CY metrics for CICYs and Toric Varieties

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Strings: Geometry and Symmetries for Phenomenology
Oaxaca

November 11, 2021

Based on:

[Anderson, Gray, Gerdes, Krippendorf, Raghuram, FR: 2012.04656]

[Larfors, Lukas, FR, Schneider: 2111.01436]

[Ashmore, FR: 2103.07472]

[For a review see FR: Phys. Rept. 839 (2020)]

Motivating Question

Question:

Can string theory describe the universe we live in? $\Rightarrow$ **Landscape**

Question:

Are there phenomena that always/never come from string theory (or even any quantum theory of gravity)? $\Rightarrow$ **Swampland**

Swampland Distance Conjecture

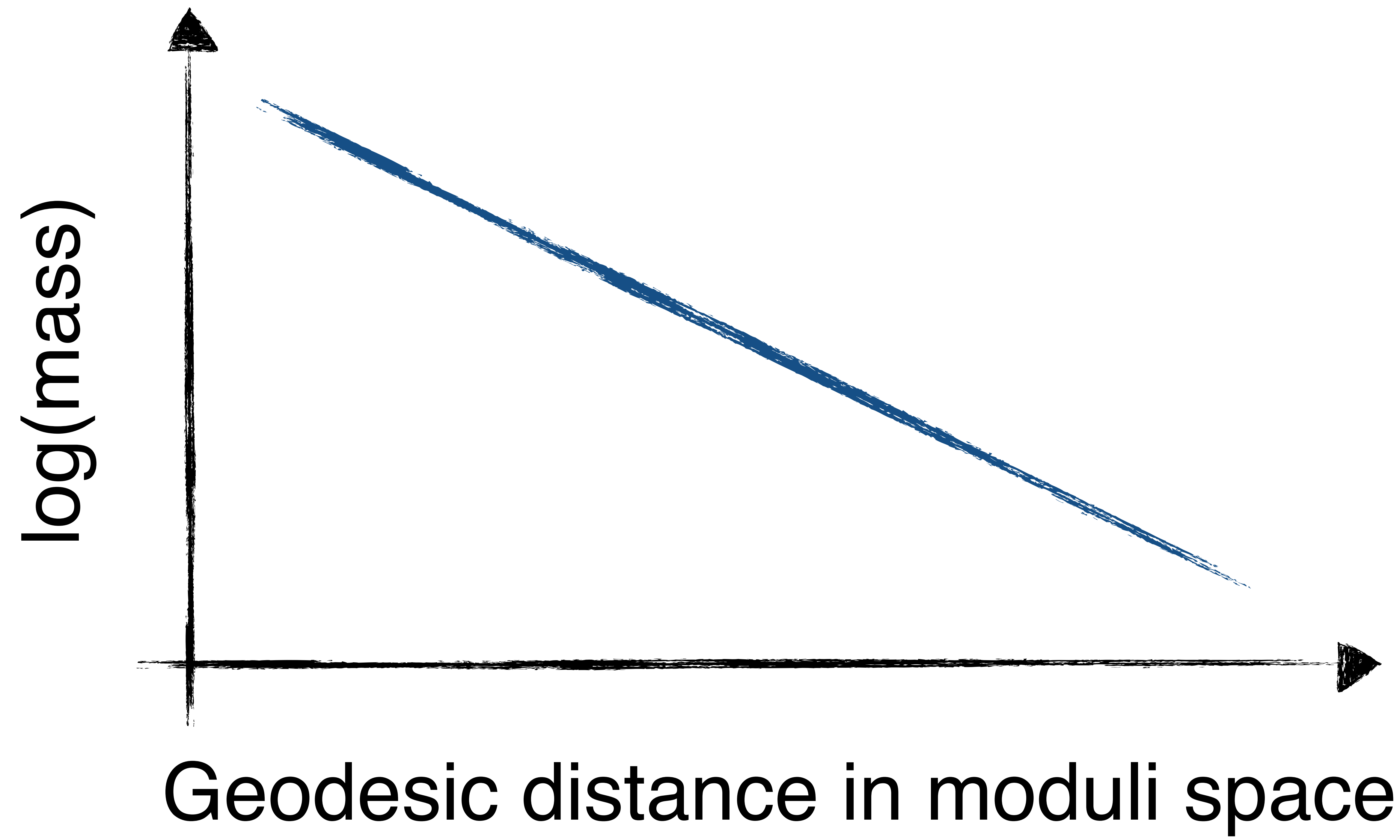
- An intuiting conjecture about a property that any string theory satisfies is the **Swampland Distance Conjecture** **[Ooguri, Vafa `06]**

Conjecture:

Compare a string theory compactified on a CY X at a point p_1 in its moduli space with the theory at a point p_0. Denote the (geodesic) moduli space distance by d.
Then, the theory at p_1 has an infinite number of light particles, with mass starting at the order $m \sim e^{-\alpha d}$ with $\alpha = \mathcal{O}(1)$ in Planck units.

Swampland Distance Conjecture

We want to check that for a specific CY (the quintic): $m \sim e^{-\alpha d}$



Compute massive KK states (schematically!)

- Starting point: **10D Klein-Gordon equation**

$$\Delta_{10D}\ \Phi_{10D} = 0 \qquad \Delta_{10D} \sim g^{MN}\partial_M\partial_N$$

- Now decompose

$$\Delta_{10D} = (\Delta_{4D};\ \Box_{6D}) \qquad \Phi_{10D} = (\phi_{4D};\ \varphi_{6D}) \qquad g^{MN} = (g^{\mu\nu};\ g^{ab})$$

- Use **6D eigenfunctions** of d'Alembertian

$$\Box_{6D}\varphi_{6D} = \lambda\varphi_{6D} \qquad \Box = d\delta + \delta d = \sqrt{\det(g)}^{\,-1}\partial_a\sqrt{\det(g)}g^{ab}\partial_b$$

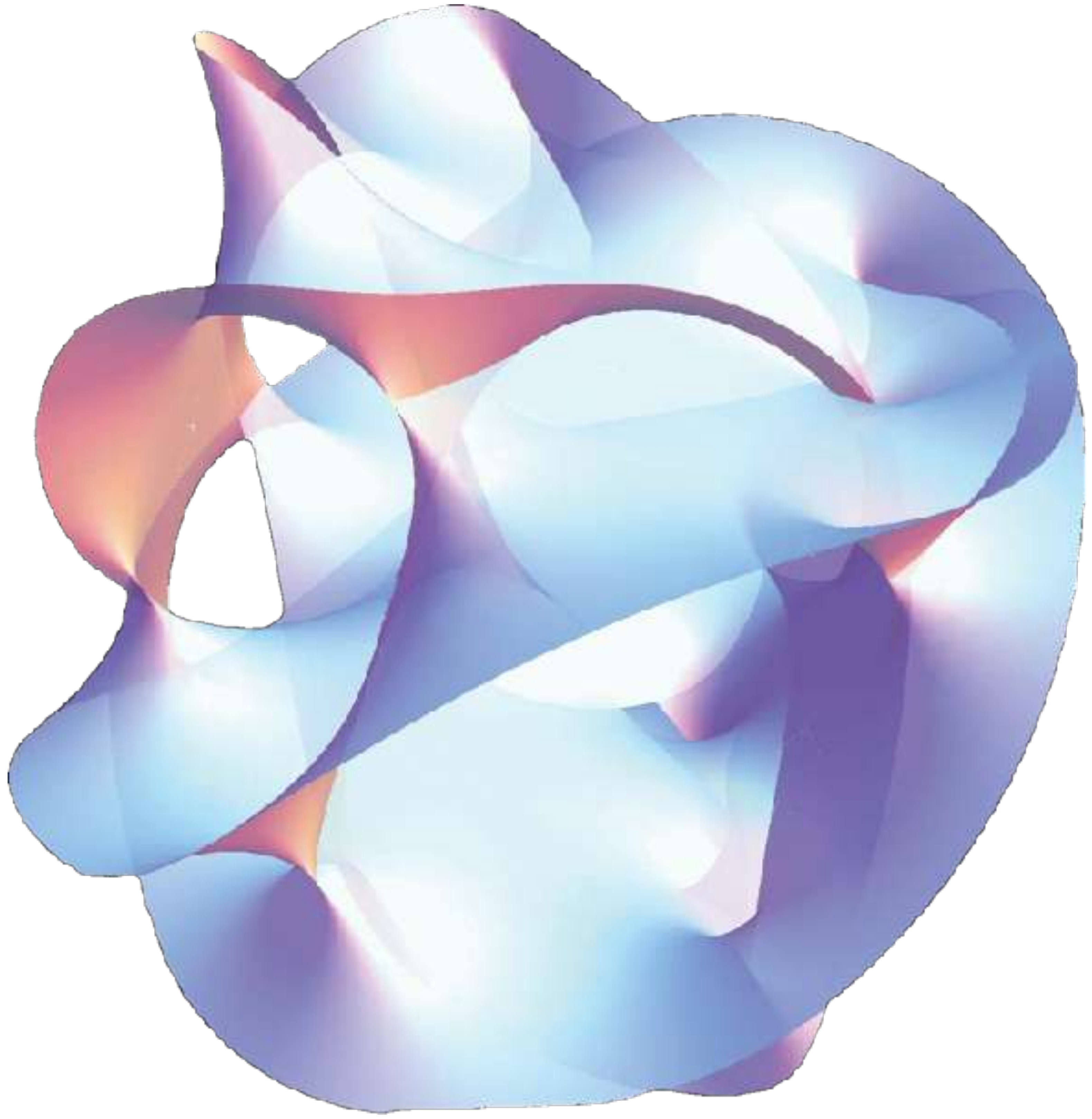
- Get **4D Klein-Gordon** field equation with a mass term

$$(\Delta_{4D} + \lambda)\phi_{4D} = 0 \qquad m^2 \sim \lambda$$

To **compute** the tower of **masses**, we **need** the **CY metric** g^{ab}

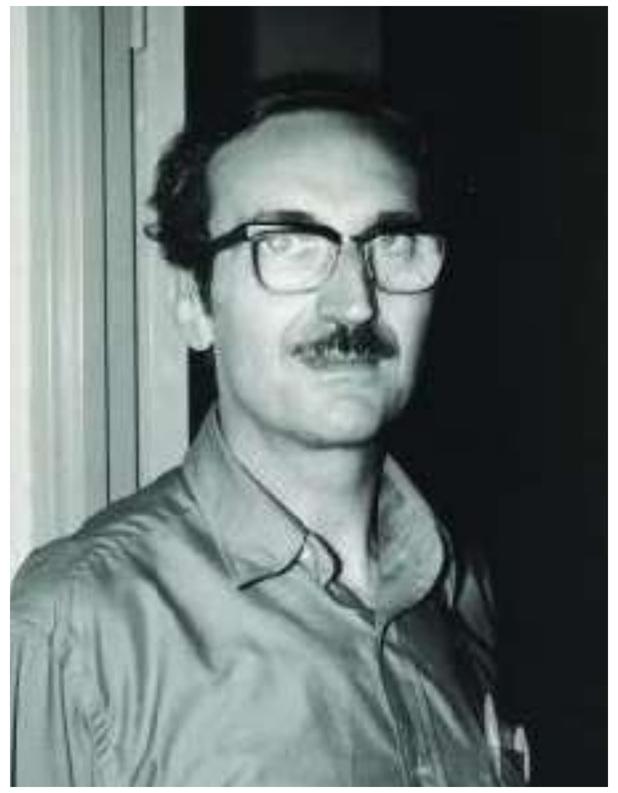
Outline

- Part I
  - CY metrics
    - Introduction
    - CY metrics from machine learning
- Part II
  - Compute massive string spectrum
  - Compute geodesic distance
- Conclusion



Calabi-Yau metrics

Calabi-Yau manifolds



Calabi

Yau

=

Complex

Kähler

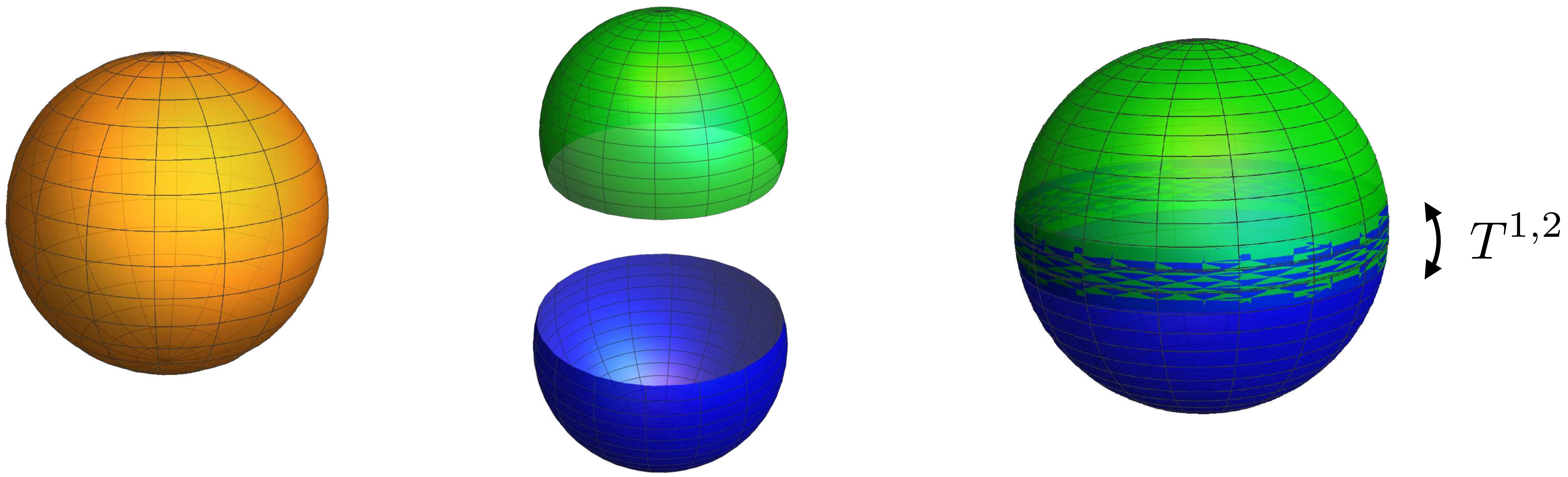
Vanishing first
Chern class

Theorem:
A complex Kähler manifold with vanishing first Chern class admits a Ricci flat Kähler metric (which is unique in its Kähler class)

Problem:
Yau's proof of this theorem is non-constructive, so we don't know what the metric looks like or how to construct it explicitly for CY 3-folds

CY Property 1 - Complex

- In general manifolds cannot be covered by a single patch
- On each patch, one can choose a local description, coordinate system, etc. But one must make sure that the descriptions can be matched on the overlap and everything can be patched to a complex manifold globally (e.g. choice $i=\sqrt{-1}$ vs $i=-\sqrt{-1}$, …)



CY Property 2 - Kähler

- The space must be Kähler
- This means that the metric can be written in terms of derivatives of a real, scalar function called the **Kähler potential** K

$$g_{a\bar{b}} = \frac{\partial}{\partial z^a}\frac{\partial}{\partial \bar{z}^{\bar{b}}} K \quad , \qquad J = \frac{i}{2}\sum_{a<b} g_{a\bar{b}}\, \varepsilon^{a\bar{b}}\, dz^a d\bar{z}^{\bar{b}} \quad , \quad z = x + iy\,, \quad \bar{z} = x - iy$$

- In general, integrating the metric to find the Kähler potential is hard. So one can either start with a Kähler potential and derive the metric, or one has to solve the differential equation $\frac{\partial J}{\partial z^a} = \frac{\partial J}{\partial \bar{z}^{\bar{b}}} = 0$.

