Some flavors of string phenomenology

Michael Ratz

November 9 2021

supported by NSF

Based on collaborations with:

Y. Almumin, M. Blaszczyk, F. Brümmer, W. Buchmüller, M.–C. Chen, M. Fallbacher, M. Fischer, S. Groot Nibbelink, K. Hamaguchi, R. Kappl, V. Knapp–Peréz, T. Kobayashi, O. Lebedev, H.M. Lee, K.T. Mahanthappa, R. Mohapatra, A. Mütter, H.P. Nilles, B. Petersen, F. Plöger, S. Raby, M. Ramos–Hamud, S. Ramos–Sánchez, G. Ross, F. Ruehle, R. Schieren, K. Schmidt–Hoberg, Y. Shirman, S. Shukla, C. Staudt, M. Trapletti, A. Trautner, P. Vaudrevange, M. Waterbury & A. Wingerter

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Disclaimers:

1. The references are not extensive.
2. Will describe only a small subset of developments.
3. Will focus on heterotic orbifolds (and try to motivate why).

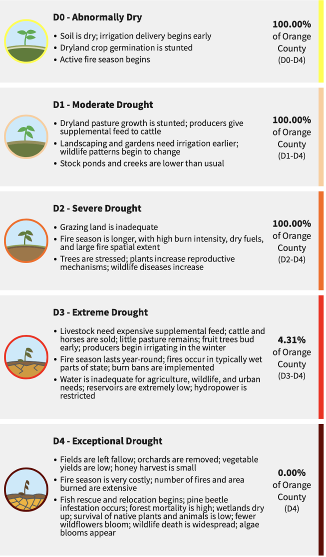
animalpath.org

Outline

&

More Disclaimers

Sorry, no swampland



D0 - Abnormally Dry

- Soil is dry; irrigation delivery begins early
- Dryland crop germination is stunted
- Active fire season begins

100.00% of Orange County (D0-D4)

D1 - Moderate Drought

- Dryland pasture growth is stunted; producers give supplemental feed to cattle
- Landscaping and gardens need irrigation earlier; wildlife patterns begin to change
- Stock ponds and creeks are lower than usual

100.00% of Orange County (D1-D4)

D2 - Severe Drought

- Grazing land is inadequate
- Fire season is longer, with high burn intensity, dry fuels, and large fire spatial extent
- Trees are stressed; plants increase reproductive mechanisms; wildlife diseases increase

100.00% of Orange County (D2-D4)

D3 - Extreme Drought

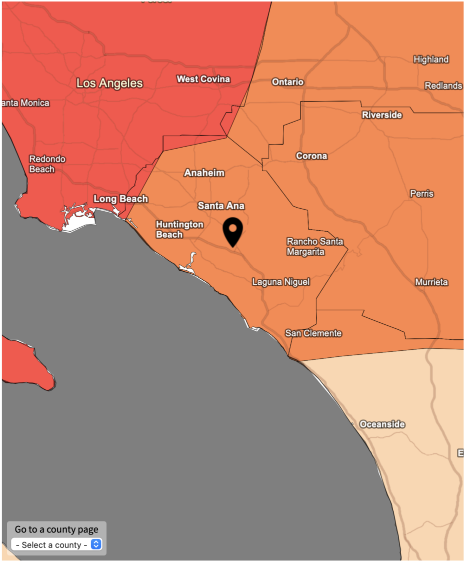
- Livestock need expensive supplemental feed; cattle and horses are sold; little pasture remains; fruit trees bud early; producers begin irrigating in the winter
- Fire season lasts year-round; fires occur in typically wet parts of state; burn bans are implemented
- Water is inadequate for agriculture, wildlife, and urban needs; reservoirs are extremely low; hydropower is restricted

4.31% of Orange County (D3-D4)

D4 - Exceptional Drought

- Fields are left fallow; orchards are removed; vegetable yields are low; honey harvest is small
- Fire season is very costly; number of fires and area burned are extensive
- Fish rescue and relocation begins; pine beetle infestation occurs; forest mortality is high; wetlands dry up; survival of native plants and animals is low; fewer wildflowers bloom; wildlife death is widespread; algae blooms appear

0.00% of Orange County (D4)



String pheno as framework for physics beyond the SM

- ☞ String theory is our arguably best shot at a consistent theory of quantum gravity

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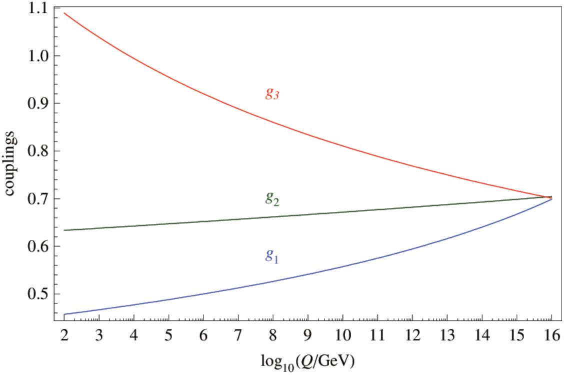

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This talk:

Considerable attention will be given to questions of masses, mixing parameters and $\mathcal{CP}$–violating phases.

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- ☞ Hierarchy $v_{\mathsf{EW}} \ll \Lambda$ may be partially stabilized by supersymmetry
- ☞ Getting the SM spectrum and gauge symmetries are only a small, yet necessary, part of the story
- ☞ Ultimately a globally consistent stringy completion of the SM may give us crucial insights on the nature of dark matter (DM), inflation (or a mechanism that replaces it) etc. but we really have to be sure that the models we construct are not doomed right from the start

Strategy & outline

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- ☞ Since all these topics are centered around symmetries, it is reasonable to consider orbifolds, which may be thought of as symmetry–enhanced points in moduli space

Flavor symmetries

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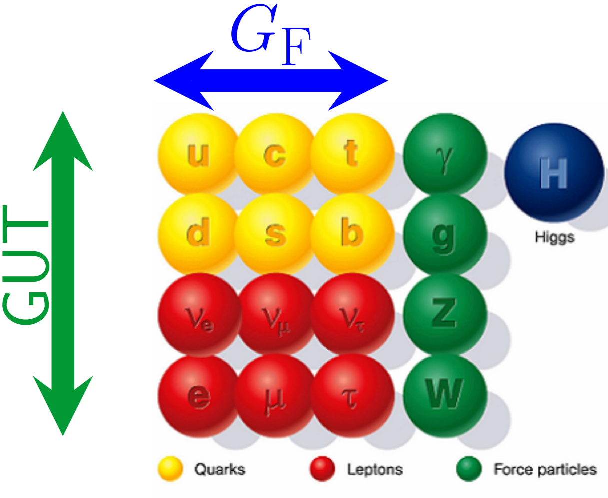
- ☞ Popular scheme in bottom–up model building: finite flavor symmetries

Why are symmetries popular/useful?

