for fractals

11:30-13:30 Lunch  
Free Afternoon recommended

17:30-19:30 Dinner  
Informal discussions

## Wednesday, December 6

7:00-9:00 Breakfast

Chair: Karlheinz Groechenig

9:00-9:40 Bernhard Bodmann (University of Waterloo, Canada)

Optimal Redundant Packet Encoding for Loss-Insensitive Linear Transmissions

9:45-10:25 Myung-Sin Song (Southern Illinois University, USA)

Entropy encoding in wavelet image compression

10:25-10:40 Coffee Break, 2<sup>nd</sup> floor lounge, Corbett Hall

10:40-11:30 Peter Massopust (Technische Universität München, Germany)

Dirichlet-Poisson Processes meet Complex B-Splines

11:30-13:30 Lunch

Chair: Marcel de Jeu

13:30-14:10 Karlheinz Groechenig (University of Vienna, Austria)

Inverse-closed Banach algebras in applied analysis

14:15-14:55 Christian Svensson (Lund University, Sweden and Leiden University, The Netherlands)

Dynamical systems and commutants in crossed products.

15:00-15:20 Johan Öinert (Lund University, Sweden)

Commutativity and ideals in generalized crossed product systems

15:20-15:40 Coffee Break, 2<sup>nd</sup> floor lounge, Corbett Hall

15:40-16:20 Sergey Neshveyev (University of Oslo, Norway)

KMS-states on Hecke algebras crossed products.

16:25-17:05 Jun Tomiyama (Tokyo Metropolitan University, Japan Women's University)

Hulls and kernels with actions of topological dynamical systems and C\*-algebras

17:30-19:30 Dinner

Informal discussions

## **Thursday, December 7**

7:00-9:00 Breakfast

Chair: Sergei Silvestrov

9:00-9:40 Ian Putnam (University of Victoria, Canada)

K-theory for Smale spaces

9:45-10:15 Christian Skau (Norwegian University of Science and Technology, Trondheim, Norway)

AF-equivalence relations and group actions

10:15-10:30 Coffee Break, 2<sup>nd</sup> floor lounge, Corbett Hall

10:30-11:10 Thierry Giordano (University of Ottawa, Canada)

Orbit equivalence for Cantor minimal  $Z^2$ -systems.

11:30-13:30 Lunch

## **Checkout by 12 noon.**

\*\* Participants are welcome to use the BIRS facilities (2<sup>nd</sup> Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 3 pm on Thursday, but are still required to checkout of the guest rooms by 12 noon. \*\*



# **Operator methods in fractal analysis, wavelets and dynamical systems (06w5027)**

**December 2 – December 7, 2006**

**Organizers:**

|                   |                                 |
|-------------------|---------------------------------|
| Ola Bratteli      | University of Oslo, Norway      |
| Palle Jorgensen   | The University of Iowa, USA     |
| David Kribs       | University of Guelph, Canada    |
| Gestur Olafsson   | Louisiana State University, USA |
| Sergei Silvestrov | Lund University, Sweden         |

## **ABSTRACTS**

(in alphabetic order by speaker surname)

**Speaker:** Florin Boca (University of Illinois)

**Title:**  $C^*$ -algebras and continued fractions

**Abstract:** An AF algebra associated with the Farey tessellation and capturing the properties of the continued fraction algorithm will be considered. The Effros-Shen rotation AF algebras arise as quotients of this algebra.

**Speaker:** Bernhard Bodmann (University of Waterloo)

**Title:** Optimal Redundant Packet Encoding for Loss-Insensitive Linear Transmissions

**Abstract:** The objective of this talk is to characterize the optimal use of redundancy in transmitting a signal that is encoded in packets of linear coefficients. The signals considered here are vectors in a finite-dimensional real or complex Hilbert space. For the purpose of transmission, these vectors are encoded in a set of linear coefficients that is partitioned in packets of equal size. We investigate how the encoding performance depends on the degree of redundancy it incorporates and on the amount of data-loss when packets are either transmitted perfectly or lost in their entirety. The encoding performance is evaluated in terms of the maximal Euclidean norm of the reconstruction error occurring for the transmission of unit vectors. The main result of this talk is the derivation of error bounds as well as the characterization of optimal encoding when up to three packets are lost.

