Northwest Functional Analysis Workshop

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The subject of Functional Analysis is now a well-established one and a strength of the Canadian Mathematical community. Western Canada, in particular, has many researches working in the field and several well-established groups.

Roughly speaking, the community can be divided into three distinct groups: C^*-algebras and non-commutative geomtery, Banach algebras and amenability and geometric functional analysis.

We were very pleased that there was an excellent level of participation, especially from graduate students and post-doctoral fellows. We were also happy that many senior visitors were able to attend. The meeting went a long way in strengthening the ties between the different groups present. We hope to continue to have this meeting on a regular basis and there seems to be a great deal of support for this idea.

Geometric Functional Analysis

Geometric Functional Analysis deals with the geometry of finite and infinite dimensional spaces.

ALEXANDER LITVAK gave a talk on "ASYMPTOTIC BEHAVIOR OF HIGH-DIMENSIONAL CONVEX BODIES". The main subject of Asymptotic Geometric Analysis is, in most general terms, the study of various geometric parameters of high-dimensional convex bodies (such as, for example, volumes) and of their projections and sections, and of the asymptotic behaviour of these parameters as the dimensions tends to infinity. Several basic examples of this approach were given, such as the concentration of measure phenomenon and Dvoretzky's theorem on almost Euclidean sections, as well as a series of new results of a similar flavor.

VLADIMIR TROITSKY talked on "MINIMAL VECTORS IN BANACH SPACES". The method of minimal vectors was introduced in 1998 by Ansari and Enflo in order to prove the existence of invariant subspaces for certain classes of operators on a Hilbert space. It turns out that the method works in general Banach spaces. A variant of the method also shows that a large class of quasinilpotent operators on arbitrary Banach spaces have hyperinvariant subspaces. As it also applies in the spaces where there are known examples of operators without invariant subspaces; this dashes out the hopes that the method of minimal vectors alone could solve the invariant subspace problem.

RAZVAN ANISCA talked on "UNCONDITIONAL DECOMPOSITIONS IN SUBSPACES OF $l_2(X)$". This work continues and generalizes the series of constructions of Banach spaces without an unconditional basis done in the framework of arbitrary Banach spaces by Komorowski and Tomczak-

Jaegermann in the 1990's. The talk presents characterizations of a Hilbert space in terms of "good" structure of all subspaces of $l_2(X)$. An additional interest stems from the connection with a long standing problem in Banach spaces on a structure of complemented subspaces of Banach spaces with unconditional basis.

BUNYAMIN SARI gave a talk "ON THE STRUCTURE OF THE SET OF SPREADING MODELS OF ORLICZ SPACES". The notion of a spreading model is an important tool in the Banach space theory. Roughly speaking, a spreading model of a Banach space X is another Banach space whose norm is obtained by stabilizing at infinity the norm on a non-degenerate sequence of vectors in X. Such a stabilization is achieved as a neat (and immediate) application of Ramsey Theorem. What type of spreading models may exist on a given Banach space X was a central problem which have been widely investigated since the spreading models were first introduced in 70's.

Abstract Harmonic Analysis

Abstract Harmonic analysis relates to the studies of Banach algebras of spaces of measures or functions associated to (unitary representations of) a locally compact group. Two locally compact groups are isomorphic if and only if certain associated Banach algebras (i.e. Fourier algebras or the group algebras) are isometrically isomorphic. Consequently, the study of various Banach algebras and their geometric properties reveal deep structural properties of the underlying group. For example, the classical result of B.E. Johnson asserts that the group algebra is amenable if and only if the underlying group is amenable. An analogous result for the Fourier algebra has been proved only very recently by Z.J. Ruan and it involves so called operator amenability. A characterization of amenability in terms of a deep combinatorial property of Følner led to a strong relationship to the recent study of amenable unitary representation of locally compact groups by M.E. Bekka.

During the BIRS workshop, reports related to the above were made by Garth Dales on homological properties of modules over locall compact groups, by Volker Runde on non-amenability of the Fourier and Fourier-Stieltjes algebra, by Monica Ilie on characterization of completely bounded homeomorphisms on the Fourier algebra viewed as an operator space and their ranges, and by Ross Stokke on quasi-approximate units on the group algebra related to Flner conditions.

C^*-algebras and Non-commutative Geometry

C^*-algebras arose as mathematical models for quantum mechanical systems. In classical mechanics, one studies a geometric object, phase space, and an algebraic one, the observables of the system. The passage from the former to the latter is simply by taking the continuous functions on the space. Thus, the algebra of observables is commutative. The passage to quantum mechanics means that one considers algebras of operators on Hilbert space which are, in general, non-commutative. This means that the phase space is lost. The program of non-commutative geometry, as proposed by Alain Connes, is to develop the tools of conventional geometry in the setting of non-commutative operator algebras. This program has had many connections with other areas of mathematics: index theory for manifolds, topology, dynamical systems and number theory.

Marcelo Laca (Victoria) gave a talk on the structure of Hecke C^*-algebras. These were first constructed by Bost and Connes and arise naturally from number theoretic information. Igor Nikolaev (Calgary) gave a talk on the C^*-algebras which arise from dynamical systems which are closely linked with Riemann surfaces. In particular, the K-theory of these algebras provide information regarding the dynamics. Inhyeop Yi (Victoria) gave a talk about the general structure and K-theory of C^*-algebras arising from hyperbolic dynamical systems and shifts of finite type.