CY Property 3 - Ricci-flat

- Calabi-Yau spaces are spaces on which a metric exists that is "flat enough", i.e. their Ricci tensor vanishes

$$
\begin{aligned}
R_{ij} = & -\frac{1}{2}\sum_{a,b=1}^{n}\left(\frac{\partial^2 g_{ij}}{\partial x^a \partial x^b} + \frac{\partial^2 g_{ab}}{\partial x^i \partial x^j} - \frac{\partial^2 g_{ib}}{\partial x^j \partial x^a} - \frac{\partial^2 g_{jb}}{\partial x^i \partial x^a}\right) g^{ab} \\
& + \frac{1}{2}\sum_{a,b,c,d=1}^{n}\left(\frac{1}{2}\frac{\partial g_{ac}}{\partial x^i}\frac{\partial g_{bd}}{\partial x^j} + \frac{\partial g_{ic}}{\partial x^a}\frac{\partial g_{jd}}{\partial x^b} - \frac{\partial g_{ic}}{\partial x^a}\frac{\partial g_{jb}}{\partial x^d}\right) g^{ab} g^{cd} \\
& - \frac{1}{4}\sum_{a,b,c,d=1}^{n}\left(\frac{\partial g_{jc}}{\partial x^i} + \frac{\partial g_{ic}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^c}\right)\left(2\frac{\partial g_{bd}}{\partial x^a} - \frac{\partial g_{ab}}{\partial x^d}\right) g^{ab} g^{cd} \\
= & \; 0
\end{aligned}
$$

- Note that ensuring g is Kähler introduces 2 more derivatives since $g_{a\bar{b}} = \frac{\partial}{\partial z^a}\frac{\partial}{\partial \bar{z}^{\bar{b}}} K$

CY Property 3 - Ricci-flat

- This fourth-order partial differential equation is extremely hard to solve
- We can improve on this. On a CY, one can write down

$$J = \frac{i}{2} \sum_{a<b} g_{a\bar{b}} \; \varepsilon^{a\bar{b}} \; dz^a d\bar{z}^{\bar{b}} \qquad \Rightarrow \qquad J^3 = -\frac{i}{8} \sqrt{\det g} \; dz_1 \; d\bar{z}_1 \; dz_2 \; d\bar{z}_2 \; dz_3 \; d\bar{z}_3$$

$$\Omega = \left(\frac{\partial p}{\partial z_4} \right)^{-1} \; dz_1 \; dz_2 \; dz_3 \quad \Rightarrow \quad |\Omega|^2 = \left| \frac{\partial p}{\partial z_4} \right|^{-2} \; dz_1 \; dz_2 \; dz_3 \; d\bar{z}_1 \; d\bar{z}_2 \; d\bar{z}_3$$

- Since the volume form is unique (up to a constant): $J^3 = \kappa |\Omega|^2$
- So instead of minimizing the Ricci tensor, we can minimize this surrogate loss, which is equivalent to Ricci flatness by the powerful theorem of Calabi-Yau, which ensures existence and uniqueness.

CY metric ansatze

- The condition $J^3 = \kappa|\Omega|^2$ can be turned into a (Monge-Ampere) PDE
- As it turns out, we can ensure the complex and Kähler property and keep the volume moduli fixed if we write

$$g_{\mathrm{CY}} = g_{\mathrm{reference}} + \partial\bar{\partial}\Phi$$

  and approximate the (scalar) function $\Phi = \Phi(\mathrm{position}, \mathrm{shape})$ with a NN
- …but actually we are in a peculiar situation here:
  We have a PDE in Φ, but we not care about Φ - rather, we care about $\partial\bar{\partial}\Phi$

CY metric ansatze

- … so we can learn $g_{\text{correction}} = \partial\bar{\partial}\Phi$ instead, turning the surrogate loss into an algebraic equation
- However, we still need to impose that the resulting metric is Kähler, which requires taking one derivative

Ricci flat (4 derivatives) ⟶ MA equation (2 derivatives) ⟶ algebraic equation (1 derivative)

- Other possibilities (can depart from Kähler and fixed volume):
  - $g_{\text{CY}} = g_{\text{NN}}$ (works the least well)
  - $g_{\text{CY}} = g_{\text{reference}} + g_{\text{NN}}$ (works better)
  - $g_{\text{CY}} = g_{\text{reference}}(\mathbb{1} + g_{\text{NN}})$ (works best; as well as the $\partial\bar{\partial}\Phi$ approach)

Reference metric

- So how do we find the reference metric?
- For $\mathbb{P}^N$, there is the Fubini-Study metric:
  - In general, the Kähler metric can be written in terms of the sections of the Kähler cone generators
  - For projective spaces, these are just the homogeneous coordinates z_i
  - The FS metric in these homogeneous coordinates is then

$$g_{\text{FS}} = \partial\bar{\partial} \ln \sigma \,, \quad \sigma = \sum_i |z_i|^2$$

  - We can use projective scalings to set one $z_i=1$ and pull the metric back to the CY via the defining equations

Reference metric

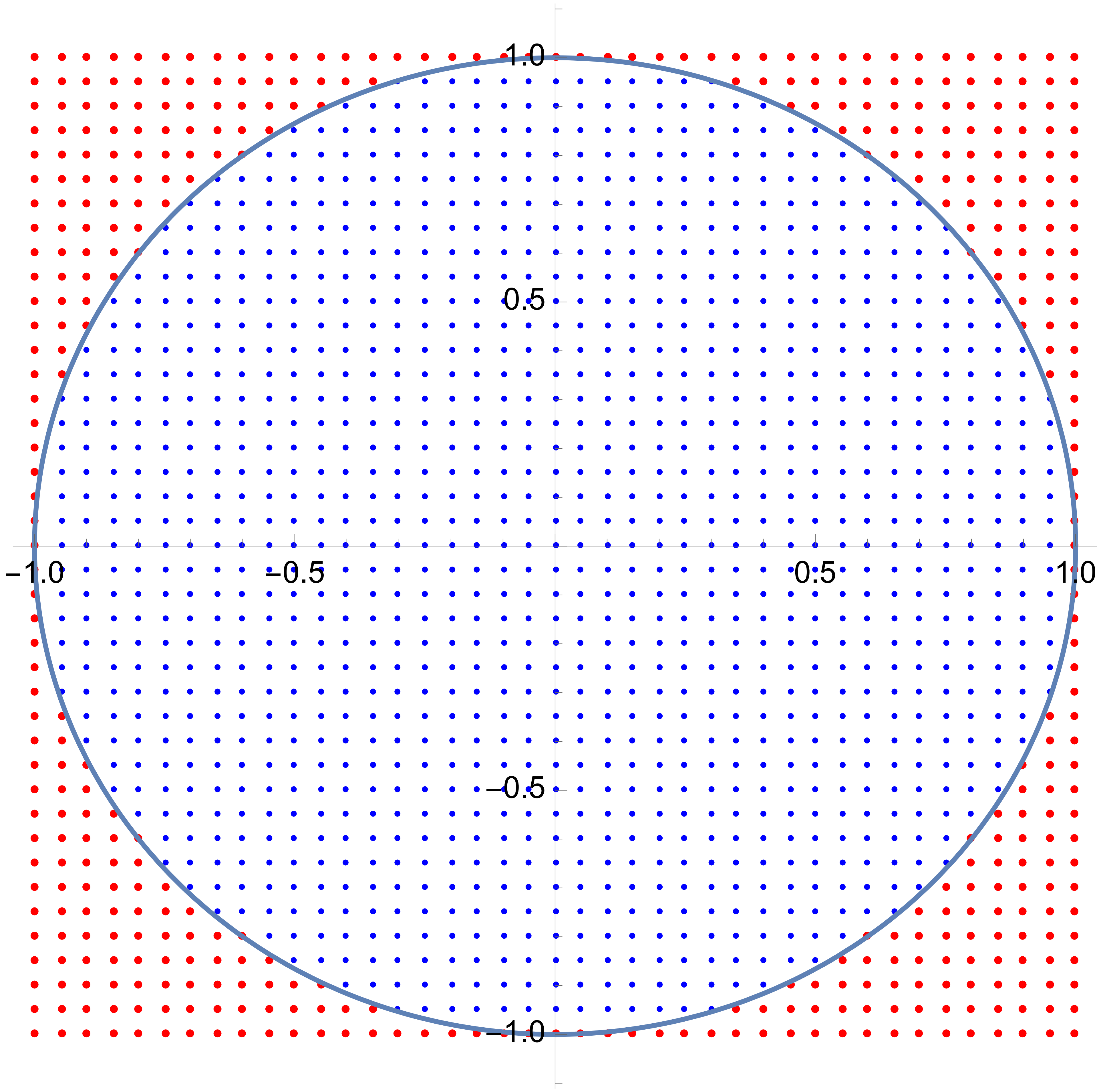
- For toric ambient spaces, there is a similar construction
  - Find the sections s_i of the Kähler cone generators of the toric variety
  - These will be expressions in the toric coordinates x_a
  - The analog of the FS metric now looks exactly like before, just in terms of the s_i rather than the z_i , i.e. we get an FS metric “in projective section space” $\mathbb{P}^{h^0(J)}$

$$g_{\text{FS}} = \partial\bar{\partial}\ln\sigma\,,\quad \sigma = \sum_i |s_i|^2$$

  - It can be shown that the dependence of s_i on x_i is such that in each patch always one $s_i{=}1$. T Moreover, not all $s_i{=}0$ simultaneously.

Numerical Integration

- To the area of a circular lake, you can throw equally spaced stones within a square of known area and count the fraction of times you hit the lake
- In the example, 41x41=1681 throws
- Out of these, 1245 hit the lake
- Hence the area is
  2 x 2 x 1245/1681=2.96



Numerical Integration

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- In the example, 41x41=1681 throws
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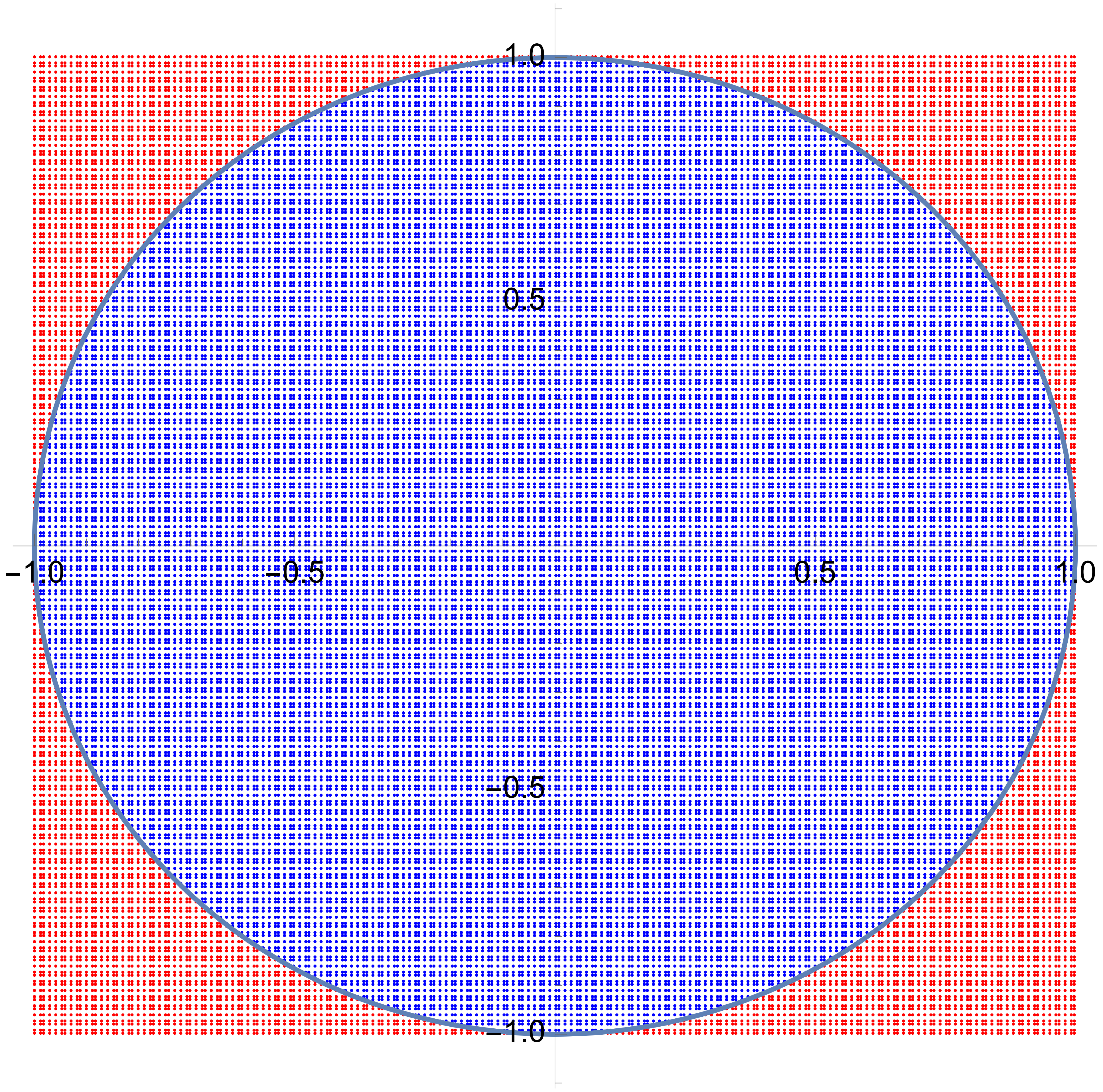
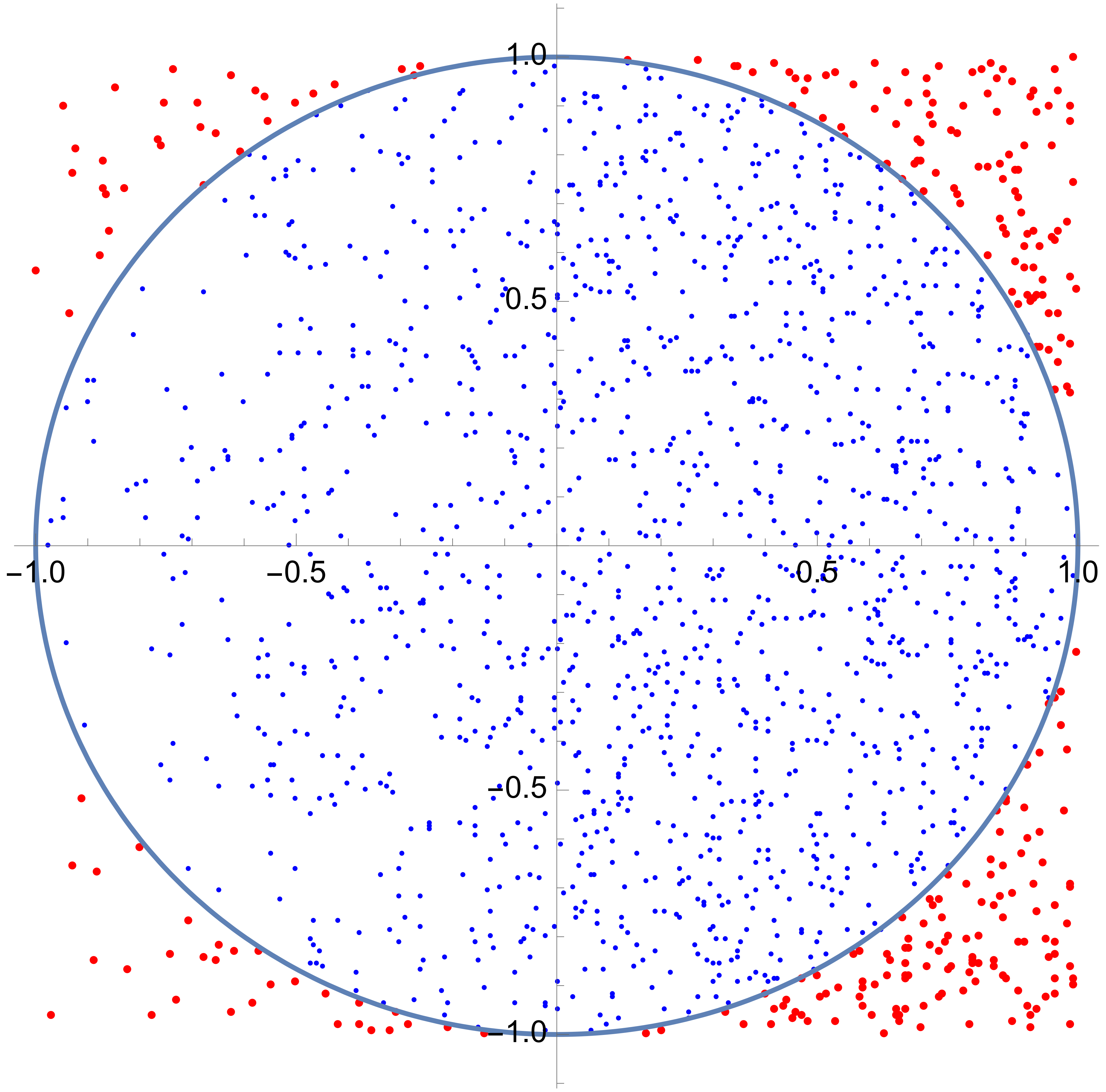
- Hence the area is
  2 x 2 x 1245/1681=2.96
- For more throws, we approach π (for 201 x 201 get 3.11)



