Generalized Geometry meets String Theory

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1 Overview of the Field, Recent Developments and Open Problems

The theme of the workshop, "Generalized Geometry meets String Theory," is embedded in worldwide research activities connecting Mathematics, Mathematical Physics and High Energy Theoretical Physics. Generalized geometry is a mathematical area rooted in differential geometry. The fundamental objects in this subject are Courant algebroids, which are vector bundles equipped with certain natural structures motivated by dynamical systems and Poisson geometry [14, 16, 31]. Generalized geometry has found applications in various subjects, including theoretical physics, where Siegel envisaged basic Riemannian aspects of the theory to understand dualities in low-energy superstrings [34]. The relation to complex geometry was conceived by Nigel Hitchin [28], who introduced the notion of generalized complex structure; a unifying framework in which structures such distinct as complex manifolds and symplectic manifolds can be studied from a natural unified perspective. One of the early successes of this theory within mathematics was the discovery by Marco Gualtieri [26, 27] that the extended deformation theory of complex structures, as developed by Sergey Barannikov and Maxim Kontsevich [7], matches the deformations of a complex structure within the class of generalized complex structures. This result can be thought of as providing an enlightening geometric meaning for an involved algebraic matter.

Building on the early work by Siegel, theoretical physicists, such as Daniel Waldram and his collaborators [9, 10], have accomplished a similar geometrization of part of the intricacies occurring in supergravity and string theory, based on generalized geometry. This has completely transformed our way of thinking about the space-time geometry on the one hand and the matter content of the universe on the other hand as the two players in Einstein's field equations. More and more of the structures modeling matter fields can be now thought of as part of the (generalized) geometry. In this way hidden symmetries (such as B-field transformations and generalizations thereof, for instance) become visible and can be used for an efficient study of supergravity theories and their solutions.

These developments have recently led to a number of emerging research areas with a visible impact on the dynamics of traditional research communities. An example is the study of the Ricci flow for generalized metrics on Courant algebroids as initiated by Mario Garcia-Fernandez and Jeffrey Streets [17, 21]. Similarly, the Einstein equations for generalized metrics on Courant algebroids are now beginning to be studied in the same spirit as the Einstein equations in Riemannnian geometry and mathematical relativity.

We understand today that despite the formidable advances in the understanding of moduli spaces of Ricci-flat Kähler metrics (and, more generally, Kähler-Einstein metrics) only a small part of the moduli spaces of string theory can be accessed on this basis. Allowing non-trivial fluxes considerably enlarges the scope of structures playing an active role, as we know from the work of Mariana Graña [25] and others. Further advancements require to admit as subjects of investigation non-Kähler complex structures, non-integrable complex structures and other geometric objects occurring in generalized geometry. Note that the enlarged scope does also allow for new, previously inaccessible, connections between Calabi-Yau manifolds by geometric transitions within larger moduli spaces (possibly fulfilling Miles Reid's fantasy [32]), conjecturelly linked to the Hull-Strominger system in heterotic string theory. New mathematical methods need to be developed to successfully treat such structures, which do not longer enjoy the same strong integrability properties as the classical ones. The integrability conditions relevant to string theory are currently being determined and analyzed within physics following generalized geometry as one of the guiding principles. We will report on the latest developments in Section 2 below. This is expected to lay the foundations of future work within mathematics extending current research in gauge theory, Kähler-Einstein metrics etc.

Courant algebroids as the typical habitat of generalized geometric structures come with various nontrivial connections to homological and higher categorical structures. These connections are the object of an increasing current research. This is already so for the simplest case of exact Courant algebroids involving the field strength of the Kalb-Ramond field as the Severa class, an element of the third de Rham cohomology over the reals. Another way of incorporating that field, but now with an additional integrality condition, is via the language of $\mathrm{U}(1)$-bundle gerbes. The latter is of a higher categorical flavor as it incorporates 2-morphisms. It has been conjectured by Severa (in his famous letters to Weinstein [33]) that infinitesimal symmetries of $\mathrm{U}(1)$-bundle gerbes should relate to Courant algebroids. Precise correspondences relating the Lie 2-algebra of infinitesimal symmetries of a $\mathrm{U}(1)$-bundle gerbe endowed with a connection to a Lie 2-algebra of sections of a corresponding Courant algebroid have been established in the literature, see [15] for a recent paper containing an overview of the latest results by Collier, the authors and others. These results afford a quasi-isomorphism between the above Lie 2-algebras (included each in short sequences of Lie 2-algebras), turning Severas conjecture into a precise statement. This type of research has paved the way for the emerging field of higher gauge theory, which has been one of the topics discussed at the workshop, as described in Section 2.

