Mathematical Criminology and Security

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1 Overview of the Field

Mathematical criminology and security is an emerging field that combines quantitative and qualitative criminology theories with mathematical analysis and methods to provide new tools for understanding and predicting criminal behavior. These tools may then be employed by law enforcement practitioners to provide evidence-based policing strategies to aid in efficient resource allocation.

Mathematical criminology to date has focused largely on the spatio-temporal dynamics of criminal events. Two general approaches have been taken: 1) understanding the general nature of these spatio-temporal dynamics by constructing mathematical models based on existing criminological theory and 2) constructing mathematical models that seek to make accurate predictions of when/where crimes are likely to occur in the future, a task that as often referred to as "predictive policing".

One of the key observations from criminology regarding spatio-temporal dynamics of crime is that urban crime clusters in spatio-temporal regions referred to as hotspots. Developing an understanding of the reasons for why hotspots form and how they will react to police intervention has been a major area for the field [28, 1, 26, 2, 30, 10, 9]. The modeling of crime hotspot formation starting from agent-based models has produced a dynamical systems and partial differential equation analysis that has helped to show that there are two types of hotspots (supercritical and subcritical) and these are either displaced or completely eliminated by police intervention. The power of these theories is that they allow for a deeper understanding of exactly how different mechanisms lead to observed crime patterns and provide a strong framework for modeling how different police intervention strategies might affect crime. The weakness of this approach is that fitting the models to actual crime data is not straightforward, and the models are therefore less useful for making specific predictions of crime events within any real-world jurisdiction.

Predictive policing emerged from the seminal work of Mohler *et al.* [14] where they modelled burglary crime via a self-exciting point process – known as a Hawkes process – in order to capture the clustering spatio-temporal dynamics indicated above in a manner that is also amenable to fitting to real crime data. This approach is very different to multi-level modelling and statistical analysis commonly done in criminology which usually only attempts to model explanatory variables or static count models; see [18] for a review of current quantitative criminology. The self-exciting point-process model has proven to be the key basis for all subsequent work in the field due to is simplicity and versatility. This approach is very powerful in terms of making specific crime predictions based on historical and recent data. However, it lacks strong explanatory power, and is not easily able to assist in understanding how new police behaviors might affect crime patterns going forward.

Alongside the mathematical work, there has been a considerable amount of quantitative and qualitative criminology research. However as pointed out by Prof. P. Jeffrey Brantingham in his talk at the workshop, despite all the research no significant improvement in the prediction of crime as been achieved using multi-level modeling. This leads to the key question in the field: have we reached the limits of predictive crime models or are we missing something in the models?

2 Recent Developments and Open Problems

We focus on developments made by mathematicians working in criminology who are using a variety of statistical, modelling, and dynamical systems/partial differential equations techniques. The area of mathematical criminology and security forms two distinct approaches: prediction and inference where the emphasis is on analysis of data, and modelling where the aim is to understand the theoretical reasons for emergence of collective behaviour (typically criminal). We briefly give a quick overview of the two areas but this is by no means an exhaustive literature review.

Recent advances have been in uncertainty quantification and data assimilation techniques for criminology/policing applications such as inferring network structures e.g., social networks [29], or links between crimes [19]. There has also been a considerable amount of work on analysing the statistical link between road networks and crime [7, 23, 3]. Other work has looked at applying data assimilation techniques from geophysical applications to bear on the problem of tracking and forecasting crime [11, 25], non-parametric methods [21, 32, 8], and other Bayesian methods for parameter estimation [16]. Analysis on the effectiveness of a predictive policing algorithm has recently been established [15] though more studies are need to be carried out to investigate further when and when not predictive policing algorithms are effective. Investigating the size of effect of police deterrence remains a major open area; see [6]. Development of change point detection algorithms to detect changes in criminal behaviour have also been developed; see [20, 33]

The development of new dynamical systems and game theory models of crime and security has been a major area of work in the past decade. There have also been two special issues in the European Journal of Applied Mathematics in 2010 (EJAM vol. 21) and 2016 (EJAM vol. 27). Typical game theoretic models [27, 12, 13] look at how rational actors interact with each other to yield some optimal pay off. Various partial differential equation models describing how criminals create hotspots and the interaction of various policing strategies (e.g "cops-on-dots") have been looked at by various authors [34, 30, 31]. Extensions to the Los Angeles model of Short *et al.* that incorporates long range travel using Lévy flights [4, 17]. Age-structured models have also been constructed [24]. The new and mathematically interesting PDE models has led to various rigorous existence analysis [22, 2] due to their quasi-linear nature.