Numerical Integration

- However, if I actually throw stones into a lake, it looks more like this
- There is a bias towards the bottom right
- If you know the bias, you can correct of it by weighting points on the left stronger than the ones on the right



Points on the CY

- In order to specify the metric on the CY numerically, we need to specify it at **points on the CY**
- In principle, we can just solve the CY equation numerically and generate points this way (this can be done using homotopy continuation)
- However, doing it this way we do not know how the points are distributed on the CY

> Theorem: **[Shiffman, Zelditch `99]**
>
> The zeros of random sections in $\mathbb{P}^N$ are distributed according to the FS metric

Points on the CY

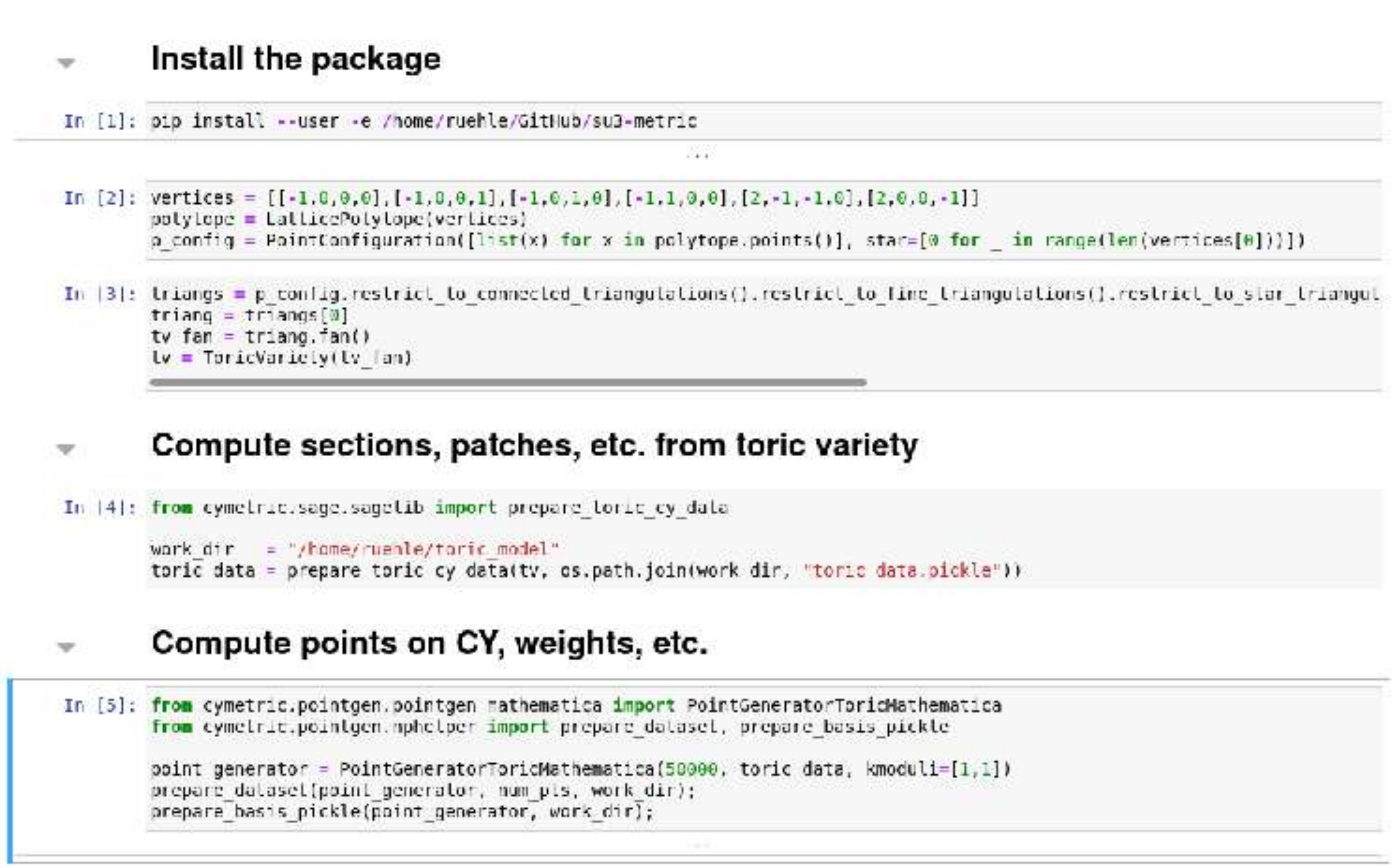
- So to find points on the CY, we can follow this procedure
- For CICYs in products of projective ambient spaces, you use effectively the Kodaira embedding of $\mathbb{P}^N$ into $\mathbb{P}^N$ via the sections of the Kähler cone generator (which are the homogeneous coordinates themselves)
  $[z_0 : z_1 : \ldots : z_N] \xrightarrow{\phi} [z_0 : z_1 : \ldots : z_N]$
- Construct random sections in the embedded $\mathbb{P}^N$, intersect them to find a collection of lines, pull back to (intersect with) CY hypersurface
- Similarly for toric varieties, use map from toric coordinates into $\mathbb{P}^{h^0(J)}$ via the sections of the Kähler cone generators
  $[x_0 : x_1 : \ldots : x_k] \xrightarrow{\phi} [s_0 : s_1 : \ldots : s_{h^{1,1}(J)}]$

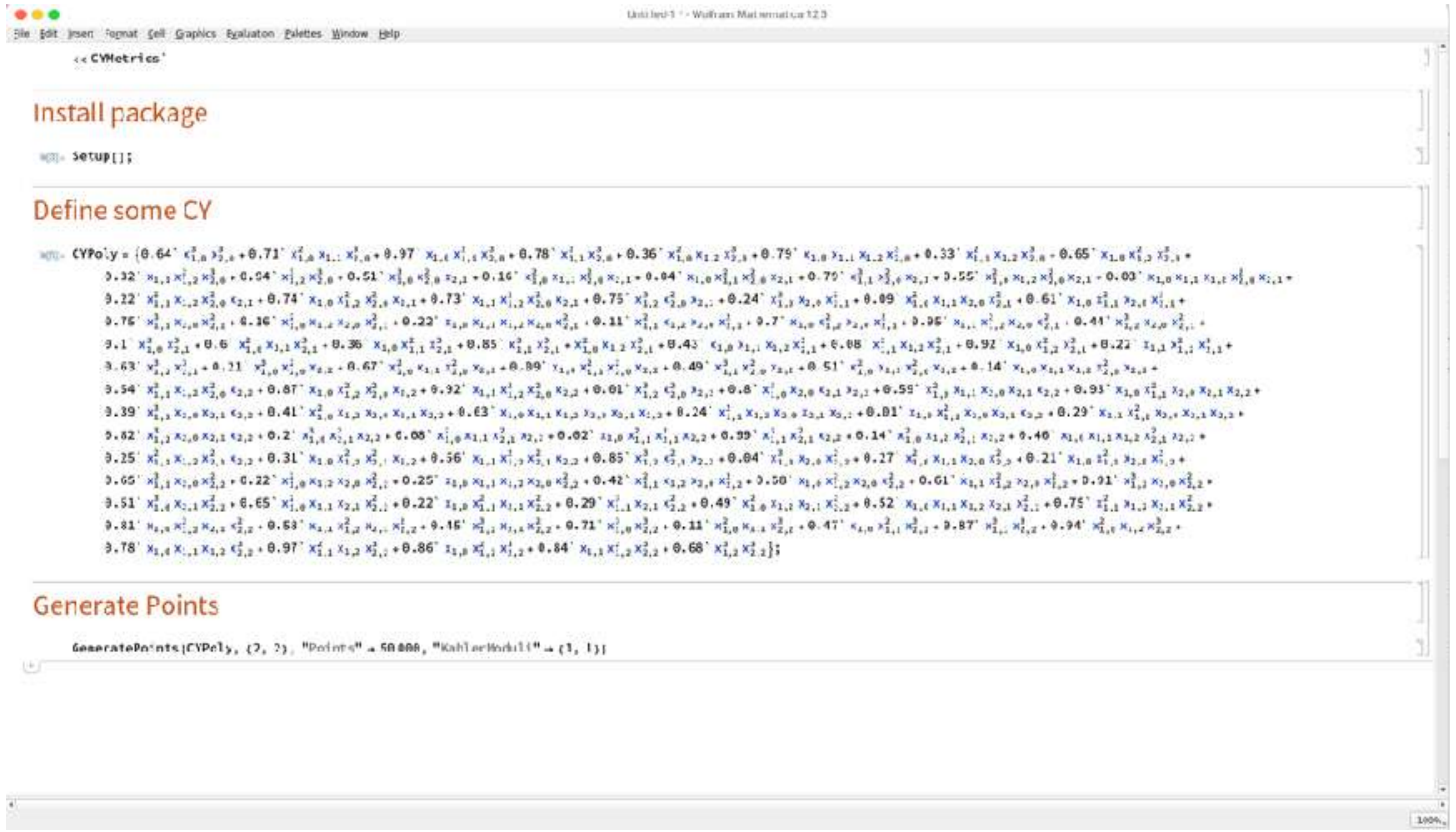
Steps to get point measure

- So to summarize you need to
  - Find the Kähler cone generators
  - Construct a map into the projectivization $\mathbb{P}\Gamma(\mathcal{A}, J_i)$
  - Find the common zeros of random iid Gaussian sections
  - Intersect these with the CY hypersurface to get points on the CY
  - Construct reference metric on the ambient space from the FS metric on $\mathbb{P}\Gamma(\mathcal{A}, J_i)$
  - Pull the metric back to the CY via the hypersurface equation
  - Construct volume measure on the CY from $\mathrm{vol}_{\mathrm{CY}} = \Omega \wedge \bar{\Omega}$
  - Compute weights for the individual points w.r.t. known volume measure

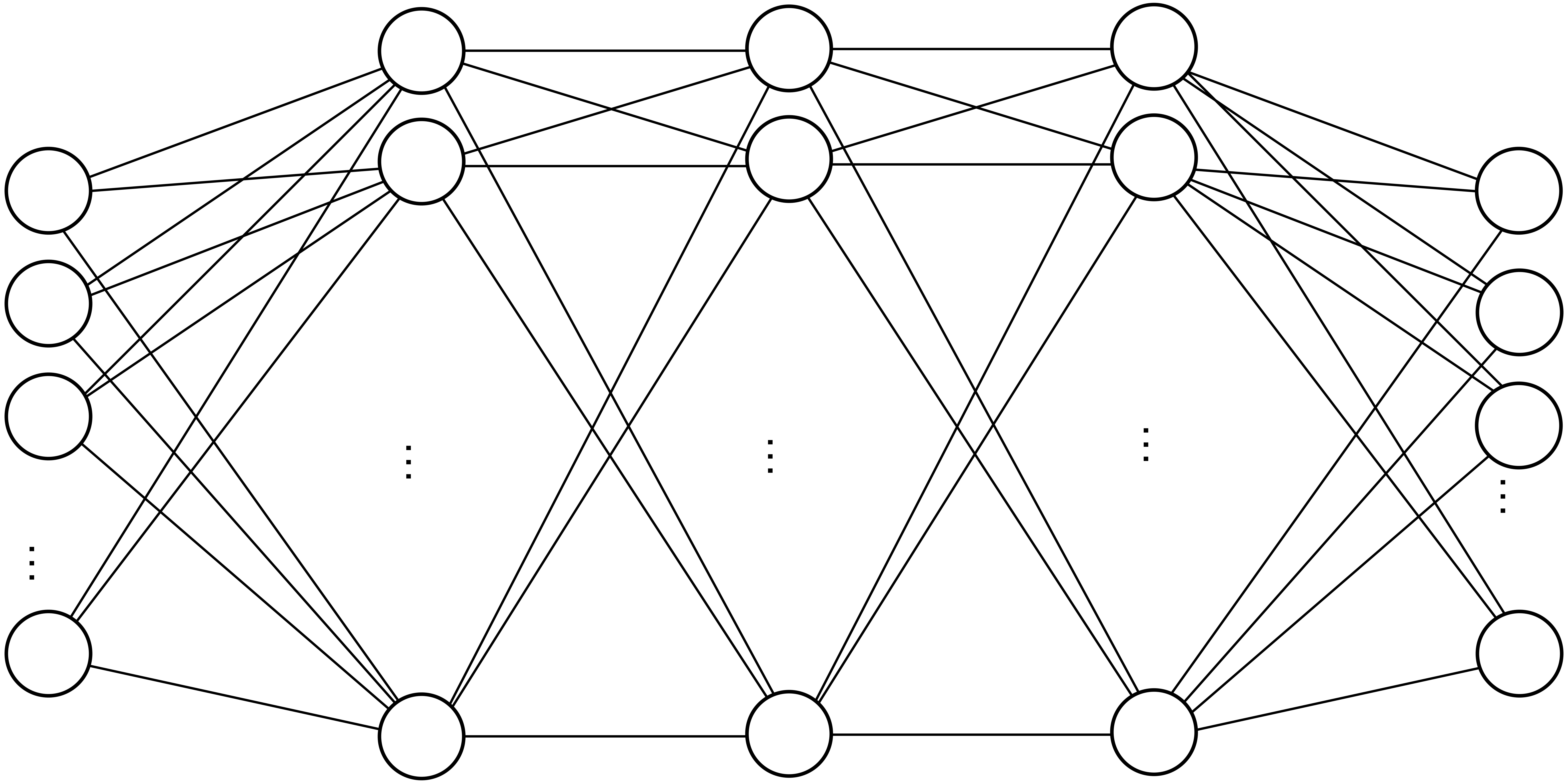
Library to do that

- Luckily this can be automated
- We provide a python code that integrates into SAGE or Mathematica to do that





[https://github.com/pythoncymetric/cymetric]