As another instance relevant to string theory of the intriguing relations to higher structures we would like to mention a major open problem related to Maxim Kontsevich's Homological Mirror Symmetry Conjecture [30]. Recall that the latter is a conjectural equivalence, verified only in special cases, between the derived category of a Calabi-Yau manifold and the (conjectural) Fukaya category of its mirror manifold. It is expected that the conjecture should have a more general version valid in the context of generalized complex structures, which would naturally include the extended deformation spaces. Further research is necessary to identify the relevant categories and their properties. It seems reasonable for this and other purposes to focus first on the special case of generalized Kähler structures. The class of this structures constitutes a far reaching generalization of ordinary Kähler structures. It has been one of the recurrent themes in our workshop, as described in Section 2. In particular, but not exclusively, generalized Kähler structures have been analyzed in the context of categorical concepts such as stacks, groupoids etc.

2 Presentation Highlights

To convey an idea of the scientific content of the workshop we have chosen to emphasize a few representative topics. These will be illustrated by mentioning particular contributions. The complete list of presentations is available at the workshop's homepage with abstracts: https://www.birs.ca/events/2024/5-day-workshops/24w5222/schedule. The speakers academic age ranged from early postdocs to senior scientists. The talks were of an excellent level and in view of the list of participants it is obvious that we could have easily filled another week with excellent talks.

In addition to the individual research talks we had an opening overview talk by Marco Gualtieri and an additional course on the geometry and topology of supergravity by Carlos Shahbazi. The course was on the free afternoon on Wednesday (introductory part) and on Friday after the end of the workshop (more advanced part). Its main purpose was to provide a service for the PhD students, especially those from the local community, but its advanced part on Friday was well attended also by postdocs and even a few senior scientists.

Generalized Kähler structures and related topics The notion of a generalized Kähler structure is the analogue of an ordinary Kähler structure (considered as a geometric structure on the tangent bundle) in the setting of geometric structures on Courant algebroids. It can be also considered as a generalization of a Kähler structure and, at least in the case of exact Courant algebroids, can be described in terms of ordinary geometric data on the tangent bundle. In fact, it is in the form of bi-Hermitian structures in which generalized Kähler structures first appeared in the physics literature [22] as new targets of non-linear σ-models with $\mathcal{N} = (2, 2)$

supersymmetry before their meaning in generalized geometry was unraveled in the mathematical literature [26]. The main new aspect of these σ-models was the appearance of Hermitian structures with non-trivial intrinsic torsion, related to the twisting of the chiral multiplets.

The Hermitian structures underlying a generalized Kähler structure occur in the study of special types of non-Kähler complex structures, as discussed in Anna Fino's talk among others. Some of the structures with torsion analyzed by Anna Fino arise as fixed points of interesting geometric flows. This is the case for the Bismut-Hermite-Einstein structures, which are fixed points of the pluri-closed flow. We recall [21] that the latter is a flow for a pluri-closed metric (and a B-field) on a complex manifold driven by the Ricci-form of the Bismut connection. It is a special instance of the generalized Ricci-flow some highlights of which are described below.

The beauty of generalized Kähler structures and their relevance to physics was emphasized right from the start of the conference, already in Marco Gualtieri's opening lecture. The description of generalized Kähler structures in terms of double stacks anticipated some of the higher structures present in later more specialized talks. Let us mention, for instance, Daniel Álvarez's talk on the generalized Kähler potential, where Morita equivalences of symplectic double groupoids were the key concepts.

It was a recurrent theme of the conference that generalized Kähler structures are intimately related to emerging higher structures as well as to central classical problems of twentieth century mathematics. Such a classical circle of ideas is Calabi's program of finding canonical metrics on complex manifolds (including Kähler metrics of constant scalar curvature). The program can be seen as a far reaching generalization of Riemann's uniformization theory. As explained by Vestislav Apostolov in his inspiring presentation, the framework of generalized Kähler geometry allows to significantly enlarge the problem of finding a Kähler metric of constant scalar curvature in a given Kähler class. One of the benefits is that holomorphic Poisson manifolds can be now be studied from the perspective of finding canonical metrics. A meaningful Calabi problem can be stated nicely for holomorphic Poisson manifolds admitting a compatible generalized Kähler structure of symplectic type (M, J, ω_0). The generalization of the Calabi problem, as proposed in [4], does then consist in finding a generalized Kähler structure of symplectic type (M, J, ω) which has constant generalized scalar curvature, is compatible with the given holomorphic Poisson structure and has the same de Rham cohomology class $[\omega] = [\omega_0]$.