- ☞ Family symmetries G_{F} relate different families
  ↷ inter–family relations



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  (Gauge origin of such symmetries will be discussed later)

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- ☞ Prominent example: A_4 ▸ details

$\mathcal{CP}$ violation from finite groups

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here:

Non–Abelian discrete (flavor) symmetry $G \leftrightarrow \xcancel{\mathcal{CP}}$

$\mathcal{CP}$ vs. outer automorphisms

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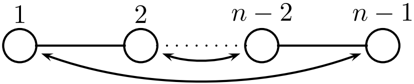

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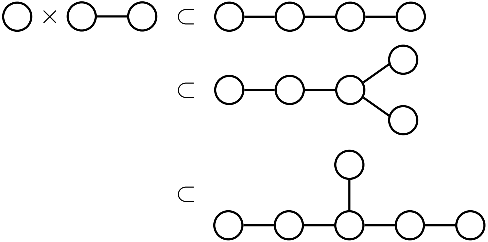

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this talk:

Chen & Mahanthappa [2009] ;Chen, Fallbacher, Mahanthappa, M.R. & Trautner [2014]

Not at all true

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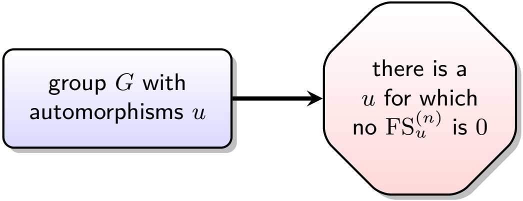
Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner [2014]

group G with automorphisms u

Three types of groups

Chen, Fallbacher, Mahanthappa, M.R. & Trautner [2014]



group G with automorphisms u

there is a u for which no $\mathrm{FS}_u^{(n)}$ is 0

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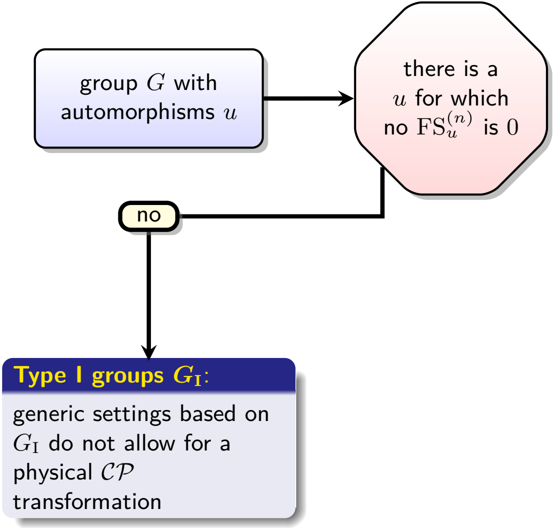

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Type I groups G_{I}:

generic settings based on G_{I} do not allow for a physical $\mathcal{CP}$ transformation

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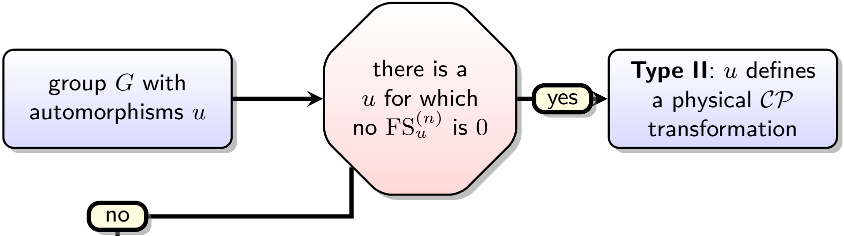

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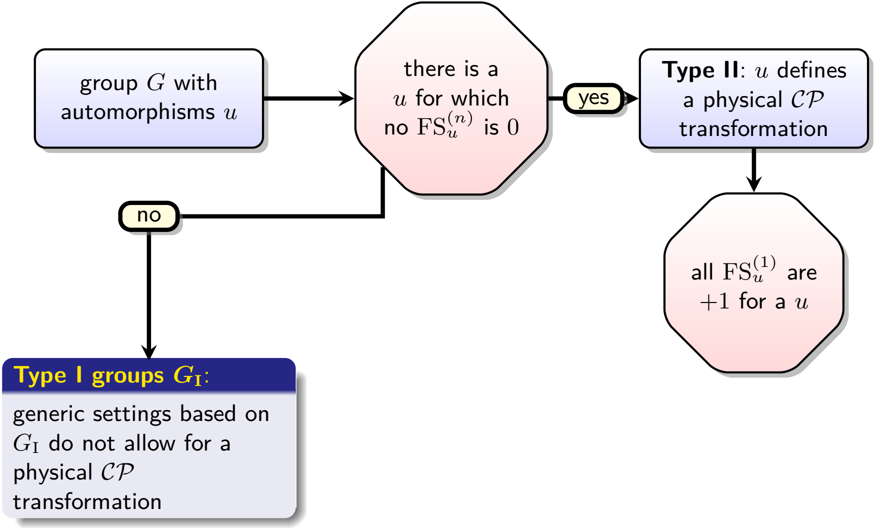

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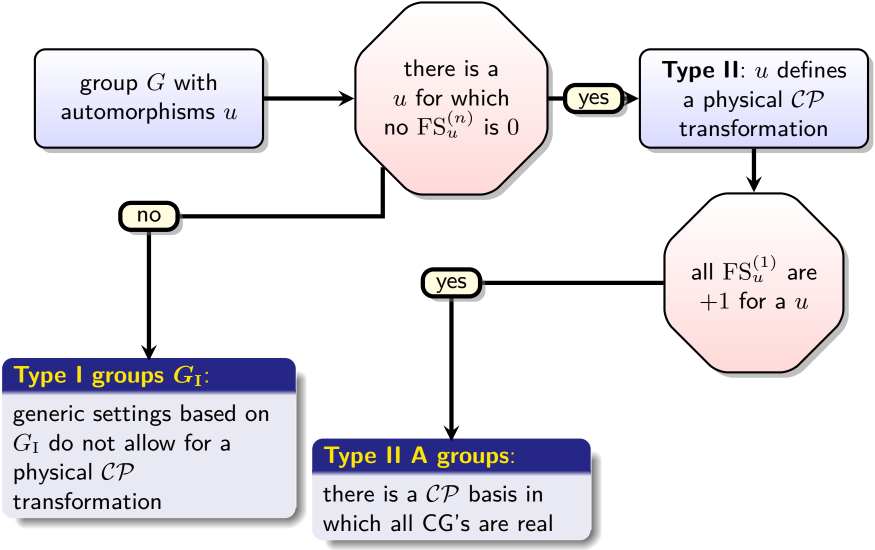

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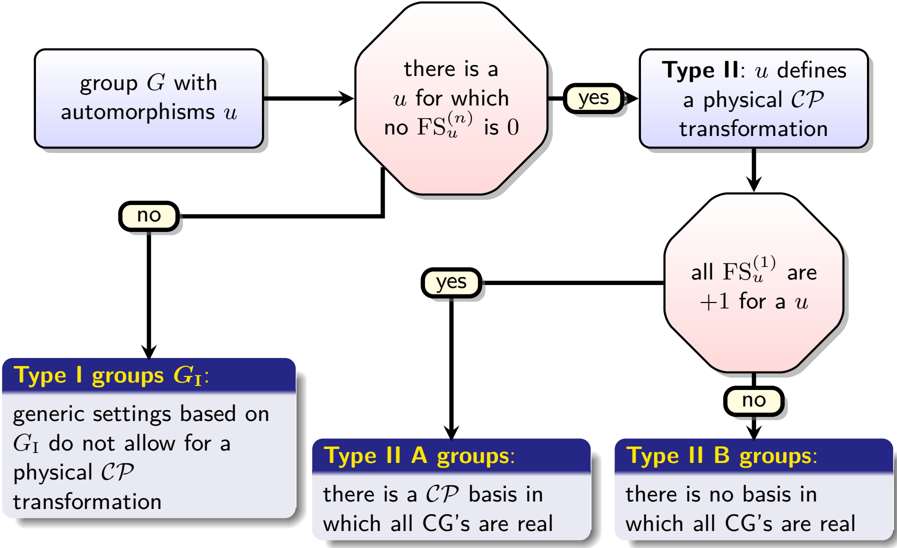

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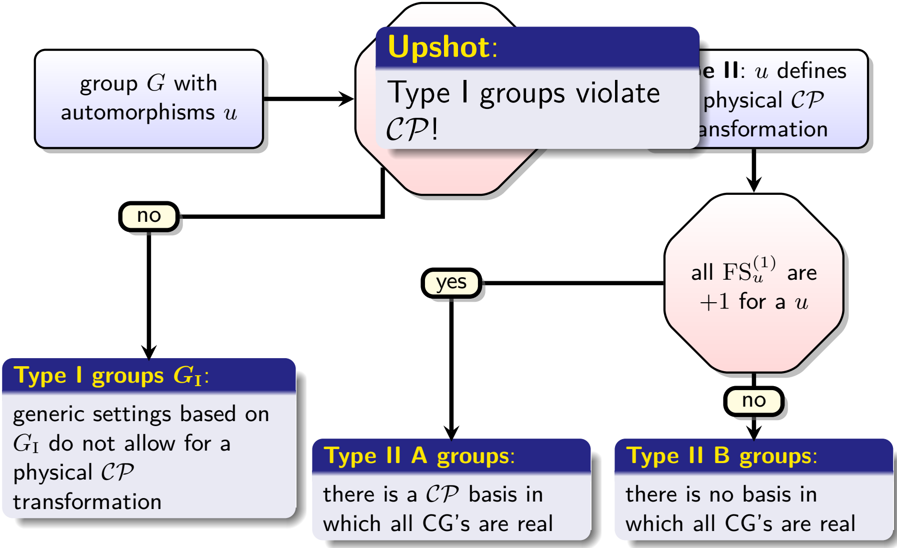

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Upshot:

Type I groups violate $\mathcal{CP}$!