Calabi-Yau metrics from Neural Networks

Calabi-Yau metrics

- The **metric** is some **unknown function**

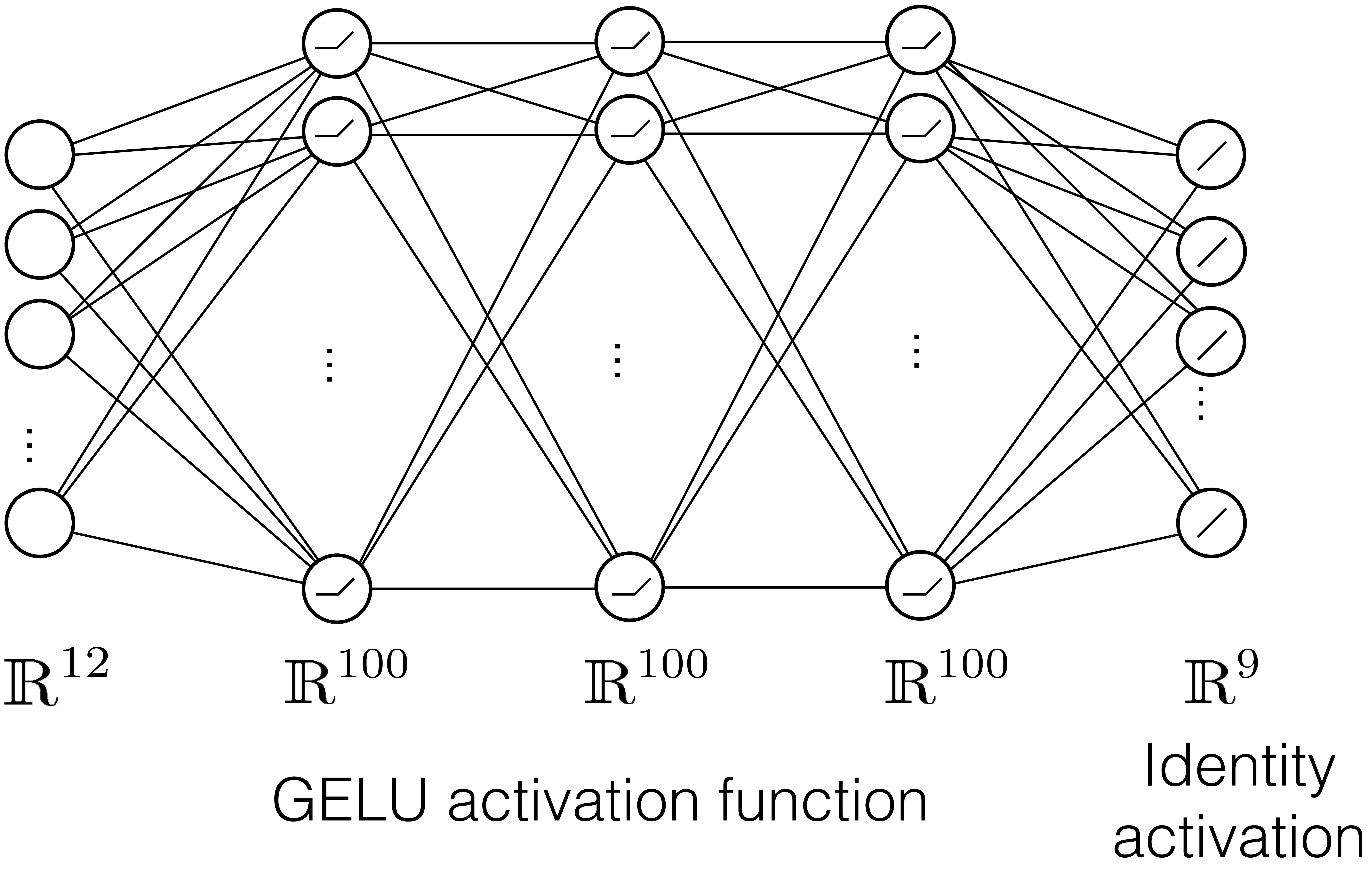
g^{ab} :	point in moduli space	$\times$	point on CY	$\rightarrow$	complex hermitian 3x3 metric

- So we are looking for a **map** (for the example of the quintic)
  - **point** in **moduli space**: 1 complex number (2 real DOFs)
  - **point** on **CY**: 5 complex numbers (10 real DOFs)
  - **complex** hermitian 3x3 **matrix**: 3 real + 3 complex (9 real DOFs)

Calabi-Yau metrics

- Simplest possibility:
  - Take a 12 x 9 matrix with 12 x 9 = 108 parameters
  - Choose the parameters to produce a Ricci-flat Kahler metric
- …but such linear maps are too simple
  - Take a 12 x 100 matrix
  - Make it non-linear by taking the tanh of each of the 100 nodes
  - Take a 100 x 100 matrix, and take again the tanh, …
  - Take a 100 x 9 matrix
  - Find the parameters in all these matrices such that the result is a Ricci-flat Kahler metric
    [Ashmore et al `19;Anderson, Gray, Gerdes, Krippendorf, Raghuram, FR `20; Douglas et al `20; Jejjala et al `20; Larfors, Lukas, FR, Schneider `21]
- What we have just described is a feed-forward NN! It is a universal approximator
  [Cybenko `89; …; Kidger,Lyons `19]

NNs for CY metrics



Library to do that

- This can also be automated
- Again, call directly from SAGE or Mathematica

Define the NN

```
In [ ]: import tensorflow as tf
tfk = tf.keras

nHiddens = [64, 64, 64]
acts     = ['gelu', 'gelu', 'gelu']
model    = tf.keras.Sequential()
model.add(tfk.Input(shape=(int(n_in))))
for nHidden, act in zip(nHiddens, acts):
    model.add(
        tfk.layers.Dense(
            nHidden,
            activation=act,
        )
    )
model.add(tfk.layers.Dense(n_out))
```

Choose the CY metric ansatz

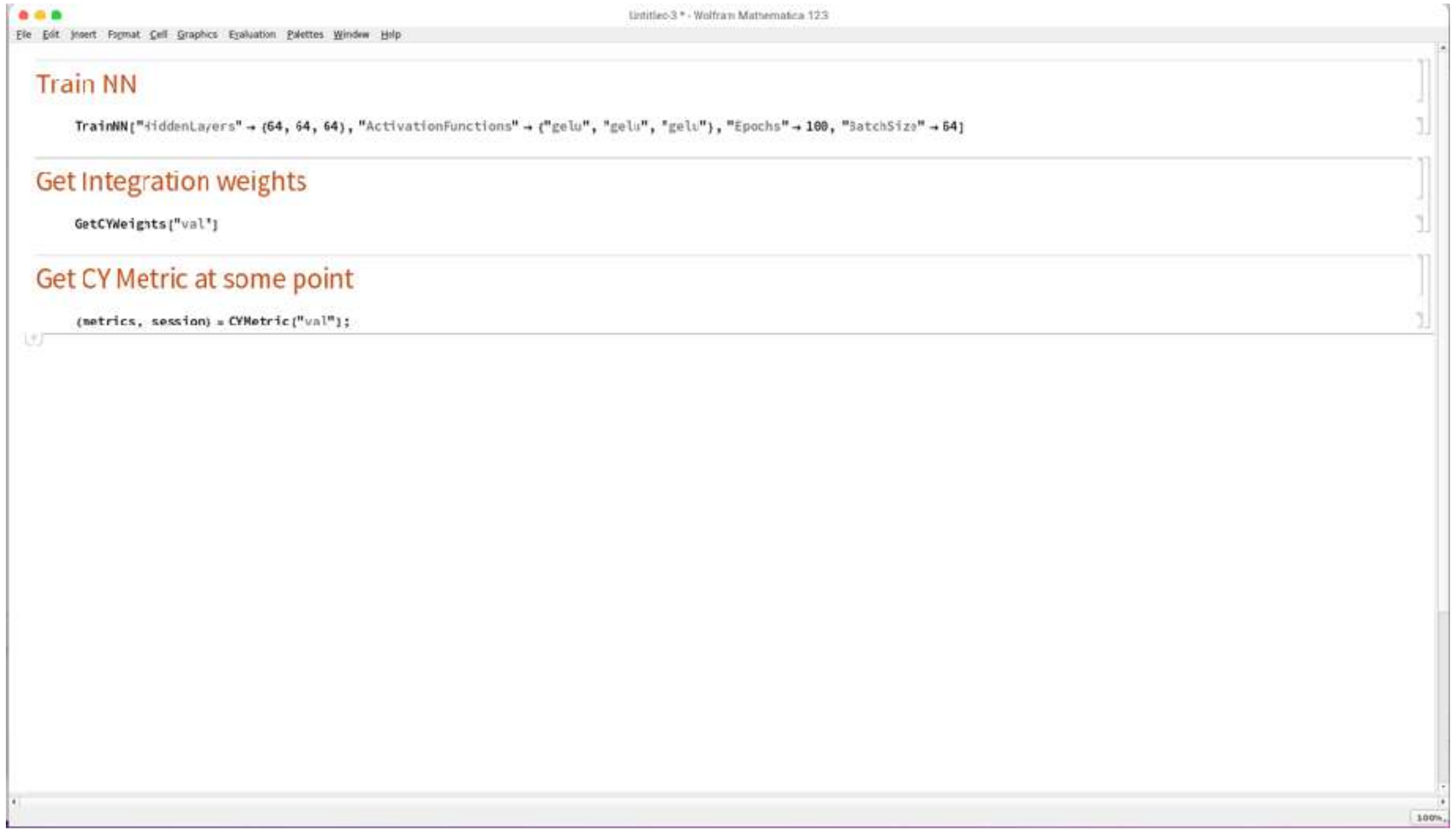
```
In [ ]: from cymetric.models.tfmodels import PhiFSModelToric
from cymetric.models.tfhelper import prepare_tf_basis
BASIS = prepare_tf_basis(np.load('basis.pickle', allow_pickle=True))

fs_model = PhiFSModelToric(model, BASIS, alpha=alpha, kappa=kappa, toric_data=toric_data)
fs_model.compile(custom_metrics=cmetrics, optimizer=tfk.optimizers.Adam())
```

train the NN

```
In [ ]: fs_model.fit(data['X_train'], data['y_train'], epochs=100, batch_size=64)
```

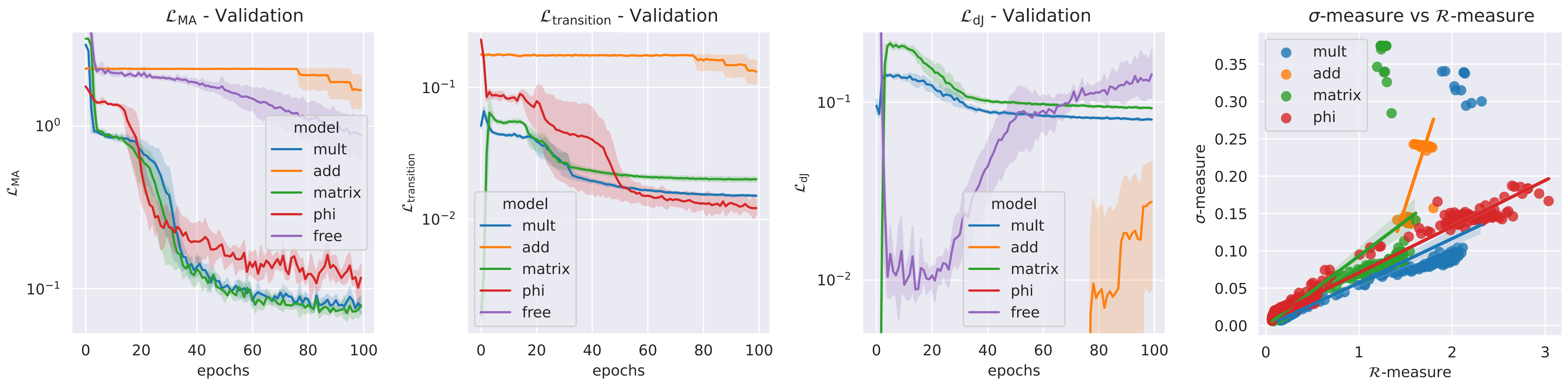
CY metric at some points

```
In [ ]: cy_metrics = fs_model(data['X_val'])
```



[https://github.com/pythoncymetric/cymetric]

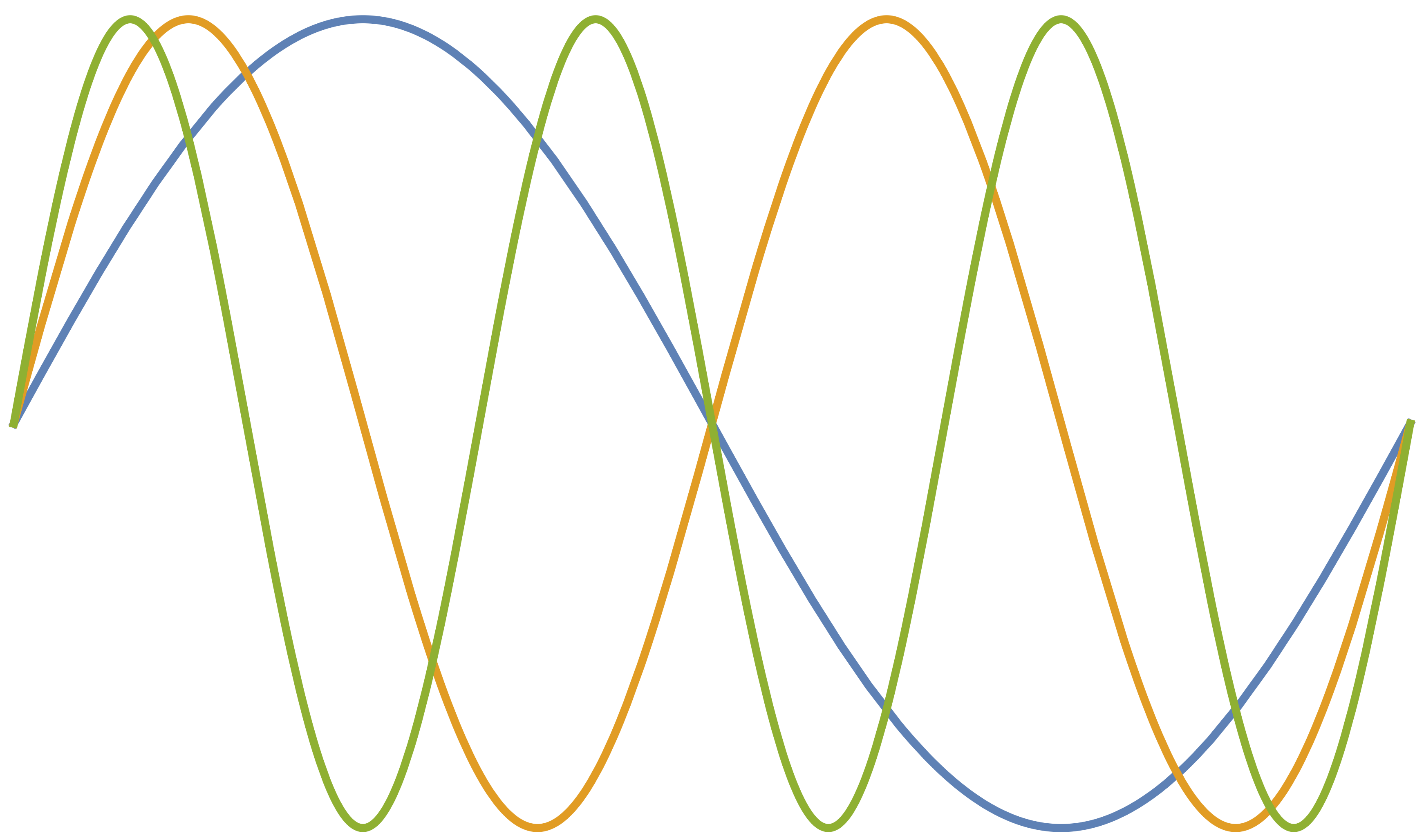
CY metric results (for quintic)



$$\mathcal{L} = \alpha_1 \mathcal{L}_{\mathrm{MA}} + \alpha_2 \mathcal{L}_{\mathrm{dJ}} + \alpha_3 \mathcal{L}_{\mathrm{transition}} + \alpha_4 \mathcal{L}_{\mathrm{Ricci}} + \alpha_5 \mathcal{L}_{\mathrm{vol}}$$

$$\sigma = \int_X \left| 1 - \frac{J^3}{|\Omega|^2} \right|$$

$$\mathcal{R} = \int_X R^{ab} R_{ab}$$



Massive strings spectrum

Compute massive string spectrum

1. **Compute** the **CY metric** at **different** points in **moduli** space
2. **Choose** a (finite subset) of an (infinite) **function basis** $\{\alpha_A\}$ on X

