BIRS Workshop
Combinatorial Hopf Algebras
28 August—2 September 2004

MEALS

Breakfast (Continental): 7:00 - 9:00 am, 2nd floor lounge, Corbett Hall, Sunday—Thursday
*Lunch (Buffet): 11:30 am - 1:30 pm, Donald Cameron Hall, Sunday—Thursday
*Dinner (Buffet): 5:30 - 7:30 pm, Donald Cameron Hall, Saturday—Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

MEETING ROOMS

All lectures are held in the main lecture hall, Max Bell 159. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
7:00-9:00	X	Continental Breakfast, 2nd floor lounge, Corbett Hall				
9:00-10:00	X	Aguiar	Cartier	Ehrenborg	Frabetti	Holtkamp
10:00-10:30	X	Coffee Break, 2nd floor lounge, Corbett Hall				
10:30-11:15	X	Schocker	Grossmann	Reiner	Hazewinkel	Garsia
11:15-11:30	X	X	X	Group Photo[1]	X	X
11:30-13:30	X	Buffet Lunch, Donald Cameron Hall				
13:00-14:00	X	Guided Tour[2]	X	free afternoon	Schmitt[3]	X
14:00-15:00	X	Stembridge	Loday	free afternoon	Coffee Break[3]	X
15:00-15:30	X	Coffee Break	Coffee Break	free afternoon	Reading[3]	X
15:30-16:15	X	Hsiao	Hivert	free afternoon	Chapoton[3]	X
17:30-19:30	Buffet Dinner, Donald Cameron Hall					X

[1] A group photo will be taken on Tuesday at 11:15 am, directly after the last lecture of the morning. Please meet on the front steps of Corbett Hall.

[2] A free guided tour of The Banff Centre is offered to all participants and their guests on Sunday starting at 1:00 pm. The tour takes approximately 1 hour. Please meet in the 2nd floor lounge in Corbett Hall.

[3] Wednesday afternoon schedule: Schmitt(13:30), Coffee(14:30), Reading(15:00), and Chapoton(16:00).

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Frédéric Chapoton** (Lyon)
Title: *pre-Lie algebras, rooted trees and posets*
Abstract:

We will recall the relation between free pre-Lie algebras and rooted trees. This is linked to the Hopf algebra of rooted trees used by Connes and Kreimer. Somehow this explains many structures of Lie algebras on all kinds of graphs.

Then we will introduce the posets of pointed partitions and the posets of hypertrees and explain what links exist or should exist with the combinatorics of rooted trees.

Speaker: **Alessandra Frabetti** (Lyon)
Title: *Hopf algebras and renormalization in physics*
Abstract:

In Quantum Field Theory, the Green functions are formal series of the coupling constants, with coefficients computed as divergent integrals associated to Feynman graphs. The meaningful Green functions can be found by applying the renormalization group action, which is based on a complicated combinatorial formula giving the renormalization of each Feynman graph. Recently, the renormalization of Feynman graphs has been very efficiently described by A. Connes and D. Kreimer in terms of a suitable Hopf algebra. We will try to motivate the use of Hopf algebras in the context of renormalization, recalling briefly the physical problem, showing the three kinds of Hopf algebras developped in this context on graphs and trees, and presenting some feed backs of the Hopf algebraic method of renormalization in physics and in mathematics.

Speaker: **Adriano Garsia** (San Diego)
Title: *The Haglund Macdonald statistics and the open problems it creates*
Abstract:

In 1990 Garsia-Haiman conjectured that a certain version $H_\mu(x;q,t)$ of the Macdonald integral forms gave the Frobenius characteristic of certain bigraded S_n modules. This came to be known as the "$n!$ conjecture". In various attempts at proving this conjecture a variety of other conjectures were formulated in the decade that passed before Haiman finally proved the n-factorial conjecture using algebraic-geometrical methods. Nevertheless the basic combinatorial problems arisen since the discovery of the Macdonald polynomials are still open. Very recently Jim Haglund has given a combinatorial interpretation for the coefficients in the monomial expansion of $H_\mu(x;q,t)$. This opens up a purely combinatorial attack to the variety of conjectures surrounding these remarkable polynomials and their Schur function expansions. In the brief time of this talk it is impossible to cover all these developments but a prominent selection will be presented.