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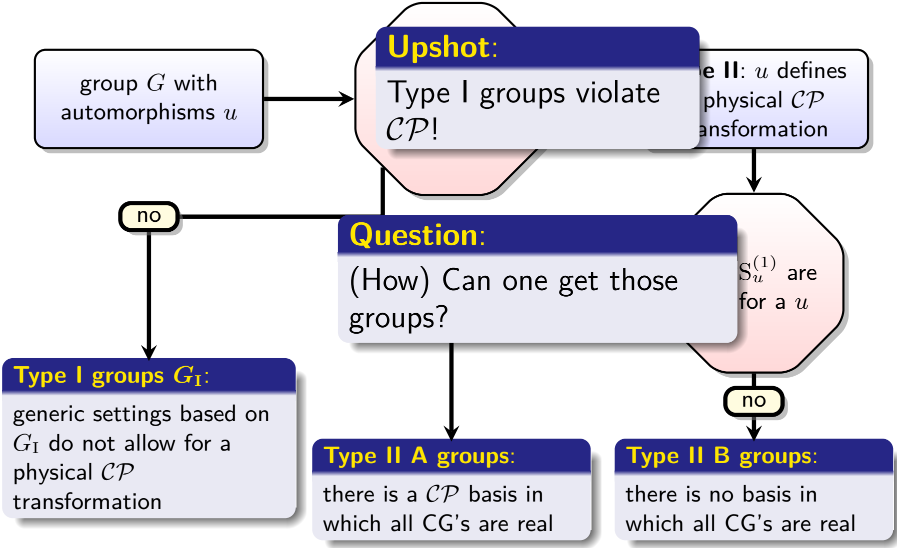

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(How) Can one get those groups?

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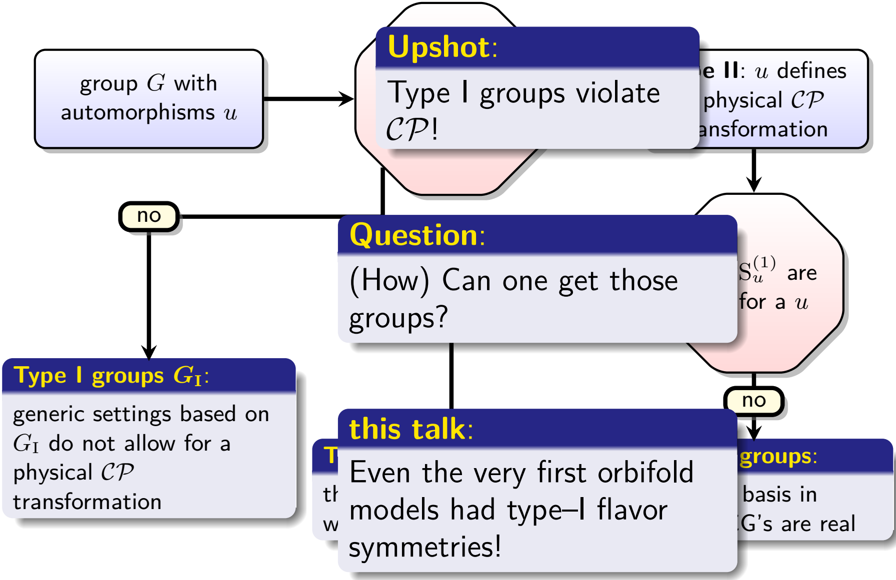

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T

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w

this talk:

Even the very first orbifold models had type–I flavor symmetries!

groups:

basis in

G's are real

Examples

will be discussed later

- ☞ type I : all odd order non–Abelian groups

group	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$	$\Delta(54)$
SG	(20,3)	(21,1)	(27,3)	(27,4)	(54,8)

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- ☞ type II B

group	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	(72,41)	(144,120)

$\Delta(54)$

from a

$\mathbb{Z}_3$ orbifold plane

First 3 family models from stringy orbifolds

Ibáñez, Kim, Nilles & Quevedo [1987]

☞ Very first stringy model of particle physics based on $\mathbb{Z}_3$ orbifold

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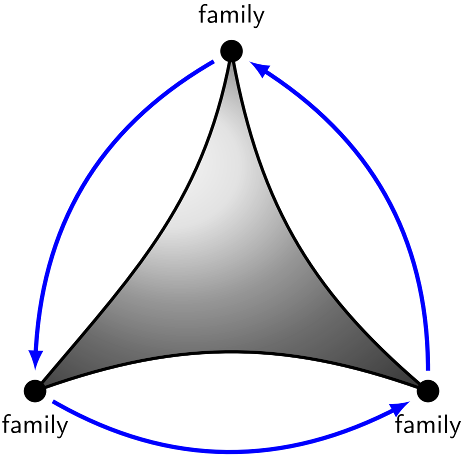
  - ☞ Permutation symmetry of fixed points/families



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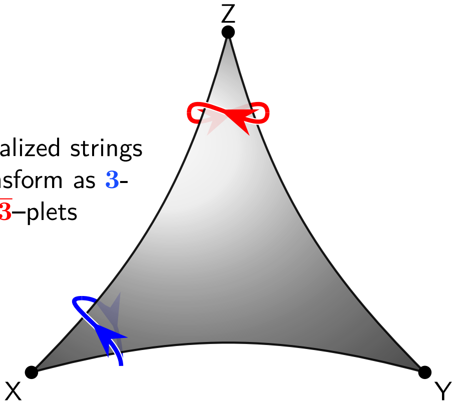
    Kobayashi, Nilles, Plöger, Raby & M.R. [2007]
  - ➥ $\Delta(54)$ flavor/family symmetry

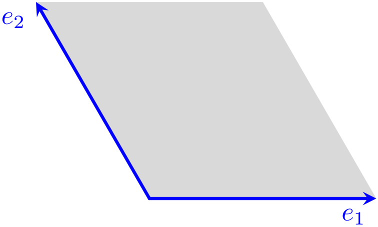


localized strings tansform as $\mathbf{3}$- or $\overline{\mathbf{3}}$–plets

$\mathbb{T}^2/\mathbb{Z}_3$ Orbifold GUT

Biermann, Mütter, Parr, M.R. & Vaudrevange [2019] , see also Guralnik & Ramgoolam [1997]

- 6d with gauge symmetry $\mathcal{G} = \mathrm{SU}(3)$ and two dimensions compactified on torus with $|e_1| = |e_2|$ with $e_1 \cdot e_2 = -{|e_1|^2}/{2}$



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- ☞ Associated gauge embedding

$$P = \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in \mathrm{SU}(3) \quad \text{where} \quad P^3 = \mathbb{1}$$

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- Residual symmetries

$$U_{(0)} \;=\; \begin{pmatrix} \mathrm{e}^{\mathrm{i}(\alpha+\beta)} & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}(\alpha-\beta)} & 0 \\ 0 & 0 & \mathrm{e}^{-2\mathrm{i}\alpha} \end{pmatrix} \quad \text{and} \quad U_{(1)} \;=\; \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

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- Altogether: $\mathrm{SU}(3) \xrightarrow{\mathbb{Z}_3^{\text{orb.}}} \Big[\mathrm{U}(1) \times \mathrm{U}(1)\Big] \rtimes \mathbb{Z}_3$