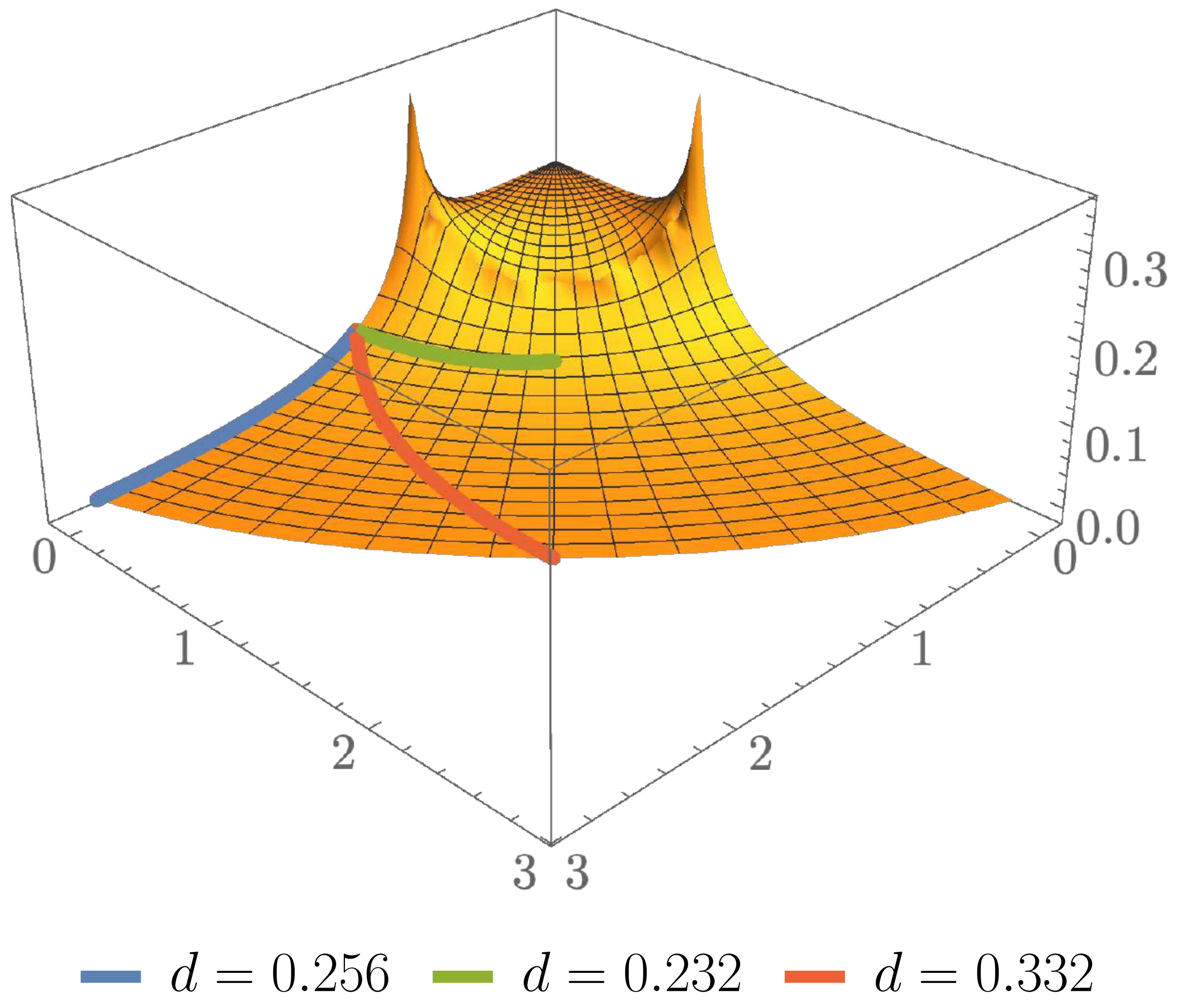
$$\{\alpha_A\} = \frac{\left\{s_\alpha^{(k_\phi)}\ \bar{s}_{\bar\beta}^{(k_\phi)}\right\}}{\left(|z_0|^2 + \ldots + |z_4|^2\right)^{k_\phi}}$$

w/ $s_\alpha^{(k_\phi)}$ degree k_ϕ monomials in z 's

3. Expand $|\varphi\rangle = \varphi^A|\alpha_A\rangle$ and **compute** $\Delta_{AB}\varphi^B = \lambda O_{AB}\varphi^B$ w/ **matrix elms**

$$\Delta_{AB} = \langle\alpha_A, \Delta\alpha_B\rangle, \quad O_{AB} = \langle\alpha_A, \alpha_B\rangle$$ (evaluated using MC integration)

- We compute at $k_\phi = 3$ $\Rightarrow$ can compute the first 1,225 eigenvalues
- For higher eigenvalues, lose accuracy due to finite truncation of basis



Geodesic Distance in Moduli Space

Geodesics

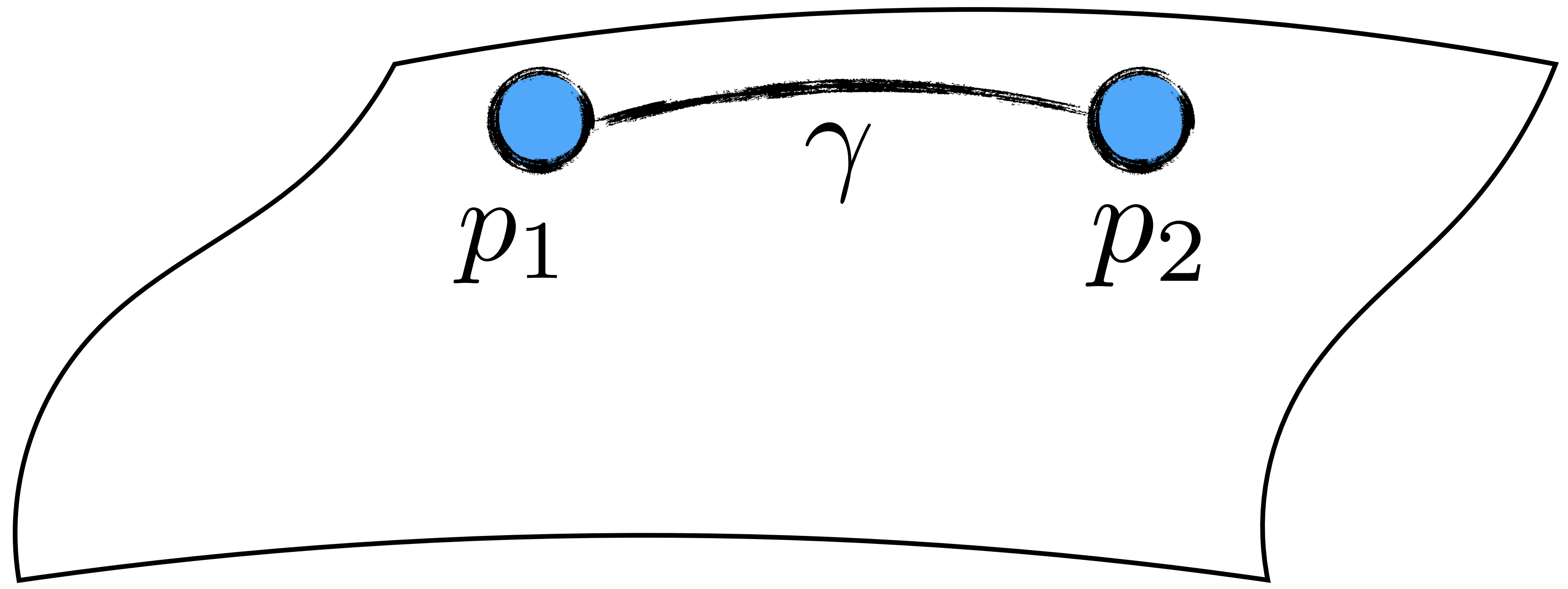
- **Geodesics** give the **shortest distance** between 2 points on a manifold
- They are solution to the **geodesics equation**

$$\ddot{\gamma}(\tau) + \Gamma^c_{ab}\dot{\gamma}^a(\tau)\dot{\gamma}^b(\tau) = 0$$

- The **geodesic distance** is then

$$d(p_1, p_2) = \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \sqrt{g_{a\bar{b}}(\gamma(\tau))\dot{\gamma}^a(\tau)\dot{\gamma}^b(\tau)}$$

(evaluate using numerical integration)



- Usually, we specify boundary values rather than initial values
  $\Rightarrow$ **solve** equations **numerically** using RK/AB **shooting** methods

Complex Structure moduli space metric

- The **CS moduli space metric** is also a **Kähler** metric
- It can be obtained from the **Kähler potential** $K_{CS} = -\ln\left(i\int_X \Omega_\psi \wedge \overline{\Omega_\psi}\right)$

  where Ω is the holomorphic (3,0) form of the CY
- To express this, **choose** a **basis** of **3-cycles**

$$A^I \cap B_J = \int_X \alpha_J \wedge \beta^I = \int_{A^I} \alpha_J = \int_{B_J} \beta^I = \delta^I_J$$

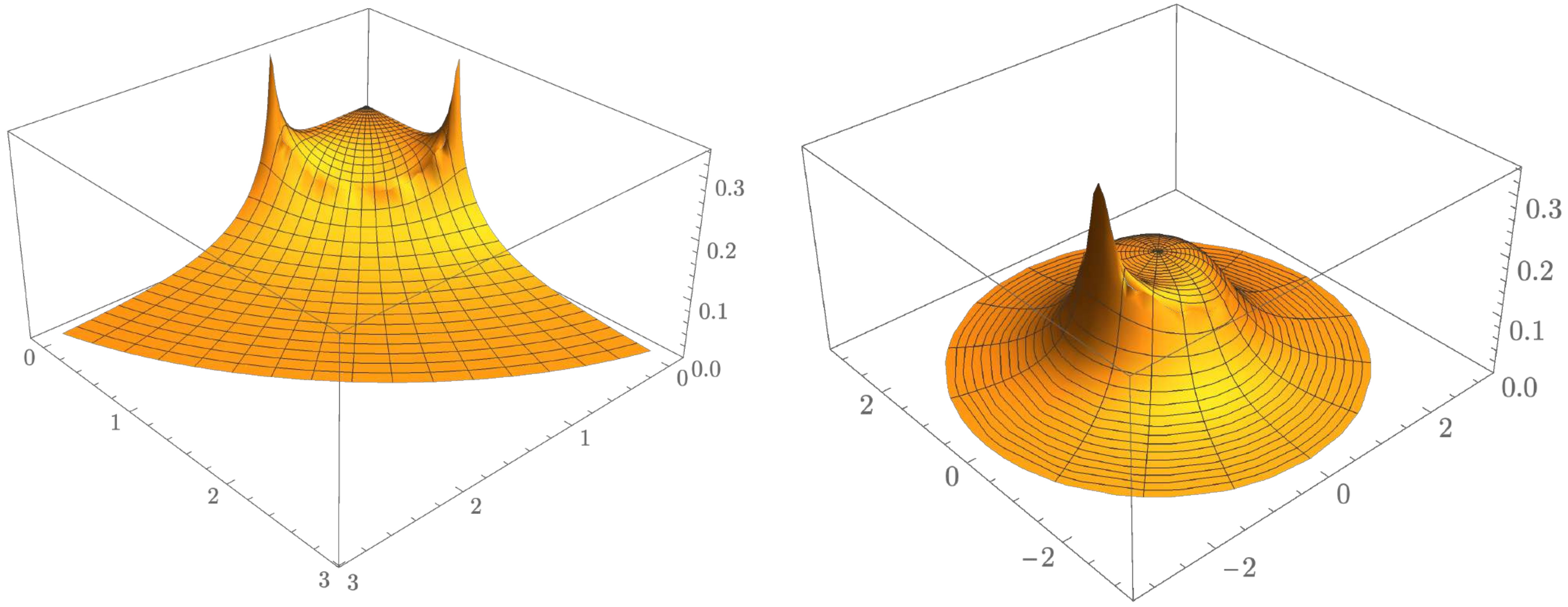
- Then, $\Omega \wedge \bar{\Omega} = z^I \bar{\mathcal{G}}_I - \bar{z}^I \mathcal{G}_I$ with $\Pi = \begin{pmatrix} \mathcal{G}_I \\ z^I \end{pmatrix} = \begin{pmatrix} \int_{B_I} \Omega \\ \int_{A^I} \Omega \end{pmatrix}$
- The so-called **periods** Π_i are **solutions** to a hypergeometric Picard Fuchs **differential equation** [Candelas,De La Ossa,Green,Parkes `91]

Complex structure moduli space metric

- In our case, we have a **single CS parameter** $\psi : z_0^5 + z_1^5 + z_2^5 + z_3^5 + z_4^5 - 5\psi z_0 z_1 z_2 z_3 z_4 = 0$
- Note:
  - The hypergeometric functions depend on ψ^5 rather than ψ ; the trafo $\psi \to e^{2\pi i/5}\psi$ can be undone by $z_0 \to e^{2\pi i/5} z_0$
  - The CY is singular at $\psi = 1$.
    This singularity is at finite distance (conifold)
  - The CY is also singular at $\psi \to \infty$.
    This is at infinite distance.
  - We can use the mirror map

$$t = \frac{z^1}{z^0} \sim -\frac{5}{2\pi i}\ln(5\psi) \quad \Rightarrow \quad g_{t\bar{t}} \sim \frac{3}{4\,\mathrm{Im}(t)^2}$$