Title: *Some Hopf Algebra Structures on Trees and their Applications*
Abstract:

Let k be a field, R be a commutative k-algebra, and $\mathrm{Der}(R)$ the Lie algebra of derivations of R. In this talk, we describe some Hopf algebras defined on the vector space spanned by rooted trees. In particular, we describe some properties of the the Hopf algebra H formed from rooted trees labeled using derivations D in $\mathrm{Der}(R)$. We also describe a construction which uses a connection on $\mathrm{Der}(R)$ to define a H-module algebra structure on R. We also give some applications of these ideas to symbolic computation and numerical algorithms.

Speaker: **Ralf Holtkamp** (Bochum)
Title: *Operads of primitive elements and their combinatorics*
Abstract:

We study P-Hopf algebras with one coassociative cooperation over different operads P. Important examples are the dendriform operad, the operad Mag freely generated by a non-commutative non-associative binary operation, and the operad of Stasheff polytopes. In order to describe the operads of primitive elements we prove an analog of the Poincare-Birkhoff-Witt theorem. We determine the generating series for these operads and show that the dimension of $\mathrm{PrimMag}(n)$ is related to the log-Catalan numbers. A similar theory may be devloped for (unitary) infinitesimal P-Hopf algebra structures.

Speaker: **Samuel Hsiao** (Michigan)
Title: *Canonical characters on quasi-symmetric functions and bivariate Catalan numbers*
Abstract:

Every character on a graded connected Hopf algebra decomposes uniquely as a product of an even character and an odd character (Aguiar, Bergeron, and Sottile, *Combinatorial Hopf algebra and generalized Dehn-Sommerville equations*, 2003, `math.CO/0310016`).

We obtain explicit formulas for the even and odd parts of the universal character on the Hopf algebra of quasi-symmetric functions. They can be described in terms of Legendre's beta function evaluated at half-integers, or in terms of *bivariate Catalan numbers:*

$$C(m,n) \;=\; \frac{(2m)!(2n)!}{m!(m+n)!n!}\,.$$

Properties of characters and of quasi-symmetric functions are then used to derive several interesting identities among bivariate Catalan numbers and in particular among Catalan numbers and central binomial coefficients.

Speaker: **Jean-Louis Loday** (CNRS, Strasbourg)
Title: *Generalized bialgebras*
Abstract:

The Poincaré-Birkhoff-Witt theorem and the Milnor-Moore theorem unravel the structure of connected cocommutative bialgebras. First, we announce an analogue for connected (not necessarily cocommutative) bialgebras. In both cases three types of algebras (i.e. of operads) are involved: (Com, As, Lie) in the classical case and $(As, 2as, B_\infty)$ in the non-cocommutative case. Second, we examine several other similar cases of triples of operads (C, A, P) having the same kind of properties (C for coalgebra, A for algebra, P for primitive). Most of the recently discovered Hopf algebras on trees (Grossman-Larson, Connes-Kreimer, Brouder-Frabetti, Loday-Ronco, Holtkamp, Goncharov) fit into this framework.

Speaker: **Nathan Reading** (Michigan)
Title: *Lattice congruences and Hopf algebras*
Abstract:

We describe a method of constructing sub Hopf algebras of the Malvenuto Reutenauer Hopf algebra of permutations (MR). The starting point is a family $\{\Theta_n\}$, where each Θ_n is a lattice congruence of the weak

thus obtained are indexed by order ideals in an infinite poset, and can be described by a pattern avoidance condition on permutations. Among the Hopf algebras arising from this construction are the Hopf algebras of planar binary trees and of non-commutative symmetric functions.