The workshop at BIRS March 2019, was formed to address and discuss three open problems in the field:

- Prediction and inference: is it possible to bring together the various models and Bayesian methods to combine different data sources and improve prediction and inference accuracy?
- Modelling:
  - why crime does not happen despite being predicted?
  - can we model other social theories for crime?
  - can we model and analyse policing bias?
- Are there other mathematical methods e.g. topological tools (Betti numbers), agent-based model analysis [11], network analysis, that might be useful in this field?

3 Presentation Highlights

- P. Jeffrey Brantingham: The Structure of Criminological Theory

  This talk opened up the workshop and looked at the structure of criminological theory where it was highlighted there might be a possible 'crisis' in explaining crime in that there have been no major improvements in multi-level statistical models commonly used in quantitative criminology. In this talk, the key problem in trying to explain variability in crime across space, time and individuals was outlined.

- Patricia Brantingham: Patterns in Crime: An Overview

  Prof. Brantingham gave an overview of core concepts in patterns in crime such as activity space, awareness space, push and pull in cities. Understanding the underlying structure of a city is very important in trying to uncover crime patterns. A key concept that was introduced was that of 'Directionality' to a crime location for individual criminals leading to 'Directionality boundaries' where crime is observed to be maximal. Open problems presented include how to combine mobility data and models to improve crime prediction, and modelling fear-of-crime.

- Jonathan Ward: Agent-based models and data assimilation

  This talk presented various mathematical tools to analyse and combine data with agent-based models (known as ABMs). ABMs are commonly used on computational sociology to model individuals as agents and their actions/interactions. Mathematically, these models are very hard to analyse and combine with data and there is an immediate need to develop new mathematical techniques for AMBs. Dr. Ward presented various techniques such as bayesian model selection, data assimilation, and equation-free methods.

- Craig Gilmour: Self-Exciting Point Processes for Crime

  This talk presented work looking at using the Hawkes process for predicting crime space and time locations. The author looked at Chicago and highlighted that a constant background crime rate does not fit the data. It is suggest that an anisotropic background crime rate with an isotropic excitation rate may provide a better fit.

- Baichuan Yuan: An Efficient Algorithm for Spatiotemporal Multivariate Hawkes Process and Network Reconstruction

  This talk presented the author's work on developing uncertainty quantification methods to infer a hidden network structure based on event count data using a Hawkes process model. The key challenge here is that to infer a large networks requires a large amount of data and computationally efficient methods. The author presented a novel approach to the problem and demonstrated his method on a real social network application. It is clear that there are some interesting future directions for research in this area from both a statistical and modelling perspective.

- Michael Porter: Spatial event hotspot prediction using multivariate Hawkes features

  Prof. Porter talked about how one might determine if two crimes are linked together using a combination of a Hawkes process to determine the probability that one crime is linked/caused by another and linkage using a logistic regression model. Open problems presented here were

  - Can the self-exciting models help estimate linkage probabilities?
  - Can we use linkage to help inform self-exciting models?

- Yao Xie: Scanning statistics for crime linkage detection

  Prof. Xie talked about her work on change point detection to detect anomalies and how to develop data driven police patrol zones optimally. For the change point detection a generalised likelihood ratio detection statistic was developed. Open problems in this area include developing good spatio-temporal-textual point processes and developing a reinforcement learning approach.

- Naratip Santitissadeekorn: Approximate filtering of intensity process for Poisson count data

  This talk presented the author's recent work on developing sequential data assimilation (filtering) techniques for the discrete-time Hawkes process on a lattice. The objective is to develop efficient tracking methods rather than reconstruction of the past. Open problems in this area including developing efficient filters for crime linkage detection and network problems.

- George Mohler: Predicting crime is easy, using crime predictions is hard

  In this talk, an overview of crime prediction algorithms was presented along with the problems associated with trying to using crime prediction software in practice. Practical problems such as how to increase officer buy-in or how to get the public or other groups more involved in using crime models were discussed. It was also discussed how one can introduce 'fairness' into the system to deal with bias in the data. Open problems such as should models be regulated and how to make use of probabilistic predictions were discussed.