    to turn CS into Kahler moduli statements



Complex structure moduli space metric

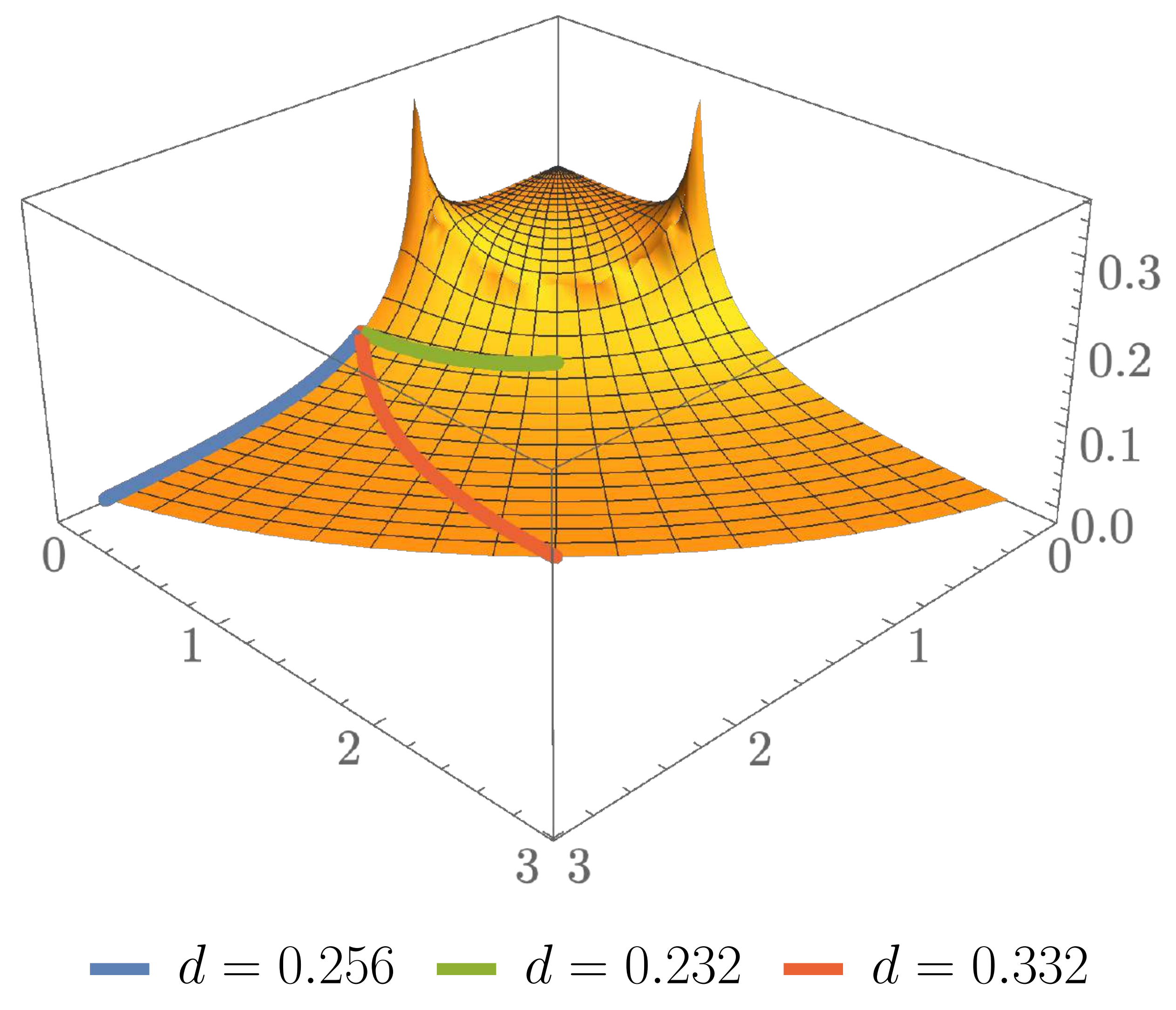
- We **compare three approaches** to computing geodesics
  - Use the **exact analytic** (hypergeometric) function
  - Use the **large volume approximation**
  - Compute the Weil-Petersson **metric numerically** following: [Keller,Lukic `91]
    - Vary the hol. 3-form $\Omega_{\psi+\varepsilon} = \Omega_\psi + \varepsilon^i \partial_{\psi_i} \Omega_\psi + \mathcal{O}(\varepsilon^2)$
    - Construct a basis of (non-holomorphic) 3-forms $\partial_{\psi_i} \Omega_\psi \in H^{3,0}(X_\psi) \oplus H^{2,1}(X_\psi)$
    - Construct a basis of vectors $\varepsilon^i \in T_\psi \mathcal{M}_{CS}$
    - Then the inner product w.r.t. the moduli space metric is (integration over X is again done using MC)

$$\langle \varepsilon_1, \varepsilon_2 \rangle = -\frac{\varepsilon_1^i \overline{\varepsilon_2^j} \int_X \partial_{\psi_i} \Omega_\psi \overline{\partial_{\psi_i} \Omega_\psi}}{\int_X \Omega_\psi \wedge \bar{\Omega}_\psi} + \frac{\varepsilon_1^a \overline{\varepsilon_2^j} \int_X \partial_{\psi_i} \Omega_\psi \wedge \overline{\Omega_\psi} \int_X \Omega_\psi \wedge \overline{\partial_{\psi_i} \Omega_\psi}}{(\int_X \Omega_\psi \wedge \bar{\Omega}_\psi)^2}$$

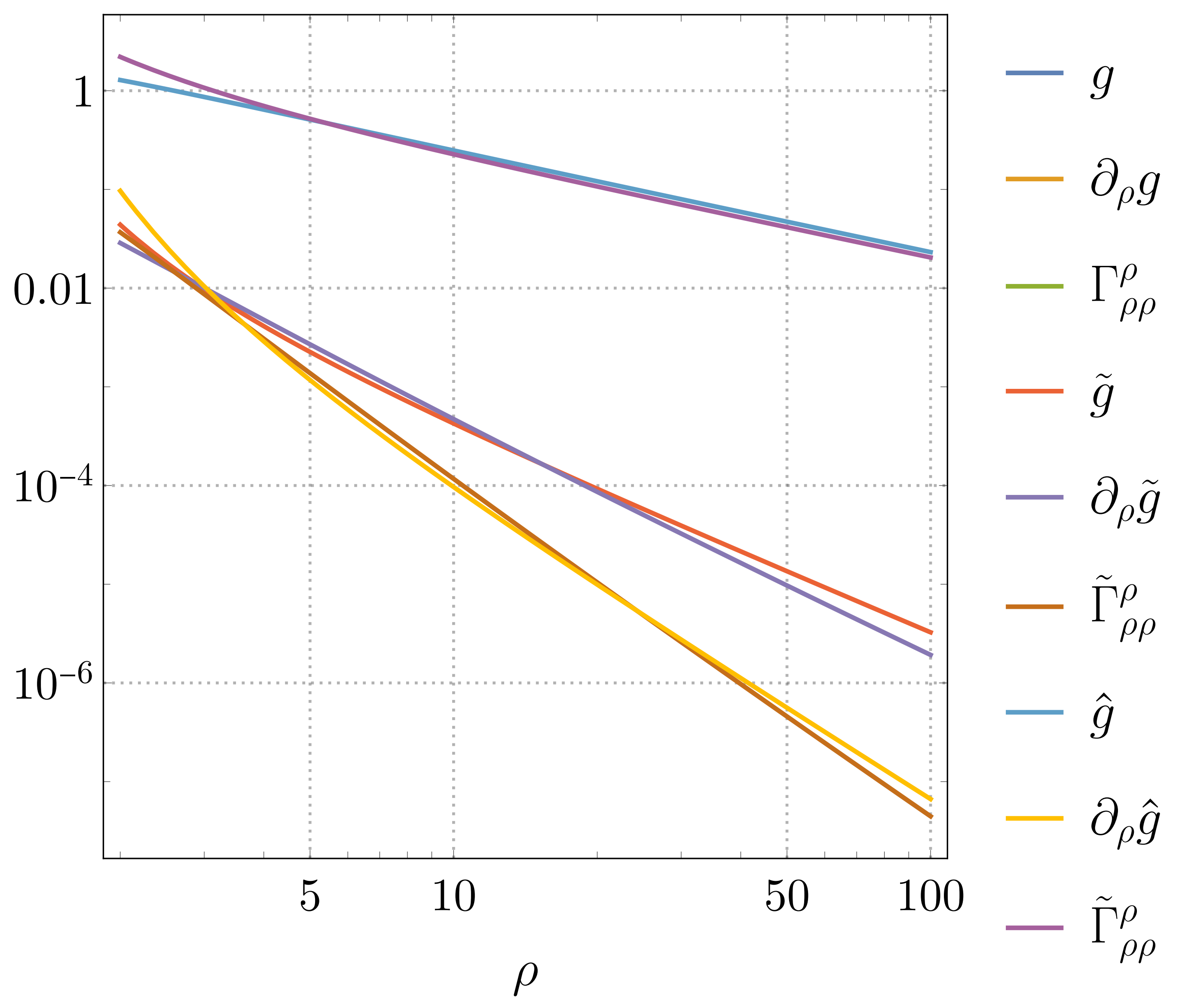
Reconstructing the metric

- The numeric computation allows us to get the moduli space metric at fixed points ψ
- We **compute** the value of the **metric** in a box around the start and end point of the geodesic trajectory **and interpolate** the full metric
  - **Fitting polynomials** / splines
  - **Use** a **NN** similar to the one used for the CY metric
  - **Use symbolic regression** (FindFormula or AIFeynman) [Mathematica; Udrescu et.al. `20]
  - **Use domain knowledge** (cheating) of underlying Picard Fuchs equation to fit a function $g(\psi) = \frac{a}{\psi^2} + \frac{b}{(\psi \ln \psi)^2}$ with L1 norm and Tikhonov regularization

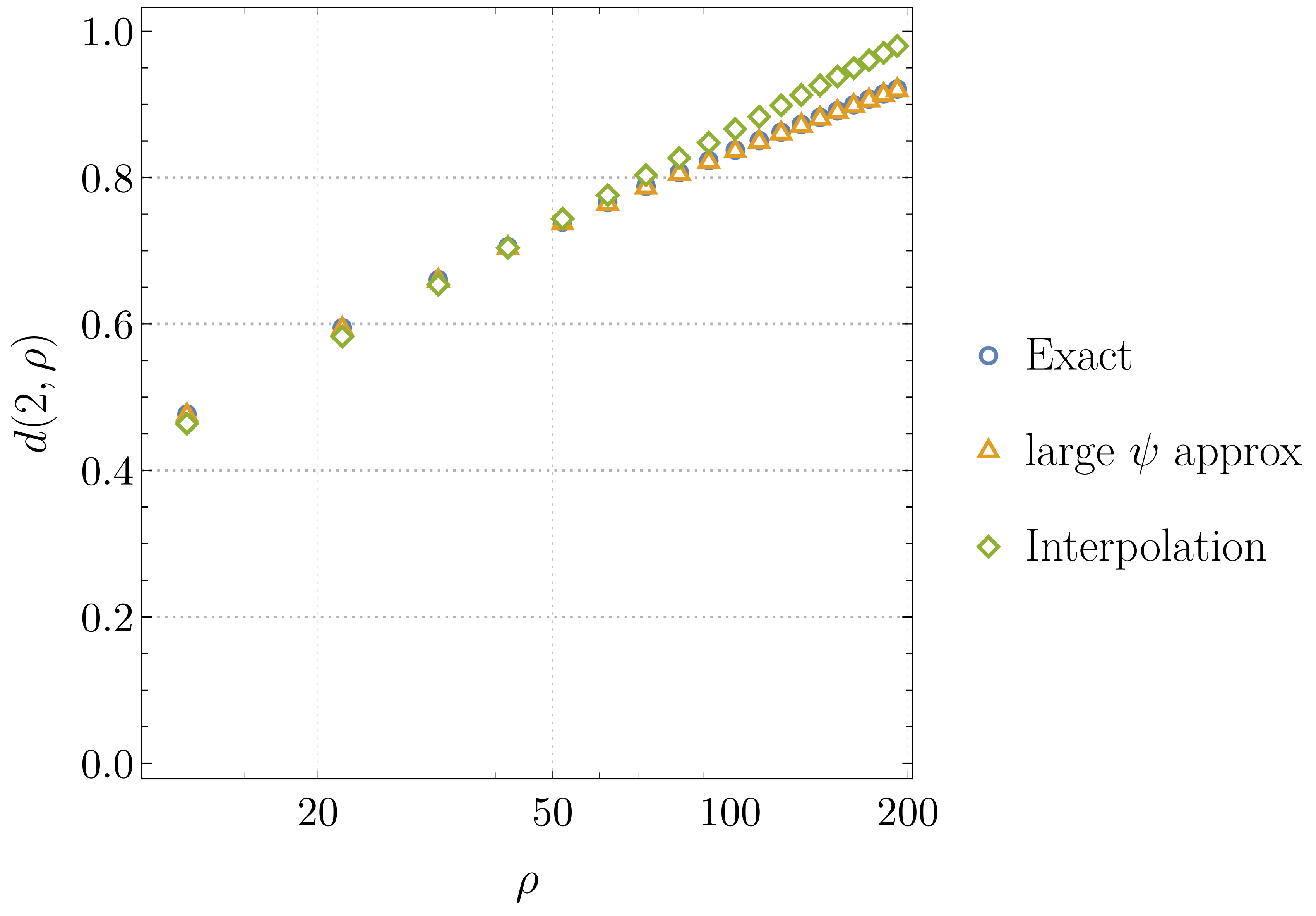
Geodesics



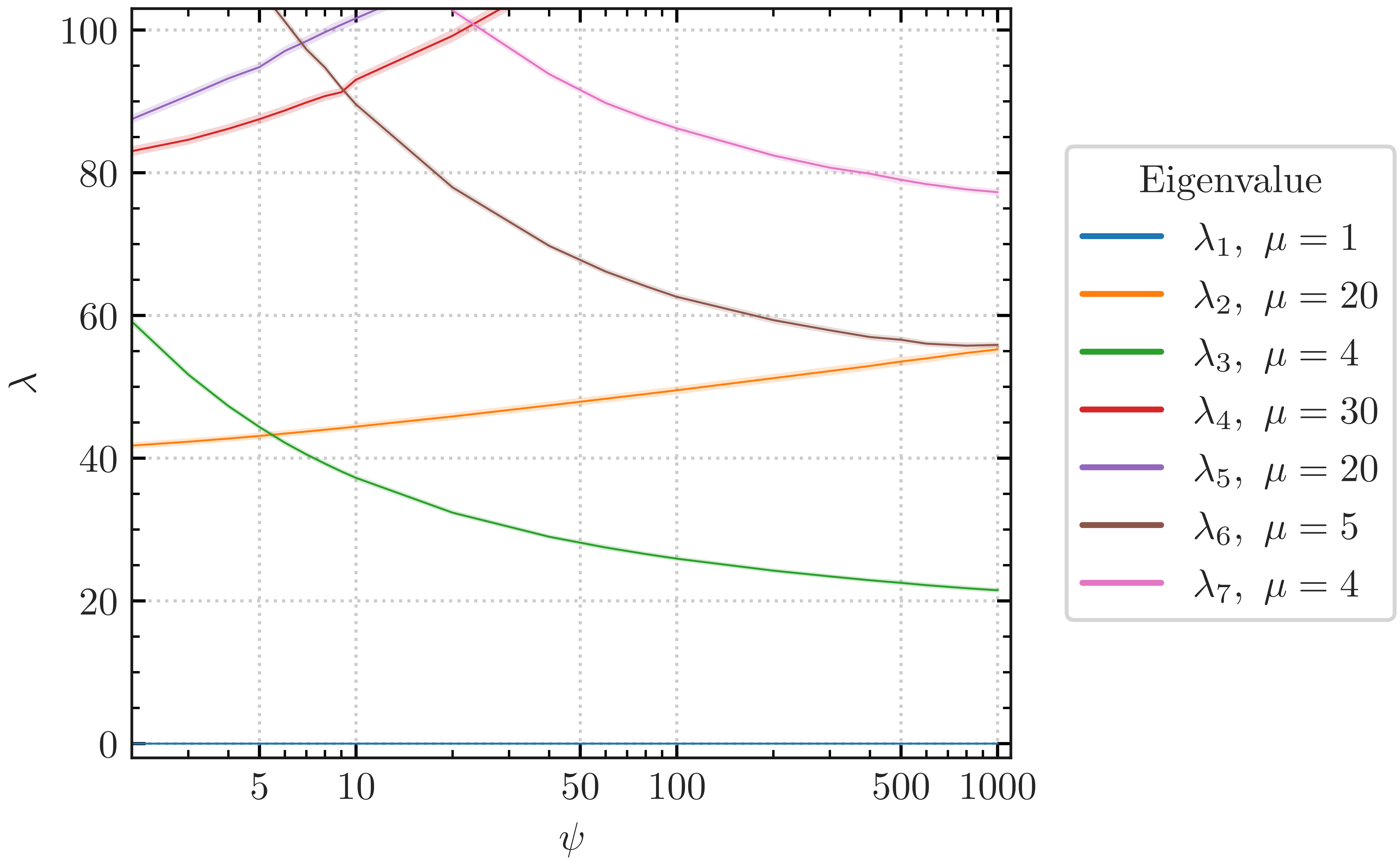
0.3
0.2
0.1
0.0
0
1
2
3
$d = 0.256$
$d = 0.232$
$d = 0.332$