Generalized Ricci flow and generalized Einstein equations Another highlight of the workshop was the presentation, by Fridrich Valach, of the most recent advances in the study of the generalized Ricci flow. While previous research had focussed on the case of exact Courant algebroids [21], we have now a fully fledged theory applicable to general transitive Courant algebroids [17, 35], which is furthermore compatible with complex geometry [19]. The evolution of the divergence operator, essential for the definition of the generalized Ricci curvature and encoded in a half-density (related to the physicists dilaton), leads to a gradient flow intepretation of generalized Ricci flow [21, 35] via the *string effective action*. Different choices of evolution equation for the dilaton field, e.g. via the conjugate heat equation, the generalized scalar curvature, or the dilaton flow, lead to a batery of interesting results, extending the fundamental theory of Hamilton-Perelman on the renown Ricci flow equation.

The generalized metrics fixed under the generalized Ricci flow have vanishing generalized Ricci curvature and generalized scalar curvature and the divergence operator is defined by a half-density. More generally, one can consider generalized metrics of zero generalized Ricci curvature without assuming that the generalized scalar curvature vanishes and allowing arbitrary divergence operators. Such metrics are known as generalized Einstein metrics. The most important cases are those when the pseudo-Riemannian metric on the base manifold induced by the generalized metric is either Riemannian or Lorentzian. The system of non-linear partial differential equations describing generalized Einstein metrics reduces to a purely algebraic system in the setting invariant solution on a homogeneous space. In this way it is possible to construct solutions and to obtain partial classification results based on algebraic methods. This is the approach presented by Liana David who explained the classification [11] of left-invariant generalized Einstein metrics on the simplest class of non-exact heterotic Courant algebroids known as odd exact Courant algebroids or Courant algebroids of type B_n over 3-dimensional Lie groups.

The link of generalized Ricci flow to complex geometry was covered in the talk by Jeffrey Streets, who introduced a system of parabollic equations known as *pluriclosed flow on string algebroids*, following the recent preprint [19]. These equations extend the theory for the *pluriclosed flow*, as introduced by Streets in Tian, and aim at constructing solutions of the Hull-Strominger system in heterotic string theory. The evolution equations are derived using the theory of string algebroids, a class of Courant algebroids which occur naturally in higher gauge theory. The main result presented by Streets was the existence of a priori C^∞ estimates for uniformly parabolic solutions. This in particular settles the question of smooth regularity of uniformly elliptic solutions of the Hull-Strominger system, generalizing Yau's C^3 estimate for the complex Monge-Ampère equation. A conjectural relationship of the flow to the geometrization of Reid's fantasy was also discussed.

Generalized geometry in string theory and supergravity The fact that generalized geometry was the proper framework to describe supergravity backgrounds with fluxes was first noted in [37], where it was shown that supersymmetric solutions require integrability of generalized complex structures. Alessandro Tomasiello reviewed many of the applications of generalized geometry in supergravity. Most of the recent advances in the field, including on-going work, were then discussed in detail in the workshop. David Tennyson showed how the moduli of the flux backgrounds are encoded in the generalized structures [38], while Michela Petrini discussed how to incorporate supersymmetry breaking in the formalism. Generalized geometry also plays a very important role in AdS/CFT correspondence, as many features of the conformal field theory are encoded in the generalized geometry. Anthony Ashmore explained how to encode the marginal deformations of the CFT in the generalised structures [5], while Daniel Waldram showed the generalised geometry realisation of RG flow of the R-charges of the gauge theory.