- Hao Li: Uncertainty Quantification for Semi-Supervised Multi-class Classification in Ego-Motion Analysis of Body-Worn Videos

  This talk presented the author's work on how to perform classification of body-worn police videos with a small amount of annotated training data. The main problem is that a massive amount of video data is collected without labelling and it is a problem how to make efficient use of human labelling (triage). For this problem he developed a novel uncertainty quantification algorithm to carry out the labelling efficiently and highlight where human labelling is required. This was then demonstrated on real data.

- Nancy Rodriguez-Bunn: Modelling Riot Dynamics

  This talk presented the author's work on modelling the 2005 French riots using a non-homogeneous Poisson process and a Susceptible-Infected-Recovered model. Open problems in this area are how to effectively model and forecast riot activity.

- Chunyi Gai: Existence and stability of spike solution in SIRS model with diffusion

  This talk looked at a variant of the Susceptible-Infected-Recovered model from epidemiology with spatial diffusion that can lead to stationary spatial spikes in the infected population. A stability analysis of the spikes was carried out. An open problem is to analyse spatio-temporal spikes.

- Toby Davies: Street networks and their role in crime modelling

  This talk looked at the link between the street network and burglary crime. Several challenges were highlighted such as how 'Directionality' can be reconciled with the street network, community structure, and analysis of immunity when crime does not happen despite being predicted. It was also discussed how to link a network-based model to continuum models.

- Wen-Hao Chiang: Multi-armed bandit problem on rescue resource allocation

  This talk presented the problem of how to use hot-spot prediction for early resource allocation leading to the problem of exploration versus exploitation. The key challenge is how to develop efficient algorithms to solve this problem.

- Ian Brunton-Smith: Collective efficacy and crime in London: The importance of neighbourhood consensus

  In this talk, the criminology theory for collective efficacy was presented and new opportunities for mathematical modelling of collective efficacy was suggested. For instance, can mechanistic models help understand the formation/dynamics of spatio-temporal consensus of collective efficacy? can we understand the impact of external events like terror attacks?

- Maria R D'Orsogna: Santa Monica, the train and proposition 47

  In this talk an analysis of violent crime in Santa Monica and the possible reasons for the recent increase (a new train line or proposition 47 law change) were investigated. Open problems here are in trying to determine the key causes of violent crime patterns to change.

4 Scientific Progress Made

The workshop ran several discussion groups to focus on key areas of interest in order to map out interesting future directions:

- Bias - There have been various criticisms of the use of predictive policing algorithms and the possibility they perpetuate biases; see for instance [5]. Common worries are that these biases lead to either too little police in an area or too much police (leading to incidental arrests reinforcing crime hotspots). So far most of these issues have not been looked at mathematically from a modelling point-of-view. Such an analysis could look at the effect of timescales and different communities.

- Networks and neighbourhood/community effects: this session discussed how one might model various physical transportation networks and link this with social networks.

- Directionality: this session looked at how one could model mobility/direction of crime to travel paths, and directionality boundaries and porosity. The discussion group also looked at geographic profiling and how one could incorporate this and directionality in self-exciting models, and combine with mobility data.
- Prevention and Intervention - here prevention is defined as long-term and intervention as short-term. The discussion group looked at the open problem of how age-structured models could be used to predict 40year crime waves and to investigate intervention strategies e.g. at what age should society intervene? The main challenge is to overcome the low probabilities of events and either saying intervene on everyone or no one.
- Modelling criminology (learning) theories - An introduction to other theories for criminal behaviour were presented and discussions focused on what mathematical modelling could do to combine these theories and explore them in a systematic fashion.

5 Outcome of the Meeting

The meeting highlighted three main interesting directions for the future of mathematical criminology:

- The first area for future work is in prediction and inference where recent developments in uncertainty quantification and data assimilation could be combined with network models to help improve explaining the variability in crime.
- The second area for future work highlighted is in modelling and mathematical analysis of other types of criminology (Learning) theories such as social disorganisation, differential association, control theory, ANOMIE, conflict theory, labelling theory etc. The aim of the models would be to combine various qualitative theories to see if they can explain spatio-temporal crime patterns.
- The third area was how to build a community in this area and organise workshops to develop the field of mathematical criminology.

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