Algebraic Dynamics and its Connections to Difference and Differential Equations (Online)

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First of all, we wish to thank BIRS for supporting this 5-day workshop which had to be done in an online format due to travel restrictions caused by the pandemic.

1 Overview of the Workshop

This workshop brings together (in an online setting) researchers in Algebraic Dynamics and Algebraic Differential and Difference Equations. Each is a highly interesting area in its own and an important connection between them lies in the model theory of difference fields. There have been several significant results in both sides recently. One the one hand, we have results involving the classification of periodic subvarieties of split polynomial maps, canonical heights, equidistribution, and unlikely intersections. On the other hand, we have results on difference Galois theory, algebraic independence of Mahler functions, and Painlevé irreducibility of certain Schwarzian equations. This workshop comes at the right time to further the exchange of ideas, collaboration, and advancement of both areas.

There are 12 talks in total featuring a wide range of topics. All the talks are done through Zoom. We strive to commit to diversity and inclusion: at least one third of the talks is given by members of underrepresented groups is mathematics. Although the number of talks is only around 60% the number in a "normal" BIRS 5-day workshop in order to accommodate speakers and participants from different time zones, many of the talks attract more participants than the typical capacity of 40 people in the BIRS lecture hall at Banff. After each talk, many participants choose to remain in the "main room" to continue the discussion with the speaker. Many other participants can also join the "breakout room" in which they have casual chats about life at different parts of the world during this challenging time.

2 Presentation Highlights

Unlike a normal workshop in which participants would travel to the Banff Centre and devote the whole week for the workshop activities, this time everyone still needs to continue with duties at his/her home institution as well as familial responsibilities. In spite of this, each talk is prepared carefully and delivered in the most engaging way possible. For this reason, we feel that every single talk has to be part of the highlight of this workshop. In chronological order, these talks are:

- "Difference equations over fields of elliptic functions" by Ehud Deshalit (Einstein Institute of Mathematics).

- "Billiards and the arithmetic of non-arithmetic groups " by Curtis McMullen (Harvard).

- "On the Zariski dense orbit conjecture" by Junyi Xie (IRMAR - Université de Rennes 1).

- "Schwarzian equation, automorphic functions and functional transcendence" by Joel Nagloo (CUNY).

- "Elliptic surfaces and R-divisors " by Laura DeMarco (Harvard).

- "Equivariant currents and heights on the boundary of the ample cone of a K3 surface" by Simion Filip (Chicago).

- "A couple of conjectures in arithmetic dynamics over fields of positive characteristic" by Dragos Ghioca (UBC).

- "Model theory of group actions on fields" by Piotr Kowalski (Uniwersytet Wrocławski).

- "Definable Galois theory and holomorphic vector bundles" by Anand Pillay (Notre Dame).

- "On an arithmetic criterion for holonomicity" by Vesselin Dimitrov (UToronto).

- "Algebraic independence of solutions of linear difference equations" by Charlotte Hardouin (IMT - Université Paul Sabatier.)

- "Finite orbits and canonical heights for large groups of automorphisms" by Serge Cantat (Université de Rennes 1).

3   Further Comments

Although the main drawback of the online format is that participants cannot devote as much time and energy, there are several notable advantages. The first obvious one is the elimination of carbon footprint and environmental impact of travel. The second is the increase in the number and diversity of participants: while most participants are from Europe and North America due to the more favorable time zones, there are also several participants from China, Japan, and other places.

Dr. Di Vizio at CNRS sent us the following comments after the workshop, "..., it is easier to follow on Zoom than on the live video that are recorded at BIRS usually. So I think that we, all of us, have to think about how to organize hybrid online-in presence events, which should be the norm in the future. I'm digressing...I was skeptical about the whole on line things, but actually it worked nicely for me: I followed the talks better than usual (but I was not jet-lagged and there were less of them, which is good). Mathematically, I liked the talks. There were 4 of 5 of them which were of particular interest for me and that I found outstanding..."

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