Higher gauge theory and generalized geometry Another interesting aspect covered in the workshop was the relation between generalized geometry and higher gauge theory. Ever since the seminal work by Ševera [33], it was expected that infinitesimal symmetries of $\mathrm{U}(1)$-gerbes should relate to Courant algebroids. Precise correspondences relating the Lie 2-algebra of infinitesimal symmetries of a $\mathrm{U}(1)$-gerbe endowed with a connection to a Lie 2-algebra of sections of a corresponding exact Courant algebroid have been established in the literature (see [15] and reference therein). The presentation by Roberto Tellez during the workshop, covered the latest develpments on this topic, including application to the theory of the Hull-Strominger system in heterotic supergravity. Building on previous work by Sheng-Xu-Zhu [42], Tellez presented recent work with Álvarez Cónsul and Garcia-Fernandez extending this correspondence to the case of transitive Courant algebroids of *string type*. He also presented a Chern correspondence for holomorphic principal 2-bundles [36], described by means holomorphic multiplicative gerbes on principal bundles for complex reductive Lie groups. The main result of the talk was the construction of the moduli space of holomorphic principal 2-bundles with holomorphic connective structure, as a derived 2-stack. This results pave the way towards a Donaldson-Uhlenbeck-Yau type theorem in higher gauge theory, via the Hull-Strominger system, and have potential applications to the construction of higher versions of Donaldson-Thomas invariants, as proposed in the physics literature [6].

3 Scientific Progress Made

In this section we illustrate the advances recently obtained by the participants of the workshop by highlighting some specific research results elaborating on the themes mentioned in Section 2.

- **Bismut-Hermite-Einstein structures.** A Bismut-Hermite-Einstein structure on a manifold M is a Hermitian structure (J, g) such that the corresponding fundamental 2-form ω is pluriclosed (that is $\partial\bar{\partial}\omega = 0$) and the Ricci form of the Bismut connection is zero. These structures can be considered as a toy model for the Hull-Strominger system and, in a sense, as *half-generalized Kähler structures*. Fundamental aspects of theory have been developed in [**?**, **?**], where it is shown that they relate to algebraic stability conditions via holomorphic Courant algebroids. Thanks to the work of Anna Fino and her collaborators [8] we have now explicit non-Kähler constructions of such structures on mapping tori in dimension bigger than two. The main extra assumption for the theorem is that the torsion of the Bismut connection is required to be parallel. Under this assumption the universal cover of (M, J, g)

is shown to be a product of a Ricci-flat Kähler manifold and a Lie group with bi-invariant Bismut Ricci-flat Hermitian structure. Non-trivial examples of such manifolds (M, J, g), that is which are neither Bismut-flat nor products, are constructed in [8] as mapping tori making use of isometries of particular non-flat Calabi-Yau manifolds such as K3 surfaces. This settles a question raised in [21]. More generally, one can ask for Bismut-Hermite-Einstein structures on compact non-Kähler manifolds that have not only non-zero Bismut curvature but also non-parallel torsion, or even full holonomy $SU(n)$ for the Bismut connection.

- **Global stacky description of generalized Kähler structures.** A new idea described in the BIRS workshop was the recent preprint [1] of Alvarez–Gualtieri–Jiang which provides a "global" description of generalized Kähler structures in the general case. This originates from a new description of generalized Kähler geometry as a relationship between holomorphic Courant algebroids equipped with the structure of a Manin triple. Manin triples, originally introduced by Drinfeld for the classification of Poisson Lie groups, were generalized to arbitrary Courant algebroids by Liu-Weinstein-Xu. While it is well-known that a generalized Kähler structure involves two complex manifolds $X_{\pm}$, each equipped with a holomorphic Manin triple, it recently became clear that these two Manin triples, together with their complex conjugates, are related by a square of morphisms in the double category of Manin triples. This preliminary result of [1], expressing generalized Kähler geometry as a square in a higher category of Manin triples, together with the idea that a Manin triple is an infinitesimal object corresponding to a symplectic double Lie groupoid, leads to the main result: that the four related Manin triples integrate to a square of morphisms in the double category of holomorphic symplectic double Lie groupoids. Here the vertical and horizontal morphisms are given by vertical and horizontal Morita equivalences, while the square of morphisms are "filled" by a 2-morphism or homotopy consisting of a holomorphic symplectic double Lie bimodule. The generalized Kähler metric is finally determined by a Lagrangian submanifold of this last bimodule.

  One of the interesting applications of the above result is to the positive resolution of a conjecture made by physicists Gates, Hull, and Rocek in 1984 concerning the existence of a scalar function, the generalized Kähler potential, which determines the metric. Since, in the above description, a generalized Kähler metric is determined by a real Lagrangian submanifold, it becomes immediately clear that it can be described locally, in Darboux coordinates, as the graph of the derivative of a real scalar-valued function, as required by the conjecture.