**Speaker:** Berndt Brenken (University of Calgary)

**Title:** Topological quivers as multiplicity free relations

**Abstract:** For a discrete directed graph a certain graph  $C^*$ -algebra is invariant under a procedure that yields a multiplicity free graph. We show the analogue of this holds for topological quivers; a certain  $C^*$ -algebra, namely the unaugmented Cuntz-Pimsner  $C^*$ -algebra, of a topological quiver remains Morita equivalent to the  $C^*$ -algebra of an associated multiplicity free topological quiver.

**Speaker:** Jonas D'Andrea (University of Colorado)

**Title:** Fractal wavelets of Dutkay-Jorgensen Type for the Sierpinski gasket space

**Abstract:** Several years ago, D. Dutkay and P. Jorgensen developed the concept of wavelets defined on a sigma-finite fractal measure space, developed from an iterated affine system. They worked out in detail the wavelet and filter functions corresponding to the ordinary Cantor fractal subset of  $\mathbb{R}$ . In this talk we examine the construction of Dutkay and Jorgensen as applied to the fractal measure space corresponding to the Sierpinski Gasket fractal. We develop a variety of high-pass filters, and as an application use the various families of wavelets to analyze digital images.

**Speaker:** Kenneth R Davidson (University of Waterloo)

**Title:** Operator algebras for multivariable dynamics I

**Abstract:** Let  $X$  be a locally compact Hausdorff space with  $n$  proper continuous maps of  $X$  into itself. To this we associate various topological conjugacy algebras; and two emerge as the natural candidates for the universal algebra of the system, the tensor algebra and the semicrossed product. I will discuss the reasons, including dilation theory, representations and  $C^*$ -envelopes. Generalized notions of wandering sets and recursion will be used to characterize when these algebras are semisimple.

**Speaker:** Dorin Dutkay (Rutgers University)

**Title:** Covariant representations, scaling functions and affine fractals

**Abstract:** (This is a joint work with Palle Jorgensen)

We show how some operator algebra constructions of covariant representations can be used to analyze orthogonality in wavelet theory, to construct super-wavelets, and to obtain orthogonal Fourier bases for affine fractal measures.

**Speaker:** Søren Eilers (University of Copenhagen)

**Title:** Classification of  $C^*$ -algebras associated to irreducible shift spaces

**Abstract:** A construction by Matsumoto allows an invariant association of  $C^*$ -algebras to any shift space. Somewhat exceptionally, these  $C^*$ -algebras are not always simple when the shift space is irreducible, and in previous work, mainly with Carlsen, we have endeavored to explain what dynamical information is stored in the ideal structure in those cases. I intend to review and discuss this problem in light of a recent classification result for certain non-simple Matsumoto  $C^*$ -algebras obtained in joint work with Restorff and Ruiz.

**Speaker:** Thierry Giordano (University of Ottawa)

**Title:** Orbit equivalence for Cantor minimal  $Z^2$ -systems

**Abstract:** In 1959, H. Dye introduced the notion of orbit equivalence and proved that any two ergodic finite measure preserving transformations on a Lebesgue space are orbit equivalent. He also conjectured that an arbitrary action of a discrete amenable group is orbit equivalent to a  $Z$ -action. This conjecture was proved by Ornstein and Weiss and its most general case by Connes, Feldman and Weiss by establishing that an amenable non-singular countable equivalence relation  $R$  can be generated by a single transformation, or equivalently is hyperfinite, i.e.,  $R$  is up to a null set, a countable increasing union of finite equivalence relations.

In the Borel case, Weiss proved that actions of  $Z^n$  are (orbit equivalent to) hyperfinite Borel equivalence relations, whose classification was obtained by Dougherty, Jackson and KeCHRIS.

In 1995, Giordano, Putnam and Skau proved that minimal  $Z$ -actions on the Cantor set were orbit equivalent to approximately finite (AF) relations and their classification was given. Since then some special classes of minimal free actions of  $Z^2$  on the Cantor set were shown to be affable (i.e., orbit equivalent to AF-relations).

In this talk I will indicate the main steps of the proof of the general result obtained in a joint effort with H. Matui, I. Putnam and C. Skau and whose statement is the following:

Theorem: Any minimal, free  $Z^2$ -action on the Cantor set is affable.

**Speaker:** Karlheinz Groechenig (University of Vienna)

**Title:** Inverse-closed Banach algebras in applied analysis

**Abstract:** A Banach algebra  $A$  is called inverse-closed in a larger Banach algebra  $B$ , if every element in  $A$  that is invertible in  $B$  is already invertible in the smaller algebra  $A$ . For instance, the algebra of absolutely convergent Fourier series is inverse-closed in the algebra of continuous functions on the torus. This is the classical Wiener Lemma. In the talk I will present several results about inverse-closed Banach algebras in applied analysis.