Speaker: **Victor Reiner** (Minnesota)
Title: *The weak order on tableaux*
Abstract:

(Joint work with Muge Taskin)

This talk will discuss a partial order on the standard Young tableaux with n cells, analogous to the weak Bruhat order permutations, or to the Tamari order on binary trees. It was first introduced by A. Melnikov in connection with the geometry of orbital varieties. We will explain its connection to the Poirier-Reutenauer Hopf algebra on tableaux, and discuss some of its good and bad properties.

Speaker: **Manfred Schocker** (Oxford)
Title: *A decomposition of the Tits algebra of the symmetric group*
Abstract:

Tits' notion of projection operator yields the structure of a semigroup on the set of faces F of a Coxeter complex (or, more generally, of an arbitrary hyperplane arrangement over a real vector space). The natural action of the underlying Coxeter group W preserves this semigroup structure and the ring of invariants of W in the integral semigroup ring of F is isomorphic to the descent algebra of W.

I shall analyse the module structure of the "Tits algebra" kF over an arbitrary field k in the case where W is the symmetric group. This includes a construction of primitive idempotents and projective indecomposable modules as well as a description of the Cartan invariants, of the quiver and of the module structure of the Loewy layers. I shall also discuss the impact of these results on the descent algebra.

The approach is combinatorial in nature (rather than geometric) and uses a coproduct and a second product on the direct sum of the Tits algebras of various symmetric groups, generalising well-known concepts of the theory of descent algebras.

Speaker: **John Stembridge** (Michigan)
Title: *P-partitions and quasi-symmetric functions*
Abstract:

This will be a survey talk in which we will (1) review the combinatorial and algebraic properties of Schur S-functions and Q-functions as motivation for (2) the theory of P-partitions (both ordinary and enriched) and (3) their associated (Hopf) algebras of quasi-symmmetric functions. If time permits, we will discuss our original motivation for developing the theory of enriched P-partitions: heap expansions for stable Schubert polynomials.

Title: *Hopf algebras, Lie algebras, algebraic groups and their use in the renormalization in quantum field theory*

Abstract:

This talk will present an alternative description (obtained in collaboration with Marcus Berg) of the Hopf algebraic methods used by Connes and Kreimer in their reformulation of quantum field renormalization. Our approach uses extensively Vinberg algebras (alias pre-Lie algebras) and brings together the combinatorics of Feynman diagrams and algebro-differential geometry (in the line of the original approach of Vinberg). There are obvious connections with the lectures of Frabetti and Chapoton.

Speaker: **Marcelo Aguiar** (Texas A&M University)

Title: *The smash product of symmetric functions*

Abstract:

We introduce a product on the space of symmetric functions that interpolates between the classical "internal" and "external" products (which are constructed in terms of tensor products and induction of representations). This product is best understood in terms of Hopf algebraic constructions (the "smash product").

The smash product exists as well on the space of non-commutative symmetric functions. At this level it interpolates between Solomon's product (the "descent algebra") and the usual product of non-commutative symmetric functions (as defined by Thibon et al). The dual coproduct can be described in terms of the multiplicative formal group law for alphabets.

The smash product also exists on larger spaces such as the algebra of Solomon-Tits and the algebra of Malvenuto-Reutenauer.

This is joint work with Walter Ferrer and Walter Moreira.

Speaker: **Florent Hivert and Jean-Christophe Novelli** (Université de Marne-la-Valé)

Title: *Colored quasi-symmetric function and representation theory*

Abstract:

We introduce analogues of the Hopf algebra of Free quasi-symmetric functions with bases labelled by colored permutations. As an application, we recover in a simple way the descent algebras associated with wreath products $\Gamma \wr \mathfrak{S}_n$ and the corresponding generalizations of quasi-symmetric functions, the Poirier quasi-symmetric algebra and the dual Mantaci-Reutenauer algebra (also considered by Poirier). We then show that this last algebra (and its dual) can be interpreted as the Grothendieck rings of the tower of some algebras, namely the specialization at $q = 0$ of the Ariki-Koike algebras in the Shoji's presentation.