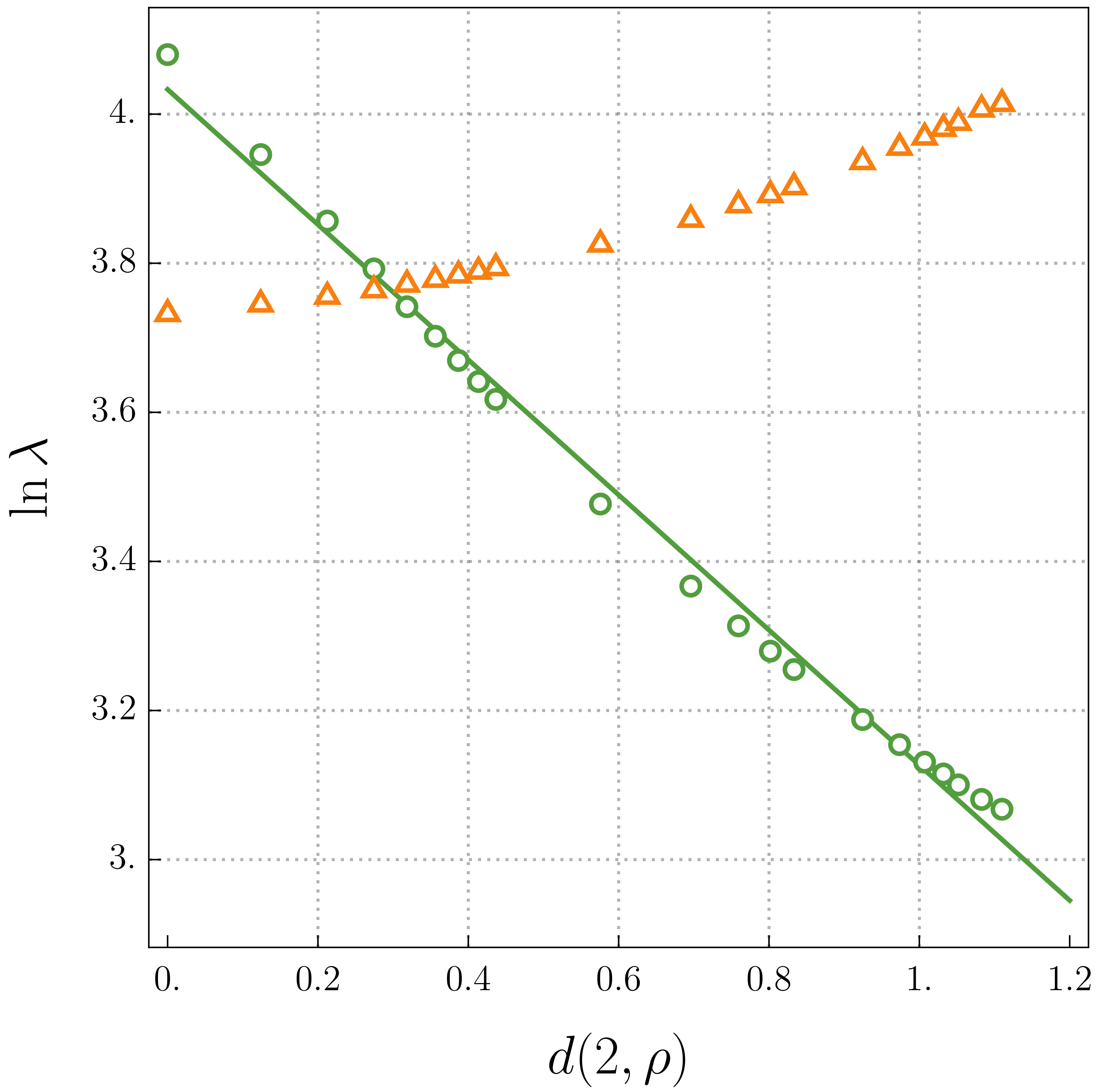
1
0.01
10^{-4}
10^{-6}
5
10
50
100
ρ
g
$\partial_\rho g$
$\Gamma^\rho_{\rho\rho}$
$\tilde{g}$
$\partial_\rho \tilde{g}$
$\tilde{\Gamma}^\rho_{\rho\rho}$
$\hat{g}$
$\partial_\rho \hat{g}$
$\hat{\Gamma}^\rho_{\rho\rho}$



1.0
0.8
0.6
0.4
0.2
0.0
$d(2, \rho)$
20
50
100
200
ρ
Exact
large ψ approx
Interpolation



100
80
60
40
20
0
5
10
50
100
500
1000
ψ
λ
Eigenvalue
λ1, μ = 1
λ2, μ = 20
λ3, μ = 4
λ4, μ = 30
λ5, μ = 20
λ6, μ = 5
λ7, μ = 4

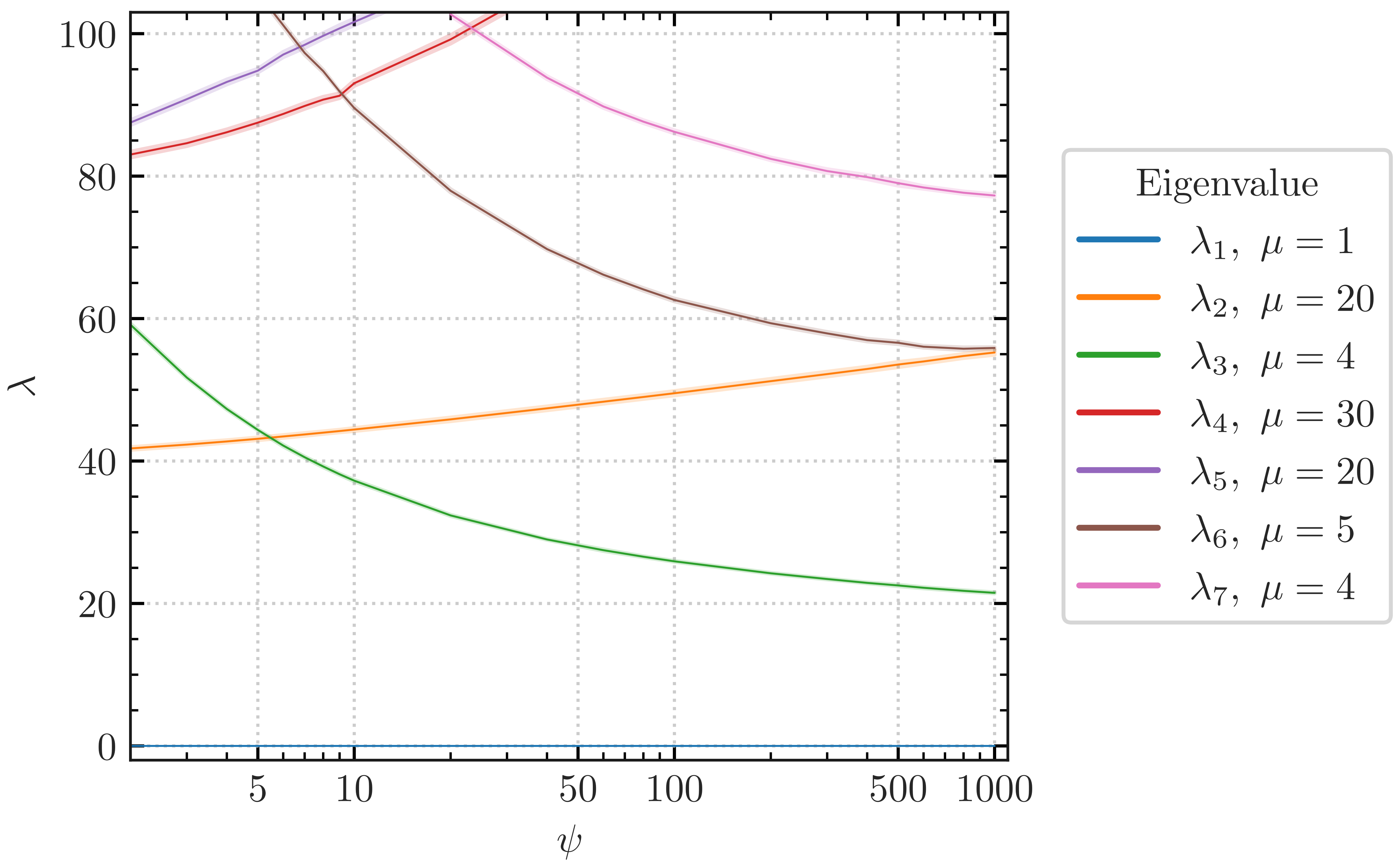


4.
3.8
3.6
3.4
3.2
3.
0.
0.2
0.4
0.6
0.8
1.
1.2
ln λ
d(2, ρ)

Results

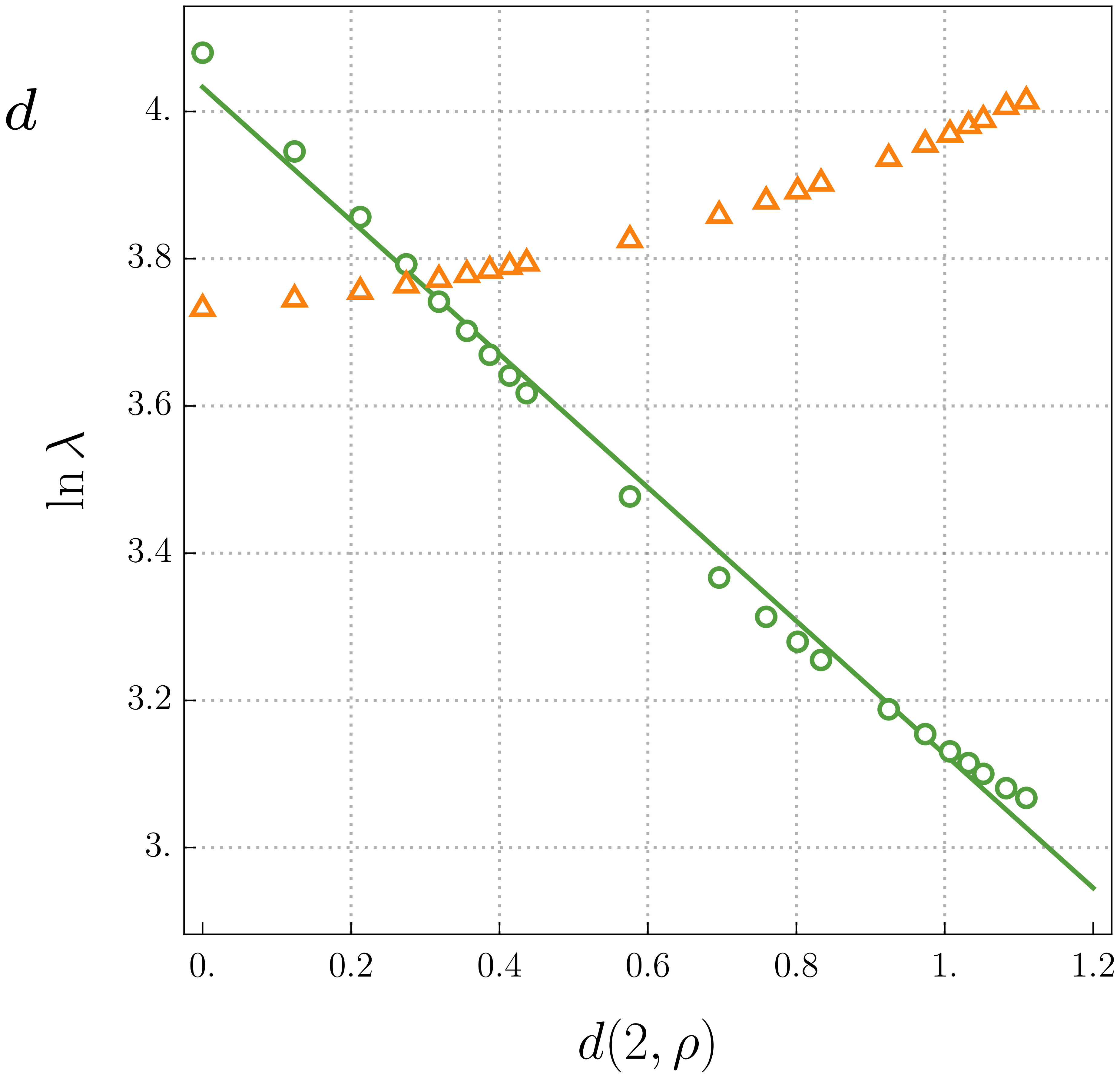
Results - I

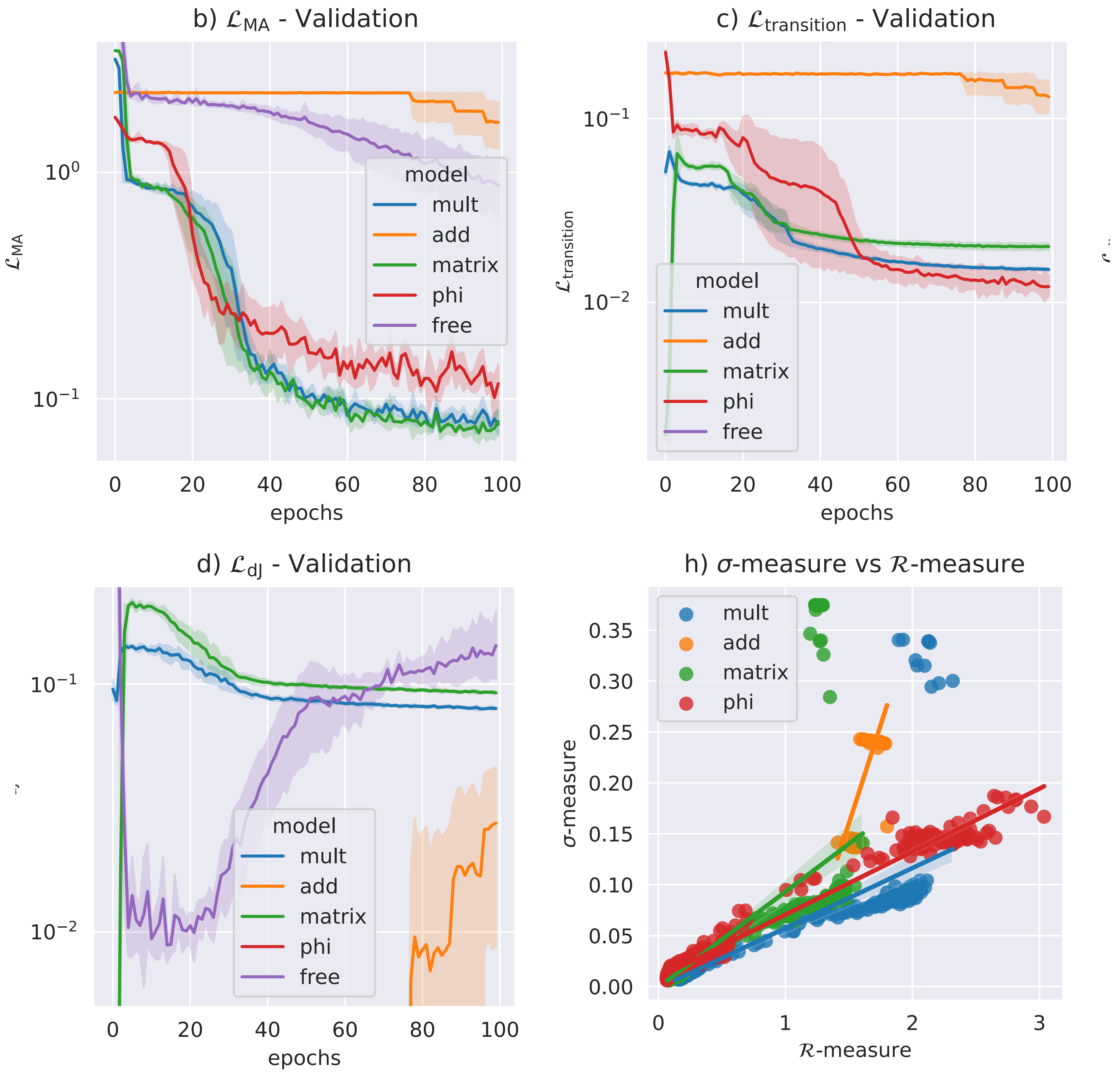
- One massless scalar (ψ)
- **Spectrum degenerate** w/ degeneracies given by **irreps of** CY **symmetry group**
- **Eigenvalues** (mass levels) **cross** in contrast (but not contradiction) to
  - No-crossing theorems in QM
  - Eigenvalue repulsion in RMT for hermitian matrices
- **States** with **large multiplicity** become **heavier**, states with **small multiplicity** become **lighter**