(a) Wiener's Lemma for twisted convolution and for the rotation algebras

(b) Algebras of infinite matrices with off-diagonal decay are inverse-closed in the algebra of all bounded operators

(c) Inverse-closedness plays an essential role in quantitative studies of the finite section method to solve operator equations.

(d) We present a new construction of inverse-closed matrix algebras by approximation properties.

**Speaker:** Cristina Ivan (University of Hannover, Germany)

**Title:** Spectral triples for fractals

**Abstract:** The purpose of this talk is to present two possible ways of associating a spectral triple to a fractal such that it encodes geometric data of the fractal. The spectral triple is obtained in both constructions as a countable sum of unbounded Fredholm modules. In the first construction (joint work with Erik Christensen) each summand is a spectral triple for a set consisting of just two points (a "two-point" spectral triple). It was Alain Connes who first constructed in this way a spectral triple for the middle Cantor set in the unit interval. Connes showed how the metric, the Hausdorff measure and dimension are encoded by this spectral triple and its associated Dixmier trace. Connes' construction has been studied in details and extended to certain classes of fractals by Guido and Isola. Together with Erik Christensen we have investigated to which extent such a spectral triple may encode geometric data of a general compact metric space. We showed that for any compact metric space it is possible to associate a spectral triple which is a countable sum of "two-point" spectral triples and which reflects the Minkowski dimension of the space and the metric induced by the spectral triple is equivalent to the given one. We did explicit computations for the unit interval, Cantor set and Sierpinski gasket and each time we saw that the spectral triple and its Dixmier trace recovers metric, dimension and volume measure of the compact metric space under discussion. In the second construction (joint work with Erik Christensen and Michel Lapidus) each summand is based on a curve in the space. Several fractals, like a finitely summable infinite tree, and the Sierpinski gasket fit naturally within this framework. In these cases, we show that our spectral triples do describe the geodesic distance and the Minkowski dimension as well as. Furthermore, in the case of the Sierpinski gasket, the associated Dixmier-type trace coincides with the normalized Hausdorff measure. It is important to mention the advantage of each proposal for constructing spectral triples. The advantage of the first construction is that it is modelled for a general compact metric space. The advantage of the second construction is that, when it is possible to be done, it brings more information about the topological structure of the fractal (the spectral triple will induce a nontrivial element in the  $K$ -homology of the fractal).

**Speaker:** Elias G. Katsoulis (East Carolina University)

**Title:** Operator algebras for multivariable dynamics II

**Abstract:** (This is a joint work with Ken Davidson.)

Let  $X$  be a locally compact Hausdorff space with  $n$  proper continuous maps of  $X$  into itself. To this

we associate various topological conjugacy algebras; and two emerge as the natural candidates for the universal algebra of the system, the tensor algebra and the semicrossed product. We will introduce and discuss a new concept of topological conjugacy for multidimensional systems, which we coin piecewise conjugacy. We will prove that the piecewise conjugacy class of the system can be recovered from the algebraic structure of either the tensor algebra or the semicrossed product. Various classification results follow as a consequence. For example, for  $n=2,3$ , the tensor algebras are (algebraically or even completely isometrically) isomorphic if and only if the systems are piecewise topologically conjugate.

**Speaker:** Palle Jorgensen (The University of Iowa)

**Title:** Wavelets on Fractals

**Abstract:** (This is a joint work with Dorin Dutkay)

Joint work between the Speaker and Dorin Dutkay, recently led to wavelet constructions, and wavelet algorithms in Hilbert spaces built on fractals. The talk will cover some highpoints, and will be a comparative study: the case of fractals will be contrasted with the more traditional wavelets, those of  $L^2(\mathbb{R}^d)$ . As a conclusion we note several instances of dichotomies; e.g., measure classes, regions of convergence, stability to mention three. Two computational features will be addressed:

(a) Approximation of the father/mother functions by subdivision schemes, and

(b) matrix formulas for the wavelet coefficients. For (a) we show that the variety of data when  $L^2$ -convergence holds is much smaller in the case of fractals than is the case for  $L^2(\mathbb{R}^d)$ -wavelets.