$\Delta(54)$ from $\mathbb{Z}_3$ orbifold planes

Beye, Kobayashi & Kuwakino [2014, 2015]

☞ The $\mathrm{SU}(3)$ gauge symmetry arises at the self–dual radius

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origin clarified

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- ☞ Detailed understanding of *gauge* origin of $\Delta(54)$ flavor symmetry
- ☞ Part of a so–called eclectic symmetry (see later)

Nilles, Ramos–Sánchez & Vaudrevange [2021]

$$G_{\text{ecl}} \;=\; \Delta(54)\cup T'\cup\mathbb{Z}_9^R\cup\mathbb{Z}_2^{\mathcal{CP}}$$

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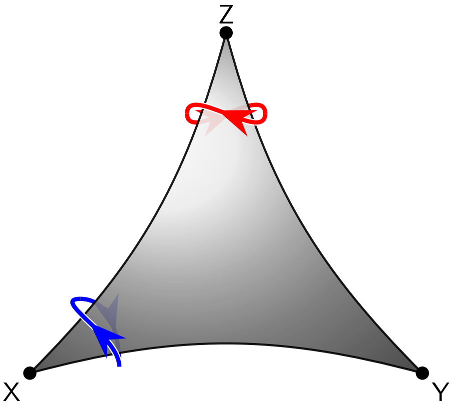
☞ $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry



localized strings tansform as $\mathbf{3}$- or $\overline{\mathbf{3}}$–plets

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- ☞ $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry
- ☞ Explicit model

Carballo-Pérez, Peinado & Ramos-Sánchez [2016]

#	irrep	$\Delta(54)$	label
3	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$\mathbf{3}_{11}$	Q_i
3	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$	$\mathbf{3}_{11}$	$\overline{u}_i$
3	$(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$	$\mathbf{3}_{11}$	$\overline{d}_i$
3	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\mathbf{3}_{11}$	L_i
3	$(\mathbf{1},\mathbf{1})_{1}$	$\mathbf{3}_{11}$	$\overline{e}_i$
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	$\overline{\nu}_i$

$\Delta(54)$ from a $\mathbb{Z}_3$ orbifold plane

- ☞ $\mathbb{Z}_3$ orbifold plane without Wilson lines leads to a $\Delta(54)$ flavor symmetry
- ☞ Explicit model Carballo-Pérez, Peinado & Ramos-Sánchez [2016]
- ☞ Quarks and leptons transform as $\mathbf{3}$–plets (or $\overline{\mathbf{3}}$–plets) of $\Delta(54)$

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- ☞ Not that simple! If the representation content is very special, one *can* impose a $\mathcal{CP}$ transformation

$$\exists\ \text{out}\ :\ \mathbf{3}_i \xleftrightarrow{\text{out}} \overline{\mathbf{3}}_i \quad \text{and} \quad \mathbf{1}_i \xleftrightarrow{\text{out}} \overline{\mathbf{1}}_i$$

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- ☞ Not that simple! If the representation content is very special, one *can* impose a $\mathcal{CP}$ transformation
- ☞ At the massless level, only 3- and 1–dimensional representations occur $\curvearrowright$ a class–inverting outer automorphism exists $\curvearrowright$ a $\mathcal{CP}$ candidate exists

$\mathcal{CP}$ violation

in the

$\mathbb{Z}_3$ orbifold

$\mathcal{CP}$ violation from strings

☞ However, at the massive level $\Delta(54)$ $\mathbf{2}$–plets arise

Nilles, M.R., Trautner & Vaudrevange [2018]

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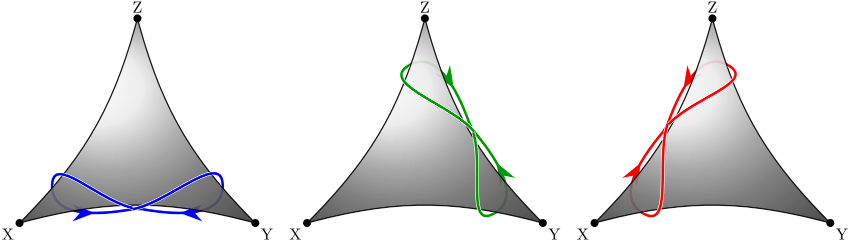
- Doublets $\mathbf{2}_1$, $\mathbf{2}_3$ and $\mathbf{2}_4$ correspond to linear combinations of strings that wind around two different fixed points in opposite directions



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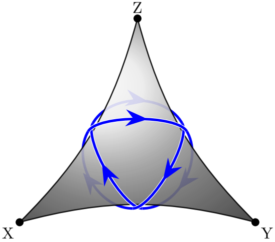
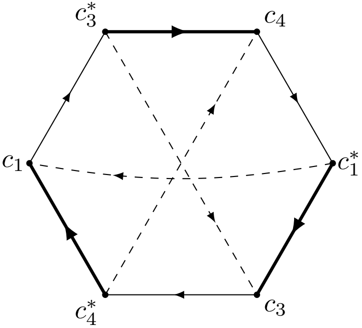
- Doublet $\mathbf{2}_2$



$\mathcal{CP}$ violation from strings

☞ Doublets save the day

Nilles, M.R., Trautner & Vaudrevange [2018]



- We follow invariant approach — Bernabeu, Branco & Gronau [1986]
- Super powerful tool: `GroupMath` — Fonseca [2021]

$\mathcal{CP}$ violation from strings

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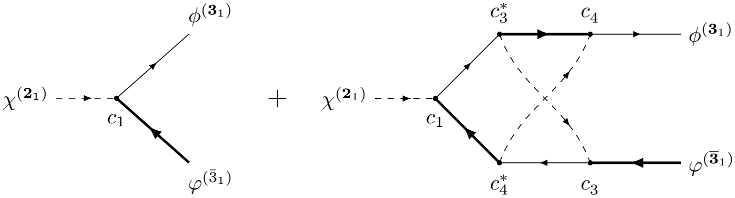
Nilles, M.R., Trautner & Vaudrevange [2018]

- ☞ Physical $\cancel{\mathcal{CP}}$ in doublet decay



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bottom–line:

$\mathcal{CP}$ violation can come from group theory in UV complete settings in which the origin of the flavor group is fully understood

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

- Type I groups can be embedded in $\mathrm{SU}(N)$

no $\mathcal{CP}$ transformation

has $\mathcal{CP}$ transformation

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

☞ Type I groups can be embedded in $\mathrm{SU}(N)$

➥ Question: at which stage gets $\mathcal{CP}$ broken?

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

- ☞ Type I groups can be embedded in $\mathrm{SU}(N)$
- ➥ Question: at which stage gets $\mathcal{CP}$ broken?
- ☞ Possible options include:
  - $\mathcal{CP}$ gets broken by the VEV that breaks $\mathrm{SU}(N)$ to G

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

- ☞ Type I groups can be embedded in $\mathrm{SU}(N)$
- ➥ Question: at which stage gets $\mathcal{CP}$ broken?
- ☞ Possible options include:
  - $\mathcal{CP}$ gets broken by the VEV that breaks $\mathrm{SU}(N)$ to G
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M.R. & Trautner [2017]

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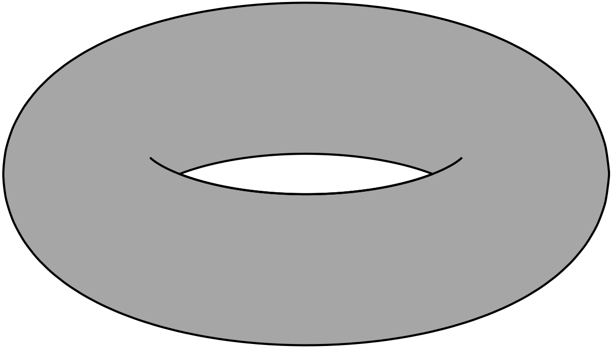
- ☞ Rather, the $\mathrm{SU}(3)$ $\mathcal{CP}$ transformation turns into an unbroken outer automorphism which does not warrant physical $\mathcal{CP}$ conservation