  The description of generalized Kähler geometry in terms of a diagram of double symplectic groupoids should have an impact on the physical 2-dimensional sigma-model which originally led to the discovery of the geometry in the first place: during the workshop, we discussed the possibility that the 2-dimensional sigma model should, in fact, be defined as a gauged nonlinear sigma model, taking into account the actions of the four symplectic groupoids in the diagram. This is somewhat analogous to the history of Chern-Simons theory, where versions of the theory were considered for a lie algebra-valued 1-form A, whereas eventually this was understood as a connection on a topologically nontrivial principal G-bundle. The same global perspective shift should occur in the generalized Kähler case, for the $N = (2, 2)$ supersymmetric sigma model.

- **Calabi problem for generalized Kähler structures.** Goto's Hamiltonian approach [23, 24] to generalized scalar curvature particularly successful in the case of generalized complex structures of symplectic type and based on the pure spinor formalism has been reconsidered and extended in the recent work [3] obtaining a more complete understanding in the language of classical bi-Hermitian structures. In particular, it is shown in [3] that Goto's generalized scalar curvature defined in terms of pure spinors is constant on generalized Kähler-Ricci solitons. In the case of generalized complex structures which are generically of symplectic type the generalized Kähler class is realized as a complexified orbit of the Hamiltonian group action. Based on these advances analogues of the Mabuchi metric and K-energy are defined in this setting and a uniqueness result for generalized Kähler metrics of constant generalized scalar curvature is obtained in [3] under a geodesic connectedness condition. These advances involving the workshop participants Apostolov and Streets connect nicely with other results and views about the generalized Kähler class discussed at the workshop.

- **Generalized Ricci flow.** The generalized Ricci flow is a strict generalization of the ordinary Ricci flow and unifies several interesting geometric flows such as the pluriclosed flow mentioned above. Until very recently, the generalized Ricci flow had been systematically studied only on exact Courant algebroids [21]. The flow is regarded as an evolution equation for a generalized metric, with the divergence operator flowing by different evolution equations, depending on the context, and representing different gauges. E.g., for the gradient flow intepretation of generalized Ricci flow, the (exact) divergence operator evolves by the *conjugate heat equation* (corresponding to constant half-density), while generalized scalar curvature monotonicity requires the *dilaton flow* for the divergence operator [43]. Building on the definition proposed in [17], this theory has very recently been extended to the case of transitive Courant algebroids [19, 35], including the relation to complex geometry. Short time existence and uniqueness for the generalized Ricci flow on transitive Courant algebroids is established in these references, along with other fundamental analytic results.

An interesting aspect of [35], based on an observation made in [21, Remark 6.9], is that the evolution equation for the divergence operator, regarded as a half-density, is taken to be minus one half of the generalized scalar curvature. This turns out to be equivalent to the *dilaton flow* for the corresponding one-parameter family of dilaton functions, implying key monotonocity formulae for the generalized scalar curvature. This way, the flow is regarded as a gradient flow for the generalized Einstein Hilbert functional, for a suitable choice of pairing on the space of pairs, given by generalized metrics and half-densities. This choice correspond physically to a particular choice of the renormalisation scheme – namely dimensional regularisation – in the calculation of the renormalisation group flow of the string nonlinear sigma model. This choice yields many key analytic properties as well, in particular that the evolution of the dilaton function contains no second order derivative terms in the metric.

As explained in Fridrich Valach's presentation, based on fundamental work by Jurco-Vysocky [29], the generalized scalar curvature can be defined as a certain full trace of a certain generalized Riemann tensor. This new insight avoids the usual component formula for the generalized scalar curvature of an exact Courant algebroid, which should be instead derived as a special case from the more fundamental definition as a full trace. It also avoids having to rely on the definition of the generalized scalar curvature in terms of a Weitzenböck formula. With the new definition as a full trace, the generalized scalar curvature can be more easily seen as a fundamental scalar curvature invariant in general. Moreover, it is directly recognized as a natural main ingredient for a generalized Einstein-Hilbert functional.