**Speaker:** Takeshi Katsura (Hokkaido University)

**Title:** Cuntz-Krieger algebras and factor maps between topological graphs

**Abstract:** A (one-sided) Markov chain is a topological dynamical system defined from a  $\{0,1\}$ -matrix. Cuntz and Krieger introduced a  $C^*$ -algebra to examine a Markov chain. Although a Cuntz-Krieger algebra is defined from a  $\{0,1\}$ -matrix, it only depends on the associated Markov chain. Later, a construction of Cuntz-Krieger algebras from Markov chains using groupoids were provided. In my talk, I introduce topological graphs which contain  $\{0,1\}$ -matrices and Markov chains as special cases. I also introduce the way to construct  $C^*$ -algebras from topological graphs which generalizes the construction of Cuntz-Krieger algebras. Using the notion of "factor maps" between topological graphs, I clarify the relation of the two constructions of Cuntz-Krieger algebras from  $\{0,1\}$ -matrices and from Markov chains.

**Speaker:** David R. Larson (Texas A&M University)

**Title:** Frames and Operator Theory

**Abstract:** A few years ago the Speaker and his collaborators developed an operator-interpolation approach to wavelets and frames using the local commutant (i.e. commutant at a point) of a unitary system. This is really an abstract application of the theory of operator algebras to wavelet and frame theory. The concrete applications of operator-interpolation to wavelet theory include results obtained using specially constructed families of wavelet sets. Our methods include the construction of certain groups of measure preserving transformations, and groups and algebras of operators, with special algebraic properties. Other results include applications of a theory of projection decompositions of positive operators, and a theory of operator-valued frames. We will discuss some unpublished and partially published results, and some brand new results, that are due to this Speaker and his former and current students, and other collaborators.

**Speaker:** Nadia S. Larsen (University of Oslo)

**Title:** Projective multi-resolution analyses arising from direct limits of Hilbert modules

**Abstract:** (This is a joint work with I. Raeburn.)

In joint work with I. Raeburn we have shown how direct limits of Hilbert spaces can be used to construct multi-resolution analyses and wavelets in  $L^2(\mathbb{R})$ . In this talk we enlarge the framework of our construction and use a direct limit of Hilbert modules over a fixed  $C^*$ -algebra to produce projective

multi-resolution analyses in the limit module. In certain cases, we prove existence of standard module frames for the limit module. For modules over the algebra of continuous functions on the product of  $n$ -copies of the circle, our methods shed light on work of Packer and Rieffel on projective multi-resolution analyses for specific Hilbert modules of functions on  $\mathbb{R}^n$ . New applications arise in the context of modules over the algebra of continuous functions on the compact infinite path space of a finite directed graph.

**Speaker:** Peter Massopust (Technische Universität München, Germany)

**Title:** Dirichlet-Poisson Processes meet Complex B-Splines

**Abstract:** (This is a joint work with Brigitte Forster.)

Complex B-splines are a generalization of ordinary B-splines to complex degrees. This results in an infinite uniform knot sequence for complex B-splines. We show that generalized fractional divided differences can be defined via the fractional Weyl-integral with complex B-splines as densities. This representation leads to a generalized Hermite-Genocchi formula over infinite dimensional simplices. The generalized Hermite-Genocchi formula then allows the extension of complex B-splines to non-uniform knots and their interpretation as probability densities for a class of stochastic processes, namely the Dirichlet-Poisson processes.

**Speaker:** Anilesh Mohari (N. Bose Centre for Basic Sciences – India)

**Title:** Translation invariant pure state in quantum spin chain and Kolmogorov property

**Abstract:** We prove a necessary and sufficient condition for a lattice symmetric translation invariant factor state to be pure. To that end we study the associated Popescu systems [Po,BJKW] representing the translation invariant state and Cuntz representation. We will also explore how this criteria is related with Kolmogorov's property [Mo] of an associated canonical Markov shift.

[BJKW] Bratteli, Ola., Jorgensen, Palle E.T., Kishimoto, Akitaka and Werner Reinhard F.: Pure states on  $\mathcal{O}_d$ , J.Operator Theory 43 (2000), no-1, 97-143.

[Po] Popescu, Gelu: Isometric dilations for infinite sequences of non-commutating operators, Trans. Amer. Math. Soc. 316 no-2, 523-536 (1989)

[Mo] Mohari, A.: Markov shift in non-commutative probability, J. Funct. Anal. vol- 199, no-1, 190-210 (2003) Elsevier Sciences.

**Speaker:** Sergey Neshveyev (University of Oslo)

**Title:** KMS-states on Hecke algebras crossed products

**Abstract:** (This is a joint work with M. Laca and N.S. Larsen)

We show that KMS-states on crossed products of abelian  $C^*$ -algebras by Hecke algebras correspond to measures scaled by Hecke operators. We then consider the  $GL_2$ -system of Connes and Marcolli and complete their analysis of the system by showing that for each value of the temperature in the critical region there exists a unique KMS-state.