▸ details

Modular
flavor
symmetries

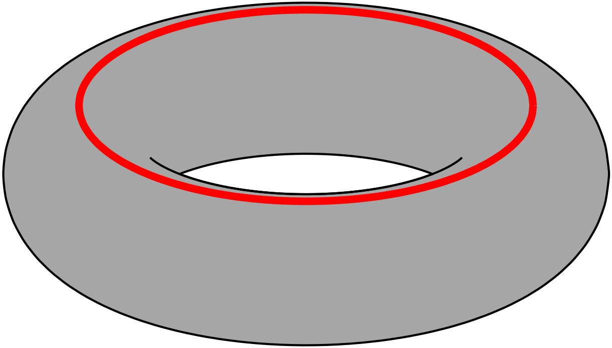
Tori

- ☞ torus=donut



Tori

- ☞ torus=donut
- ☞ two cycles



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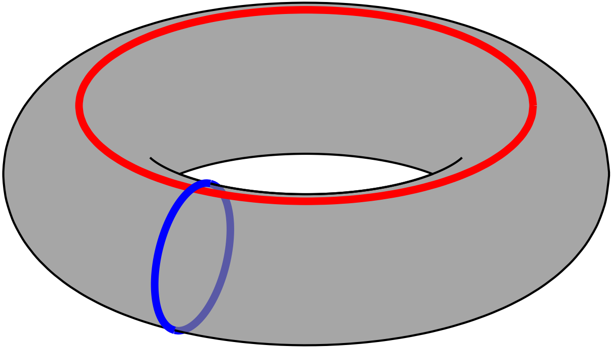
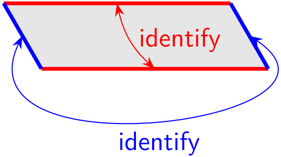


Tori

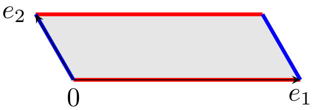
- ☞ torus can be thought of as a parallelogram (which emerges by cutting the torus open along the red and blue cycles)

Tori



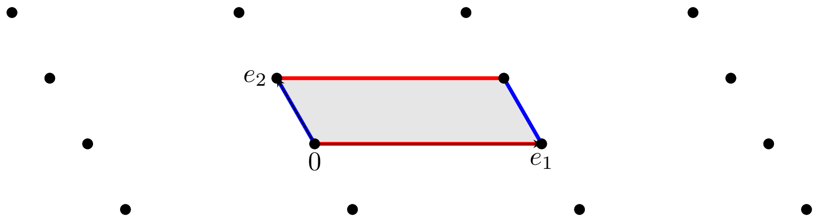
- ☞ opposite edges get identified

Tori



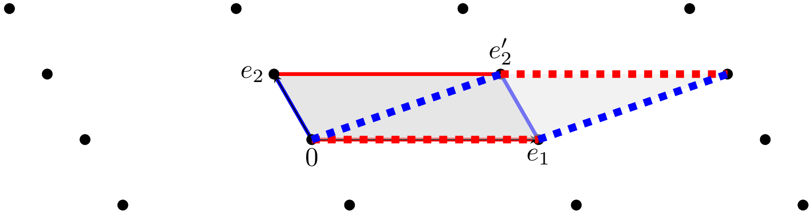
- ☞ edges define basis vectors of a lattice

Tori



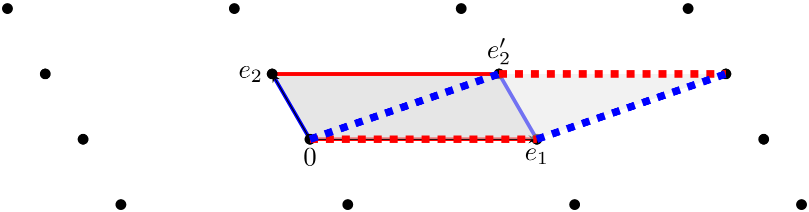
☞ torus is $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$: two points in the plane get identified if they differ by a lattice translation

Tori



☞ fundamental domain is not unique

Tori

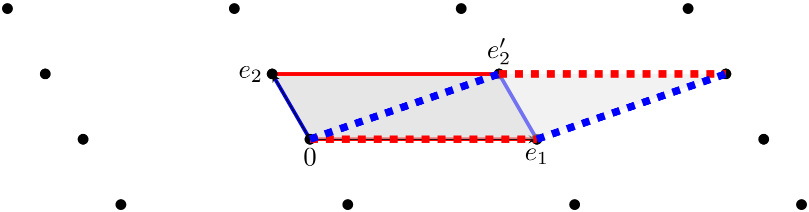


- fundamental domain is not unique
- we can build linear combinations of the basis vectors

$$\begin{pmatrix} e_2 \\ e_1 \end{pmatrix} \xrightarrow{\gamma} \begin{pmatrix} e_2' \\ e_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e_2 \\ e_1 \end{pmatrix} =: \gamma \begin{pmatrix} e_2 \\ e_1 \end{pmatrix}$$

$a, b, c, d \in \mathbb{Z}$

Tori



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- volume of fundamental domain stays the same $\Leftrightarrow \det\gamma = 1 \curvearrowright$ $\gamma \in \mathrm{SL}(2,\mathbb{Z})$ (there is a superfluous sign, so $\gamma \in \Gamma = \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2$)

$\mathrm{SL}(2,\mathbb{Z})$

- ☞ two basic transformations

$$T \;:\; e_2 \mapsto e_2' = e_2 + e_1 \qquad \curvearrowright \gamma = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} =: T$$

$$S \;:\; e_1 \mapsto e_1' = e_2 \quad \text{and} \quad e_2 \mapsto e_2' = -e_1 \quad \curvearrowright \gamma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} =: S$$

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- ☞ S and T generate $\mathrm{SL}(2,\mathbb{Z})$ and

$$S^2 \;=\; (S\,T)^3 \;=\; \mathbb{1}$$

$\mathrm{SL}(2,\mathbb{Z})$ and modular flavor symmetries

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Modular flavor symmetries:

identify finite groups with generators satisfying

$$S^2 \;=\; (S\,T)^3 \;=\; \mathbb{1}$$

and additional relations

Modular flavor symmetries

- ☞ finite subgroups $\Gamma_N := \Gamma/\Gamma(N)$ where

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2 \;;\; \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mod N \right\}$$

level

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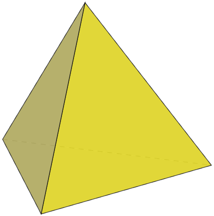

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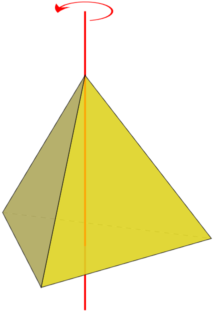

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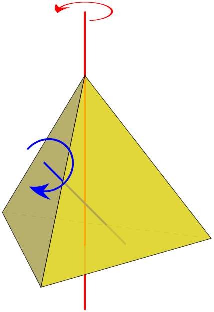