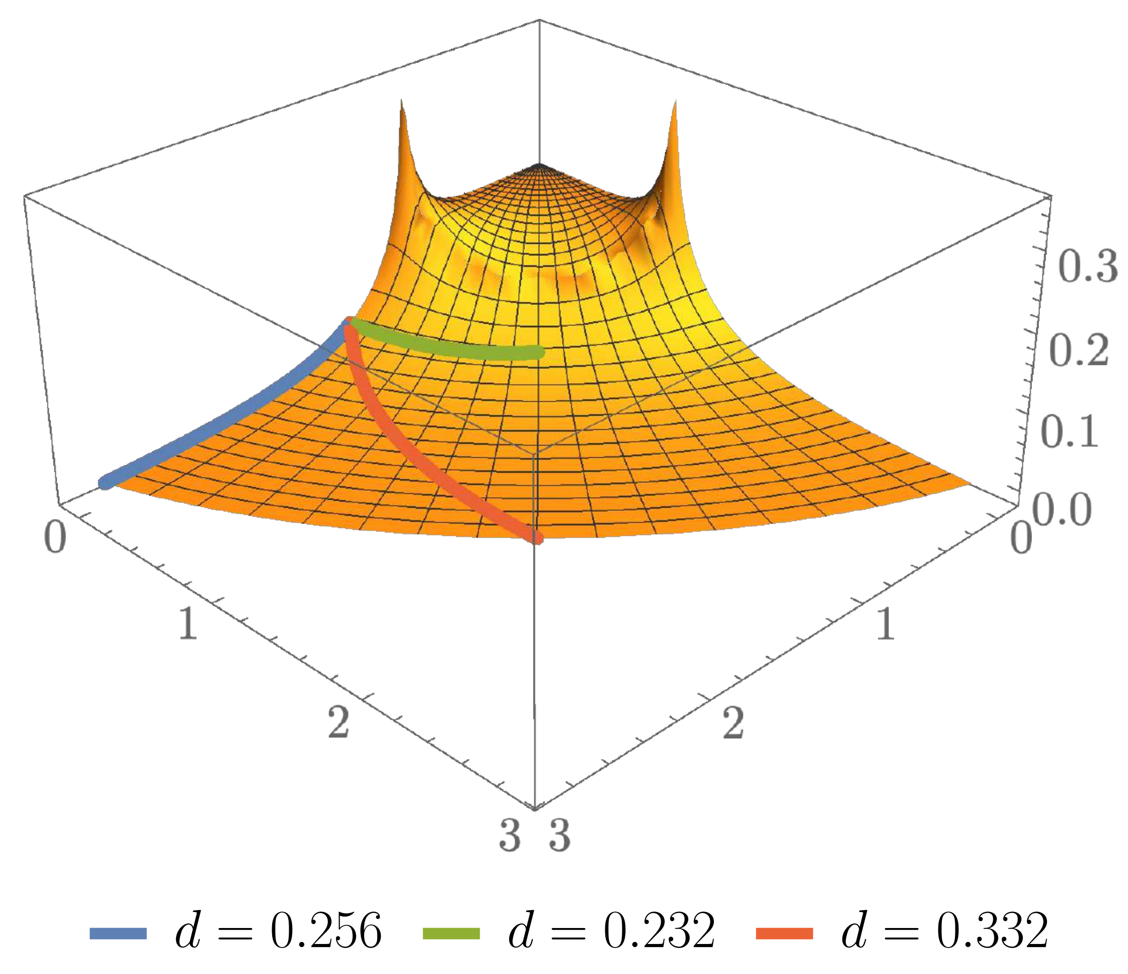


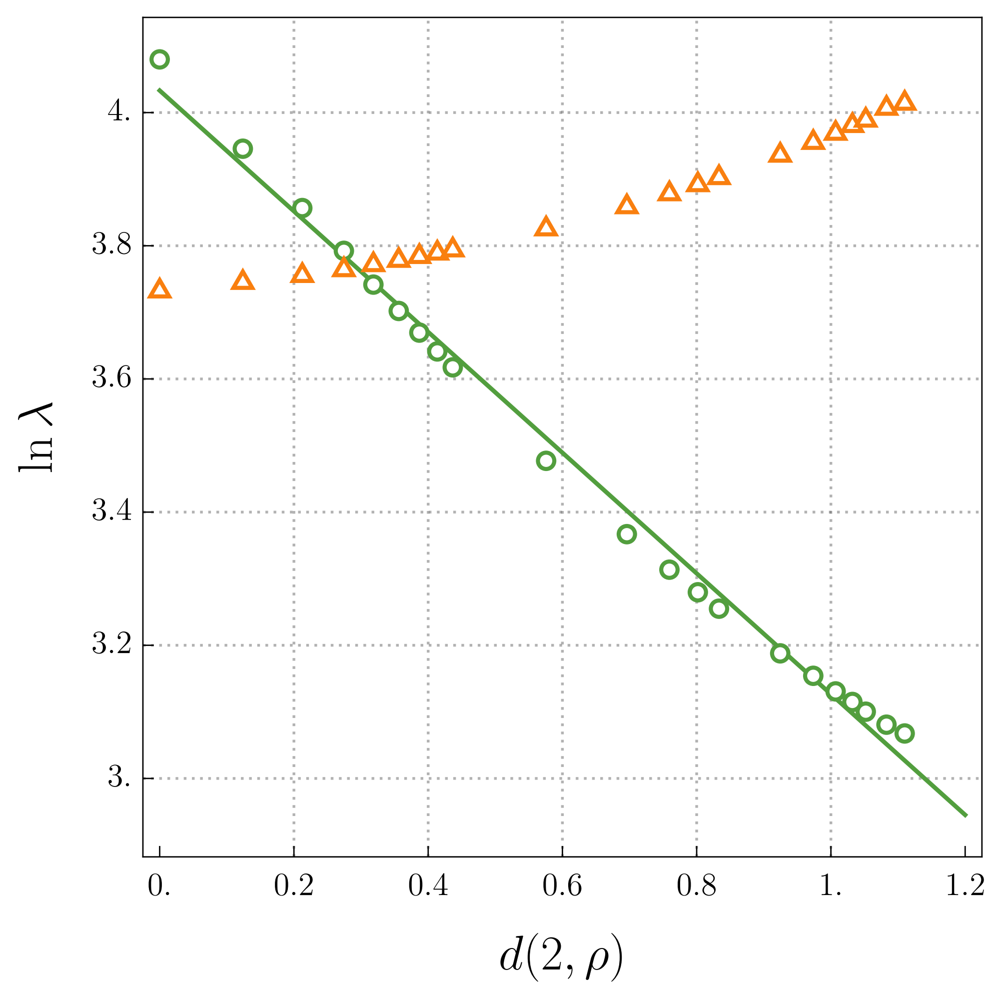
Results - II

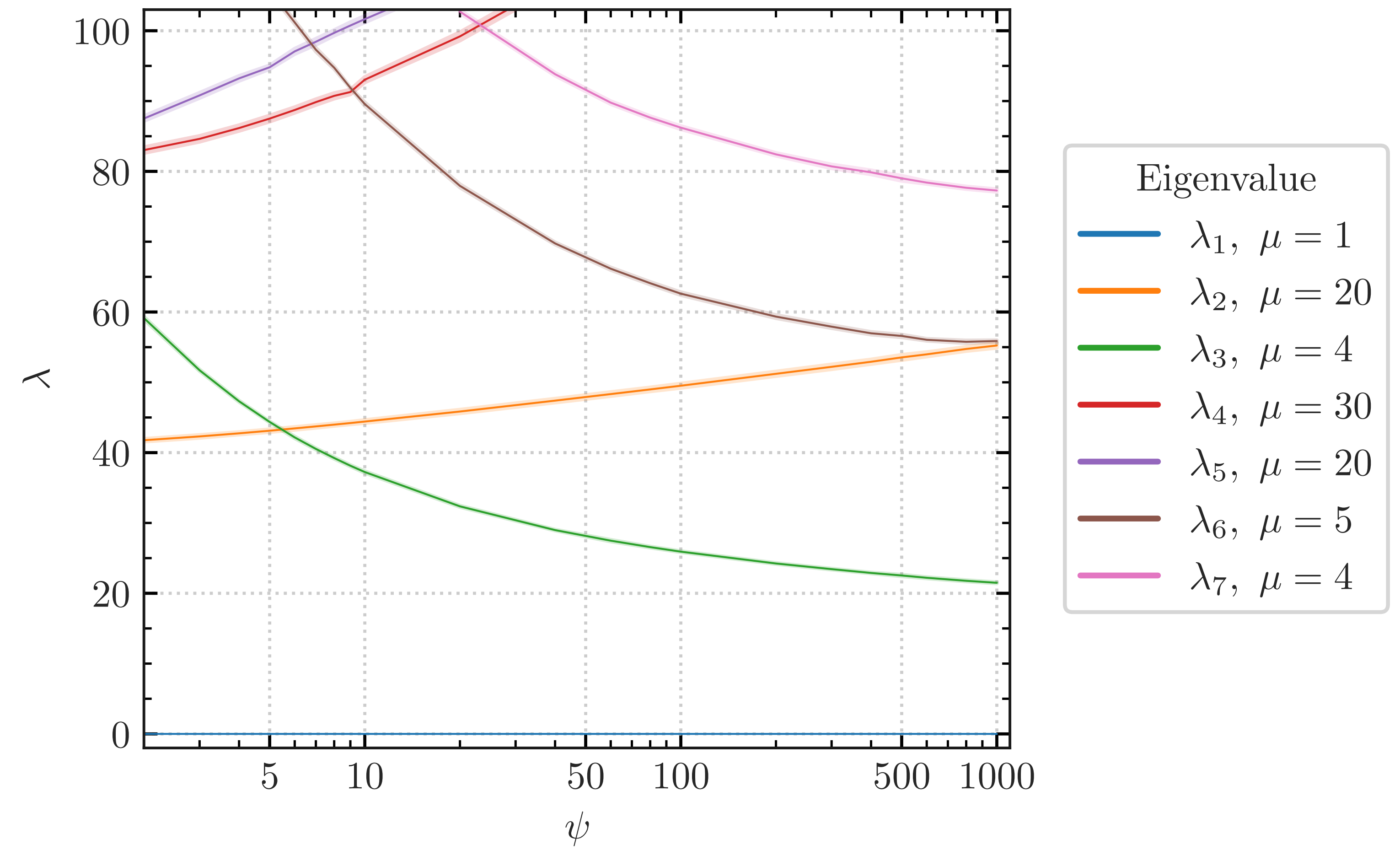
- **Scalars** become **exponentially light**
- Best fit gives $m = \lambda^{1/2} = 7.5e^{-(0.45\pm0.02)d}$
- First numerical check of the SDC
- Value very close to conjectured lower bound for $\alpha = 1/\sqrt{6}$ [Andriot,Cribiori,Erkinger `20]
- Interesting effect at sub-Planckian distances: mass of lightest state does not change for $.3\,M_P$







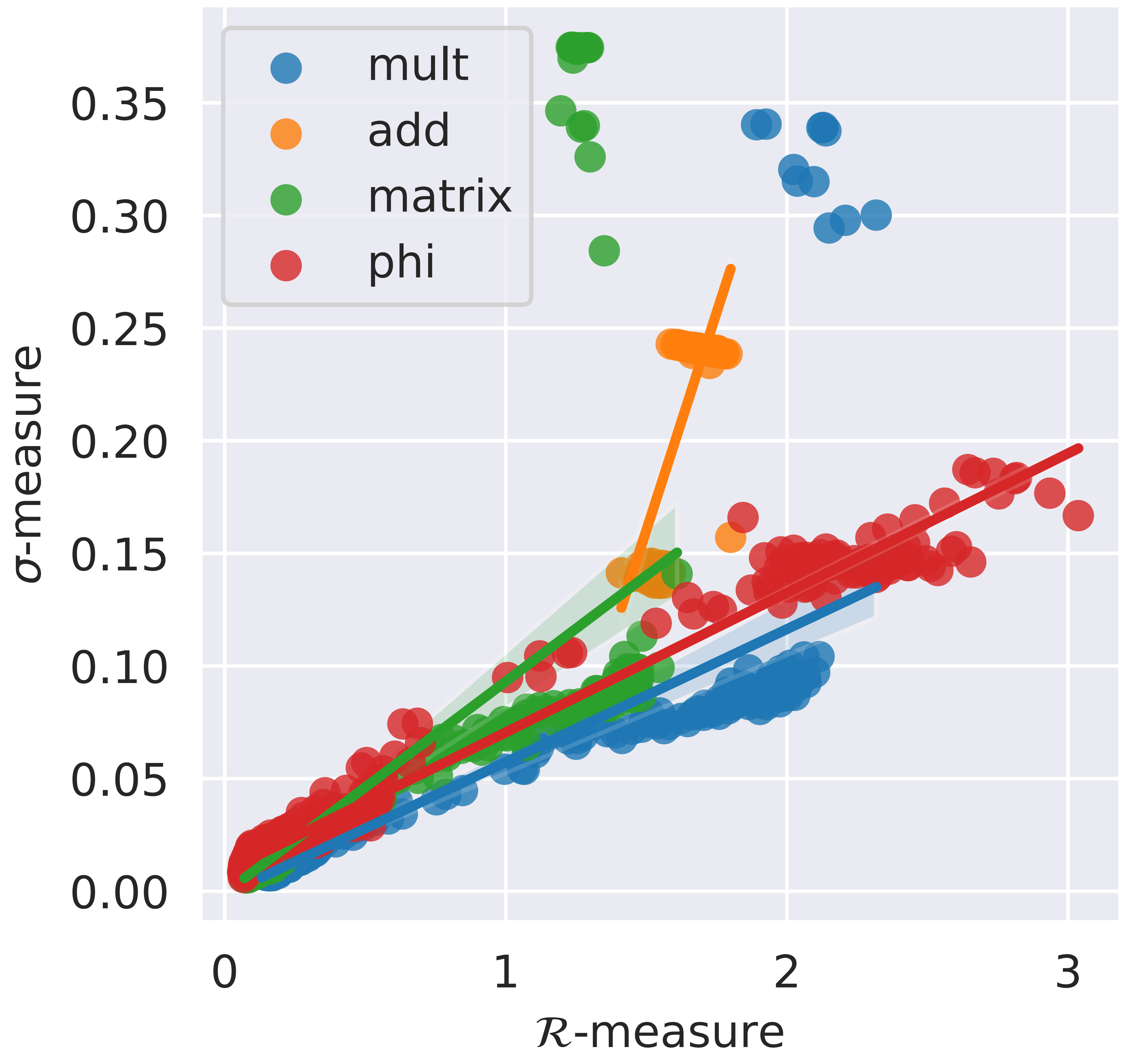




Conclusions

Conclusions - Part I

- Extended the techniques to compute CY metrics to the vast majority of known CY constructions (Kreuzer Skarke, CICYs)
- Computations possible for arbitrary number of Kähler moduli
- Develop MA NN that allows to fix the Kähler class
- Provide fast implementation that integrates into Mathematica and Sage
- Observe linear relation between MA equation



Conclusions - Part II

- Extended techniques to compute CY metrics
- Verified the SDC
   1. Use NNs to approximate CY metric and MC integration to compute massive string spectrum
   2. Use periods + mirror symmetry or numerical methods + MC integration + interpolation to compute CS moduli space metric
   3. Use PDE shooting methods to solve geodesics equations
   4. Fit masses to distance
- Surprising level-crossing and dependence on space-time symmetry group observed