**Speaker:** Johan Öinert,

**Title:** Commutativity and ideals in generalized crossed products

**Abstract:** (This is a joint work with Sergei Silvestrov)

A short review will be given of  $G$ -crossed systems and the construction of generalized algebraic crossed products following C. Năstăsescu, F. Van Oystaeyen, Methods of graded rings, LNM 1836. Springer-Verlag, 2004. Thereafter, inspired by the theory of  $C^*$ -dynamical systems, some results will be presented relating commutativity, ideals, group actions and zero-divisors in algebraic crossed product algebras.

**Speaker:** N Christopher Phillips (University of Oregon)

**Title:** Crossed products of the irrational rotation algebras by the "standard" actions of  $\mathbb{Z}/2\mathbb{Z}$ ,  $\mathbb{Z}/3\mathbb{Z}$ ,  $\mathbb{Z}/4\mathbb{Z}$ ,  $\mathbb{Z}/6\mathbb{Z}$  are AF

**Abstract:** Let  $F$  be a finite subgroup of  $SL_2(\mathbb{Z})$  (necessarily isomorphic to one of  $\mathbb{Z}/2\mathbb{Z}$ ,  $\mathbb{Z}/3\mathbb{Z}$ ,  $\mathbb{Z}/4\mathbb{Z}$ , or  $\mathbb{Z}/6\mathbb{Z}$ ), and let  $F$  act on the irrational rotational algebra  $A_\theta$  via the restriction of the canonical action of  $SL_2(\mathbb{Z})$ . Then the crossed product of  $A_\theta$  by  $F$ , and the fixed point algebra for the action of  $F$  on  $A_\theta$ , are AF algebras. The same is true for the crossed product and fixed point algebra of the flip action of  $\mathbb{Z}/2\mathbb{Z}$  on any simple  $d$ -dimensional noncommutative torus  $A_\theta$ . Along the way, we prove a number of general results which should have useful applications in other situations.

(The paper is available at [arXiv:math.OA/0609784](https://arxiv.org/abs/math.OA/0609784))

**Speaker:** Ian Putnam (University of Victoria)

**Title:** K-theory for Smale spaces

**Abstract:** Smale spaces are abstract topological dynamical systems characterized by canonical coordinates of contracting and expanding directions. These include basic sets from Smale's Axiom A systems as well as shifts of finite type. In general, they are chaotic and the underlying geometry is fractal. There are  $C^*$ -algebras associated with such objects and the aim is to compute their K-theory. For shifts of finite type, this is the usual dimension group invariant. More generally, there is a spectral sequence for this, but the answer can be given in purely dynamical terms as a kind of homology theory for chaotic systems, similar in spirit to Čech homology.

**Speaker:** Iain Raeburn (University of Newcastle, Australia)

**Title:** Direct limits, the Cuntz relations and wavelets

**Abstract:** (This is a joint work with Nadia Larsen)

A famous theorem of Mallat shows how to build a wavelet basis for the Hilbert space of square-integrable functions on  $\mathbb{R}$  starting from a quadrature mirror filter, which is a function on the unit circle satisfying an algebraic relation. From such a filter, Bratteli and Jorgensen constructed a pair of isometries satisfying the Cuntz relations well-known to operator algebraists. In this talk we will discuss an approach to Mallat's theorem which uses a direct limit construction and exploits the geometric information inherent in the Cuntz relations.

**Speaker:** Kjetil Røysland (University of Oslo)

**Title:** Transition operators on bundle maps

**Abstract:** In a joint work with Dorin Dutkay, we have studied transition operators that act on the bundle maps of a vector bundle. I will talk about the fix points of such an operator. In some situations this turn out to be a finite dimensional non commutative  $C^*$ -algebra.

**Speaker:** Mary Beth Ruskai (Tufts University)

**Title :** The relation between frames and POVM's in quantum information theory

**Abstract:** In quantum information theory a von Neumann measurement is replaced by a more general concept called a positive operator valued measure (POVM), which is essentially a partition of unity in terms of positive semi-definite operators. POVM's formed from equally weighted rank one operators define a tight frame, and any frame defines a POVM. Two special classes about which many open questions remain are known as symmetric informationally complete (SIC) bases and mutually unbiased bases (MUB).