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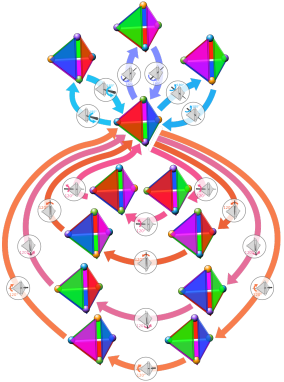
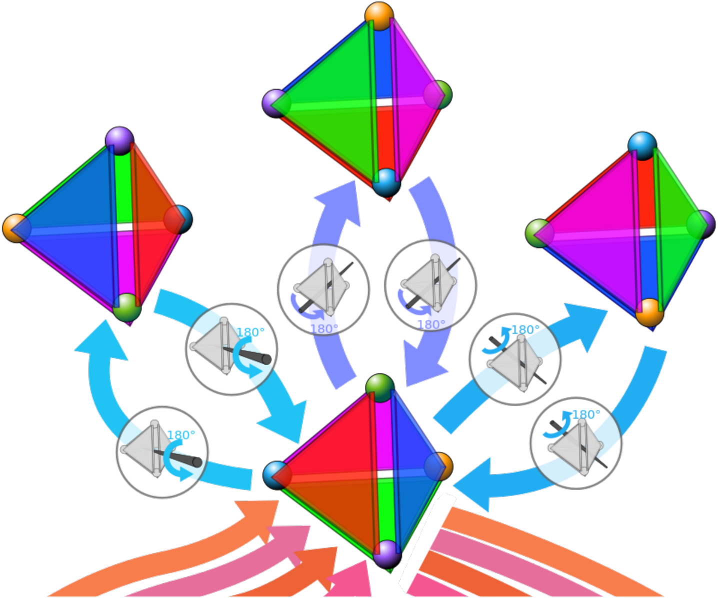


www.wikipedia.org

Modular flavor symmetries



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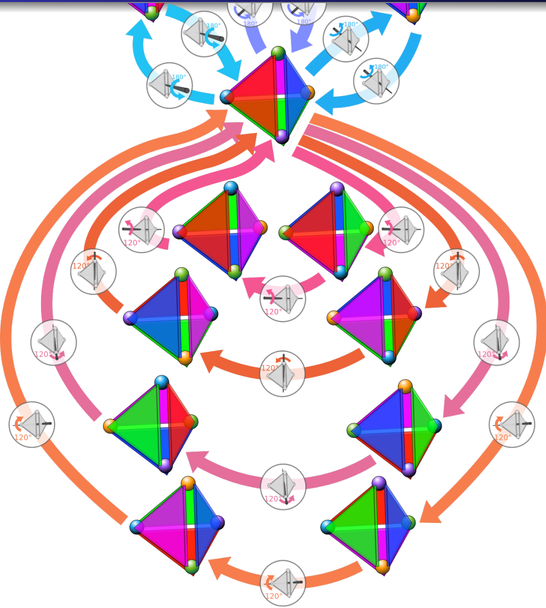

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- ☞ e.g. $\Gamma_3 \simeq A_4$ (symmetry of tetrahedron)
- ☞ complex coordinates: $\mathbb{R}^2 \simeq \mathbb{C}$
- ➥ modular transformations in complex coordinates

$$\tau \xmapsto{S} \frac{-1}{\tau} \quad \text{and} \quad \tau \xmapsto{T} \tau + 1$$

Modular forms

- traditional modular forms

$$f(\gamma\tau) = (c\tau + d)^{-k} f(\tau)$$

$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2$$

Modular forms

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$$f(\gamma\tau) = (c\tau + d)^{-k} f(\tau)$$

$k \in \mathbb{Q}$ modular weight

Modular forms & modular flavor symmetries

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$$f(\gamma\tau) = (c\tau + d)^{-k} f(\tau)$$

- ☞ modular forms of level N

$$f_i(\gamma\tau) = (c\tau + d)^{-k} \left[\rho_N(\gamma)\right]_{ij} f_j(\tau)$$

representation matrix of Γ_N

Modular forms & modular flavor symmetries

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Feruglio [2017]

Modular flavor symmetries:

What if Yukawa couplings are modular forms?

An explicit example

Feruglio [2017]

☞ lepton sector of the (supersymmetric) standard model

	(E_1^c, E_2^c, E_3^c)	L	H_d	H_u	φ
$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$	$\mathbf{1}_1$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{1/2}$	$\mathbf{1}_0$
Γ_3	$(\mathbf{1}, \mathbf{1'}, \mathbf{1''})$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$
k	$(k_{E_1}, k_{E_2}, k_{E_3})$	k_L	k_d	k_u	k_φ

An explicit example

flavon

Feruglio [2017]

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$Y = (Y_1, Y_2, Y_3)^T$ w/ Y_i modular functions (unique)

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- neutrino mass in traditional A_4 models

$$m_\nu \;=\; \frac{v_u^2}{\Lambda}\begin{pmatrix}2a & -c & -b\\ -c & 2b & -a\\ -b & -a & 2c\end{pmatrix}\qquad\text{(old)}$$

An explicit example

Feruglio [2017]

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  - we know two Δm^2 and three angles so it is nontrivial that this works.
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Chen, Ramos-Sánchez & M.R. [2020]

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Chen, Ramos-Sánchez & M.R. [2020]

➥ many more parameters

Problem with kinetic terms

Chen, Ramos-Sánchez & M.R. [2020]

☞ EFT expansion of the Kähler potential

$$K \;=\; \alpha_0\,(-\mathrm{i}\,\tau+\mathrm{i}\,\overline{\tau})^{-1}\,\left(\overline{L}\,L\right)_{\mathbf{1}}+\sum_{k=1}^{7}\alpha_k\,(-\mathrm{i}\,\tau+\mathrm{i}\,\overline{\tau})\,\left(Y\,L\,\overline{Y}\,\overline{L}\right)_{\mathbf{1},\,k}+\dots$$

canonical (up to overall factor)

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extra terms on the same footing

Problem with kinetic terms

Chen, Ramos-Sánchez & M.R. [2020]

- ☞ EFT expansion of the Kähler potential

$$K = \alpha_0 \left(-\mathrm{i}\,\tau + \mathrm{i}\,\overline{\tau}\right)^{-1} \left(\overline{L}\,L\right)_{\mathbf{1}} + \sum_{k=1}^{7} \alpha_k \left(-\mathrm{i}\,\tau + \mathrm{i}\,\overline{\tau}\right) \left(Y\,L\,\overline{Y}\,\overline{L}\right)_{\mathbf{1},\,k} + \dots$$

- ☞ Since modular flavor symmetries are nonlinearly realized there is no control over the Kähler potential

Problem with kinetic terms

Chen, Ramos-Sánchez & M.R. [2020]

- ☞ EFT expansion of the Kähler potential

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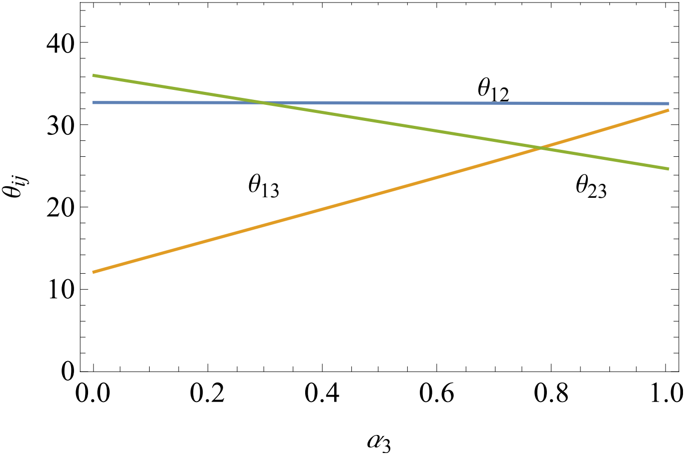
- ☞ Since modular flavor symmetries are nonlinearly realized there is no control over the Kähler potential
- ➥ More parameters than predictions in bottom–up approach

Example of corrections in modular A_4 model

Chen, Ramos-Sánchez & M.R. [2020]

- ☞ E.g. sensitivity to the α_3 coefficient



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Nilles, Ramos–Sánchez & Vaudrevange [2021], Baur, Kade, Nilles, Ramos-Sanchez & Vaudrevange [2021]
Nilles, Ramos–Sánchez, & Vaudrevange [2020c], Nilles, Ramos-Sanchez & Vaudrevange [2020a]
Nilles, Ramos-Sánchez & Vaudrevange [2020b]

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Nilles, Ramos-Sánchez & Vaudrevange [2020b]

☞ This talk: focus on a simple enough field theory model that is "stringy enough"