- **Pluriclosed flow and the Hull-Strominger system.**

  The relation between the generalized Ricci flow on transitive courant algebroid and the Hull-Strominger was covered in the talk by Jeffrey Streets. Based on the recent preprint [19], Streets introduced a system of parabollic equations known as *pluriclosed flow on string algebroids*. These equations extend the theory for the *pluriclosed flow*, as introduced by Streets in Tian, and yield gauge-fixed versions of generalized Ricci flow on transitive Courant algebroids, as in [35]. In this setup, short time existence is achieved by a suitable one-form reduction of the flow adapted to the complex structure, and new monotone quantities arise such as the *dilaton functional*. Natural fixed points for the flow are the solutions of the the Hull-Strominger system in heterotic string theory. The main advance presented by Streets was the existence of a priori C^∞ estimates for uniformly parabolic solutions. This in particular settles the question of smooth regularity of uniformly elliptic solutions of the Hull-Strominger system, generalizing Yau's C^3 estimate for the complex Monge-Ampère equation. Long time existence and convergence on Chern flat transitive holomorphic Courant algebroids, as well as a conjectural relationship of the flow to the geometrization of Reid's fantasy, were also discussed.

- **Generalized Einstein structures on Lie groups.** Considering invariant generalized Einstein metrics on Courant algebroids over homogeneous spaces considerably enlarges the classical topic of invariant Einstein metrics on homogeneous spaces, which is characterized by a close interaction between differential geomery and Lie theory. First results obtained include the algebraic formulation of the equations, some structure results and partial classifications. Already the case of left-invariant generalized Einstein metrics on exact Courant algebroids over 3-dimensional Lie groups, treated in [12], is non-trivial. As explained in Liana David's presentation, the latter classification was extended to Courant algebroids of type B_3 in [11] showing that the problem is promising even for heterotic Courant algebroids. In the case of exact Courant algebroids there has been recent progress involving some of the workshop participants [13]. As a consequence, we have now a complete description of positive definite left-invariant generalized Einstein metrics on Courant algebroids over Lie groups under the assumption that the group is either 4-dimensional or solvable. In the Lorentzian case a classification is obtained under extra assumptions. In four dimensions the assumptions made are only that the Lorentzian metric in non-degenerate on the commutator ideal and that the left-invariant tensor field induced by the (left-invariant) divergence operator is zero.

- **Moduli spaces in higher gauge theory.**

  A key advance in the understanding of moduli spaces of instantons in higher gauge theory was presented by Tellez, based on recent work with Álvarez Cónsul and Garcia-Fernandez. Regarding solutions of the Hull-Strominger system as higher instantons [2], and via a conjectural Donaldson-Uhlenbeck-Yau type theorem in higher gauge theory [20], Tellez showed that these moduli spaces can be identified with moduli spaces for holomorphic principal 2-bundles with holomorphic connective structure. This is achieved via a Chern correspondence for holomorphic principal 2-bundles [36], described by means of holomorphic multiplicative gerbes on principal bundles for complex reductive Lie groups. The geometry of this last moduli space was also uncovered, exhibiting an intricate structure of derived 2-stack (with further a conjectural -1-shifted symplectic structure). These results pave the way towards a Donaldson-Uhlenbeck-Yau type theorem for the Hull-Strominger system, and have potential applications to the construction of higher versions of Donaldson-Thomas invariants, as proposed in the physics literature [6].

- **Applications to/in string theory**

  The multiple applications of generalized geometry to string theory were reviewed in the meeting. There is now a clear understanding of the moduli spaces of string theory compactifications in the presence of fluxes [6, 38]. Generalized geometry also allowed to describe holographically properties of gauge theories [5]. There was also considerable advance in the analysis of supersymmetry breaking by fluxes using generalised geometry [39], describing a much larger set of solutions than the ones found ten years ago [40]. Also, introducing the notion pf generalised parallelisable manifolds, generalized geometry allowed to build consistent truncations of ten-dimensional supergravity which bypass the need of parallelisable manifolds [41]. This allowed to formally prove the consistency of the truncation on S^5 heavily used in holography to study deformations of conformal field theories by their dual deformations of $AdS_5 \times S^5$.

4 Outcome of the Meeting

The meeting has had an important impact on the worldwide research activity in generalized geometry and its interactions with theoretical and mathematical physics. It has helped to give new perspectives to the field and to connect the researchers integrating it. Follow up meetings extending into novel directions and new collaborations have been sparked. To name an example, let us mention the recent workshop “Generalized complex structure and homological methods” held in Hamburg, which brought together generalized complex geometry, homological algebra and higher category theory to explore connections between the fields, see https://www.conferences.uni-hamburg.de/event/464/.

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