This will be a tutorial lecture on the role of POVM's in quantum information with emphasis on open questions to which experts on frames may be able to provide some insight.

**Speaker:** Christian Skau (Norwegian University of Science and Technology, Trondheim)

**Title:** AF-equivalence relations and group actions

**Abstract:** We show that a group acting freely as homeomorphisms on a zero-dimensional space gives rise to an AF-equivalence relation if and only if the group is locally finite. Furthermore, we show that the AF-equivalence relations that occur are exactly the ones that are associated to Bratteli diagrams

that have the equal path number property. We show that the "super" order of the locally finite group is completely determined by the rational subdimension group of the AF-relation.

**Speaker:** Sergei Silvestrov (Lund University, Sweden)

**Title:** C\*-crossed Products and Shift Spaces

**Abstract:** (This is a joint work with Toke Meier Carlsen, arXiv:math.OA/0512488)

We use Exel's C\*-crossed product of non-invertible dynamical systems to associate a C\*-algebra to every shift space. We show that this C\*-algebra is canonically isomorphic to the C\*-algebra associated to a shift space in arXiv:math.OA/0505503, has the C\*-algebra defined by Toke Meier Carlsen and Kengo Matsumoto (Math. Scand. 95, 2 (2004), 145-160) as a quotient, and possesses properties indicating that it can be thought of as the universal C\*-algebra associated to a shift space. We also consider its representations, relationship to other C\*-algebras associated to shift spaces, show that it can be viewed as a generalization of the universal Cuntz-Krieger algebra, discuss uniqueness and a faithful representation, show that it is nuclear and satisfies the Universal Coefficient Theorem, provide conditions for it being simple and purely infinite, show that the constructed C\*-algebras and thus their K-theory,  $K_0$  and  $K_1$ , are conjugacy invariants of one-sided shift spaces, present formulas for those invariants, and also present a description of the structure of gauge invariant ideals.

**Speaker:** Myung-Sin Song (Southern Illinois University)

**Title:** Entropy Encoding in Wavelet Image Compression

**Abstract:** Entropy is a quantity that measures the amount of uncertainty in a probability distribution. In wavelet image compression and wavelet packet image compression, entropy encoding is performed on the image after the wavelet decomposition and thresholding. In this talk, we will discuss some engineering aspects of entropy encoding and the mathematics behind it.

**Speaker:** Christian Svensson (Lund University, Sweden and Leiden University, The Netherlands)

**Title:** Dynamical systems and commutants in crossed products

**Abstract:** (This is a joint work with Marcel de Jeu and Sergei Silvestrov, arXiv:math.DS/0604581)

Given a discrete dynamical system, one may construct an associative (non-commutative) complex algebra with multiplication determined via the action defining the system - a crossed product algebra. It turns out that for large classes of systems, one obtains striking equivalences between, in particular, dynamical properties of the system and algebraic properties of the crossed product. Quite a lot has been done in this direction for C\*-crossed products. It is satisfactory to see that many of the results obtained in purely algebraic setup are analogous to those well-known for C\*-case, and at the same time further progress on interplay between structure of crossed product and dynamics can be obtained outside the C\*-context. In this work, in particular, we describe the commutant of an arbitrary subalgebra  $A$  of the algebra of functions on a set  $X$  in a crossed product of  $A$  with the integers by a composition automorphism defined via a bijection of  $X$ . The conditions on  $A$  and on the dynamics, extending topological freeness, which are necessary and sufficient for  $A$  to be maximal abelian in the crossed product are subsequently applied to situations where these conditions can be shown to be equivalent to a condition in topological dynamics. As a further step, using the Gelfand transform we obtain for a commutative completely regular semi-simple Banach algebra a topological dynamical condition on its character space which is equivalent to the algebra being maximal abelian in a crossed product with the integers.

**Speaker:** Jun Tomiyama (Tokyo Metropolitan University, Japan Women's University)

**Title:** Hulls and kernels with actions of topological dynamical systems and C\*-algebras

**Abstract:** Let  $\Sigma = (X, \sigma)$  be a topological dynamical system in a compact space  $X$  with a homeomorphism  $\sigma$ , and let  $A(\sigma)$  be the associated C\*-crossed product. In this context we define hulls and kernels with the action  $\sigma$ , and discuss the following problems.

1. What is the meaning in C\*-theory of the Kernels of those elementary

sets for the dynamical system  $\sigma$ ?

2. What is the meaning of the Hulls of those structural ideals of the  $C^*$ -algebra  $A(\sigma)$ ?