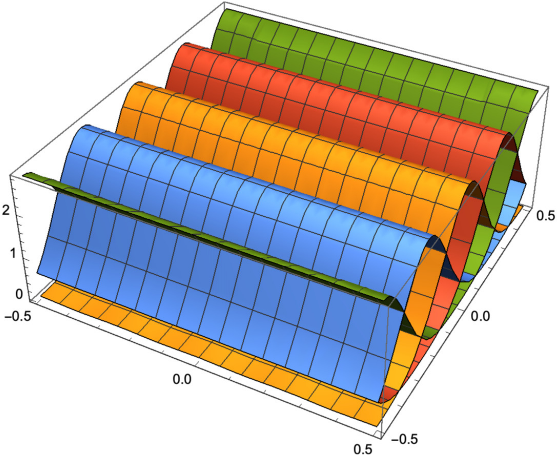
Metaplectic flavor symmetries

Magnetized tori

Cremades, Ibáñez & Marchesano [2004]

- ☞ torus with magnetic flux carries chiral zero modes

$$\psi^{j,M}(z,\tau,\zeta)=\mathcal{N}\,\mathrm{e}^{\pi\,\mathrm{i}\,M\,(z+\zeta)\,\frac{\mathrm{Im}(z+\zeta)}{\mathrm{Im}\,\tau}}\,\vartheta\begin{bmatrix}\frac{j}{M}\\0\end{bmatrix}\bigl(M\,(z+\zeta),M\,\tau\bigr)$$



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flux parameter $\curvearrowright$ # of zero modes

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“Wilson line”

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Jacobi ϑ–function

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- normalization

$$\mathcal{N} = \left(\frac{2M\;\mathrm{Im}\,\tau}{\mathcal{A}^2}\right)^{1/4}$$

area of torus
$\mathcal{A} = (2\pi R)^2\,\mathrm{Im}\,\tau$

Flux

- ☞ Flux in U(N) gauge theory w/ $N = N_a + N_b + N_c$

$$F_{z\bar{z}} = \frac{\pi \mathrm{i}}{\operatorname{Im}\tau}\begin{pmatrix} \frac{m_a}{N_a}\mathbb{1}_{N_a\times N_a} & 0 & 0 \\ 0 & \frac{m_b}{N_b}\mathbb{1}_{N_b\times N_b} & 0 \\ 0 & 0 & \frac{m_c}{N_c}\mathbb{1}_{N_c\times N_c} \end{pmatrix}$$

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- ☞ Assumption: $s_\alpha = \frac{m_\alpha}{N_\alpha} \in \mathbb{Z}$
- ☞ Differences between fluxes: $\mathcal{I}_{\alpha\beta} = s_\alpha - s_\beta$
- ☞ “Sum rule”

$$\mathcal{I}_{ab} + \mathcal{I}_{bc} + \mathcal{I}_{ca} = 0$$

Yukawa couplings

- ☞ Yukawa couplings are given by overlap integrals

$$Y_{ijk}(\widetilde{\zeta},\tau) = g\,\sigma_{abc} \int\limits_{\mathbb{T}^2} \mathrm{d}^2z\,\psi^{i,\mathcal{I}_{ab}}(z,\tau,\zeta_{ab})\,\psi^{j,\mathcal{I}_{ca}}(z,\tau,\zeta_{ca})\,\left(\psi^{k,\mathcal{I}_{cb}}(z,\tau,\zeta_{cb})\right)^*$$

gauge coupling

sign

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“Wilson lines”

Yukawa couplings

Cremades, Ibáñez & Marchesano [2004]

- ☞ Yukawa couplings be expressed as a sum of ϑ–functions

$$Y_{ijk}(\widetilde{\zeta},\tau) = \mathcal{N}_{abc}\,\mathrm{e}^{\frac{H(\widetilde{\zeta},\tau)}{2}} \sum_{m\in\mathbb{Z}_{\mathcal{I}_{bc}}} \delta_{k,i+j+\mathcal{I}_{ab}\,m} \cdot \vartheta\begin{bmatrix}\frac{\mathcal{I}_{ca}i-\mathcal{I}_{ab}j+\mathcal{I}_{ab}\mathcal{I}_{ca}m}{-\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}}\\ 0\end{bmatrix}(\widetilde{\zeta},\tau\,|\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}|)$$

$$\mathcal{N}_{abc} = g\,\sigma_{abc}\,\left(\frac{2\,\mathrm{Im}\,\tau}{\mathcal{A}^2}\right)^{1/4}\left|\frac{\mathcal{I}_{ab}\mathcal{I}_{ca}}{\mathcal{I}_{bc}}\right|^{1/4}$$

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$$\cdot\, \vartheta \begin{bmatrix} \frac{\mathcal{I}_{ca} i - \mathcal{I}_{ab} j + \mathcal{I}_{ab}\mathcal{I}_{ca} m}{-\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}} \\ 0 \end{bmatrix} \left(\widetilde{\zeta}, \tau\, |\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}|\right)$$

"collective" Wilson line

$$\widetilde{\zeta} := -\mathcal{I}_{ab}\,\mathcal{I}_{ca}\,(\zeta_{ca} - \zeta_{ab}) = d^{\alpha\beta\gamma}\, s_\alpha\, \zeta_\alpha\, \mathcal{I}_{\beta\gamma}$$

$$\text{w/ } d^{\alpha\beta\gamma} = \begin{cases} 1 & \text{if } \{\alpha,\beta,\gamma\} \text{ is even perm. of } \{1,2,3\} \\ 0 & \text{otherwise} \end{cases}$$

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$$\begin{aligned}\frac{H(\widetilde{\zeta},\tau)}{2} &:= \frac{\pi\mathrm{i}}{\mathrm{Im}\,\tau}\left(\mathcal{I}_{ab}\,\zeta_{ab}\;\mathrm{Im}\,\zeta_{ab}+\mathcal{I}_{bc}\,\zeta_{bc}\;\mathrm{Im}\,\zeta_{bc}+\mathcal{I}_{ca}\,\zeta_{ca}\;\mathrm{Im}\,\zeta_{ca}\right)\\ &= \frac{\pi\mathrm{i}}{\mathrm{Im}\,\tau}\,|\mathcal{I}_{ab}\,\mathcal{I}_{bc}\,\mathcal{I}_{ab}|^{-1}\,\frac{\widetilde{\zeta}\;\mathrm{Im}\,\widetilde{\zeta}}{\mathrm{Im}\,\tau}\end{aligned}$$

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Abel & Owen [2004]

- ☞ Obviously no sum for $\mathcal{I}_{bc}=1$
- ☞ There might still be a sum for $\gcd(\mathcal{I}_{ab},\mathcal{I}_{ca},\mathcal{I}_{bc})=1$

Almumin, Chen, Knapp-Pérez, Ramos-Sánchez, M.R. & Shukla [2021]

Yukawa couplings for general flux parameters

Almumin, Chen, Knapp-Pérez, Ramos-Sánchez, M.R. & Shukla [2021]

- Using elementary number theory one can reduce the Yukawa coupling to a single ϑ–function

$$Y_{ijk}(\widetilde{\zeta},\tau) = \mathcal{N}_{abc}\,\mathrm{e}^{\frac{H(\widetilde{\zeta},\tau)}{2}}\,\Delta^{(d)}_{i+j,k} \cdot \vartheta\begin{bmatrix}\frac{\mathcal{I}'_{ca}\,i-\mathcal{I}'_{ab}\,j+\mathcal{I}'_{ca}\,(\mathcal{I}'_{ab})^{\phi\left(|\mathcal{I}'_{bc}|\right)}\,(k-i-j)}{\lambda}\\ 0\end{bmatrix}\left(\frac{\widetilde{\zeta}}{d},\lambda\,\tau\right)$$

Euler ϕ–function

$\lambda = \mathrm{lcm}\bigl(|\mathcal{I}_{ab}|,|\mathcal{I}_{ca}|,|\mathcal{I}_{bc}|\bigr)$

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$$\Delta^{(d)}_{i+j,k} := \begin{cases} 1\,, & \text{if } i+j=k \mod d\\ 0\,, & \text{otherwise}\end{cases}$$

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$$\mathcal{I}'_{ij} = \mathcal{I}_{ij}/d$$

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- Only $\mathrm{lcm}\big(|\mathcal{I}_{ab}|,|\mathcal{I}_{ca}|,|\mathcal{I}_{bc}|\big)$ independent coupling, e.g. a model with $\big(\mathcal{I}_{ab},\mathcal{I}_{ca},\mathcal{I}_{bc}\big)=(1,2,-3)$ has as many independent couplings as a model with $\big(\mathcal{I}_{ab},\mathcal{I}_{ca},\mathcal{I}_{bc}\big)=(3,3,-6)$

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bottom–line:

Magnetized tori with $\lambda = \mathrm{lcm}(\#$ of flavors$)$ exhibit a $\widetilde{\Gamma}_{2\lambda}$ modular flavor symmetry

Metaplectic transformations

cf. also Liu, Yao, Qu & Ding [2020]

☞ Double cover of $\mathrm{SL}(2,\mathbb{Z})$: the so–called metaplectic group $\widetilde{\Gamma}=\mathrm{Mp}(2,\mathbb{Z})$

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- Double cover of $\mathrm{SL}(2,\mathbb{Z})$: the so–called metaplectic group $\widetilde{\Gamma}=\mathrm{Mp}(2,\mathbb{Z})$
- Generators $\widetilde{S}$ and $\widetilde{T}$ of $\widetilde{\Gamma}$ satisfy the presentation

$$\widetilde{S}^8=(\widetilde{S}\,\widetilde{T})^3=\mathbb{1}\quad\text{and}\quad\widetilde{S}^2\widetilde{T}=\widetilde{T}\,\widetilde{S}^2$$

- Our choice

$$\widetilde{S}=(S,-\sqrt{-\tau})\quad\text{and}\quad\widetilde{T}=(T,+1)\;,\qquad S,T\in\Gamma$$

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$$\widetilde{\Gamma}=\left\{\widetilde{\gamma}=(\gamma,\varphi(\gamma,\tau))\mid\gamma\in\Gamma,\;\varphi(\gamma,\tau)=\pm(c\,\tau+d)^{1/2}\right\}$$

- Multiplication rule

$$(\gamma_1,\,\varphi(\gamma_1,\tau))\,(\gamma_2,\,\varphi(\gamma_2,\tau))=(\gamma_1\gamma_2,\,\varphi(\gamma_1,\gamma_2\,\tau)\varphi(\gamma_2,\tau))$$

Metaplectic flavor symmetries from magnetized tori

☞ Yukawa couplings transform nontrivially under modular transformations

Metaplectic flavor symmetries from magnetized tori

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- ☞ Naive expectation: zero–mode wavefunctions get mapped to a linear combination of zero–mode wavefunctions

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- ☞ However, not true for odd flux parameters M

Ohki, Uemura & Watanabe [2020], Kikuchi, Kobayashi, Takada, Tatsuishi & Uchida [2020]

Metaplectic flavor symmetries from magnetized tori

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  Ohki, Uemura & Watanabe [2020], Kikuchi, Kobayashi, Takada, Tatsuishi & Uchida [2020]
- ☞ Yet this does not indicate an inconsistency. Rather, the true transformation involves either Scherk–Schwarz phases or equivalently a shift of the so–called Wilson line parameter ζ

  Kikuchi, Kobayashi & Uchida [2021], Almumin, Chen, Knapp-Pérez, Ramos-Sánchez, M.R. & Shukla [2021], Tatsuta [2021]

Connection to bottom–up model building

- ☞ Metaplectic flavor symmetries have been studie in bottom–up model building

Liu, Yao, Qu & Ding [2020], Ding, Feruglio & Liu [2021]

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- ☞ Realistic fits of the neutrino masses have been achieved in the bottom–up approach. . .

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Connection to bottom–up model building

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➥ More efforts required to endow phenomenologically promising bottom–up constructions with a UV completion

Eclectic
flavor
symmetries

Eclectic flavor symmetries in heterotic orbifolds

Nilles, Ramos–Sánchez & Vaudrevange [2021], Baur, Kade, Nilles, Ramos-Sanchez & Vaudrevange [2021]

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- Roughly speaking

$$G_{\text{eclectic}} = G_{\text{traditional}} \cup G_{\text{modular}} = \Delta(54) \cup T' = \mathsf{GL}(2,3)$$

double cover of A_4

$\mathbb{Z}_3$ orbifold

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  - R symmetries (including non–Abelian discrete R symmetries)
  - $\mathcal{CP}$ symmetries and $\mathcal{CP}$–like transformations

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☞ So far no explicit realization of successful bottom–up models

Quasi–eclectic flavor symmetries

Chen, Knapp-Perez, Ramos-Hamud, Ramos-Sanchez, M.R., & Shukla [2021]

- ☞ Using some of the ingredients of the eclectic scheme one may make bottom–up models predictive

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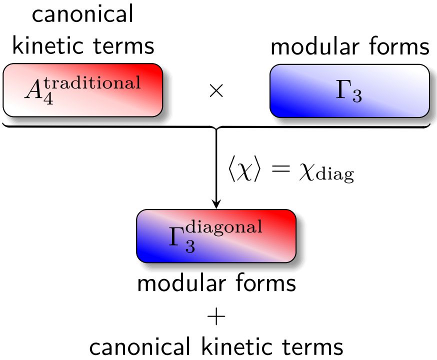
- ☞ Basic ingredient: representations which transform nontrivially under both $G_{\text{traditional}}$ and G_{modular}
- ☞ The "diagonal" subgroup of $G_{\text{traditional}}$ and G_{modular} can be sufficiently predictive



canonical kinetic terms

modular forms

$$\underbrace{A_4^{\text{traditional}} \times \Gamma_3} \xrightarrow{\langle\chi\rangle = \chi_{\text{diag}}} \Gamma_3^{\text{diagonal}}$$

modular forms

+

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Generation Flow

History

- ☞ Nelson and Strassler showed in field theory that one can obtain one chiral SM generation from states that are vector–like under G_{SM}

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this talk:

Stringy realization of Nelson–Strassler and Razamat–Tong scenarios

The Nelson–Strassler and Razamat–Tong scenarios

Strassler [1996], Nelson & Strassler [1997], Razamat & Tong [2021]

- ☞ Supersymmetric model with gauge group $\mathrm{SU}(2)_s$ and a global (or weakly gauged) $\mathrm{SU}(6) \subset \mathrm{SU}(5)_{\mathrm{GG}} \times \mathrm{U}(1)$ symmetry and matter content

$$(\overline{\mathbf{6}},\mathbf{2}) \oplus (\mathbf{15},\mathbf{1}) \to (\overline{\mathbf{5}},\mathbf{2})_1 \oplus (\mathbf{1},\mathbf{2})_{-5} \oplus (\mathbf{10},\mathbf{1})_{-2} \oplus (\mathbf{5},\mathbf{1})_4$$

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- ☞ When $\mathrm{SU}(2)_s$ undergoes "s–confinement", the $(\overline{\mathbf{6}}, \mathbf{2})$ form "baryons" transforming as $(\overline{\mathbf{6}} \times \overline{\mathbf{6}})_{\mathrm{a}} = \overline{\mathbf{15}} \to \overline{\mathbf{10}} + \overline{\mathbf{5}}$

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- Given appropriate trilinear couplings the vector–like states can acquire mass

Razamat & Tong [2021]

The $4 \rightsquigarrow 3$ model

- ☞ Unconfined spectrum

#	irrep	label
4	$(\mathbf{10},\mathbf{1})$	T
2	$(\overline{\mathbf{5}},\mathbf{1})$	$\overline{F}$
1	$(\overline{\mathbf{5}},\mathbf{2})$	$\overline{F}'$
1	$(\mathbf{1},\mathbf{2})$	ϕ
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The $2 \rightsquigarrow 3$ model

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4	$(\overline{\mathbf{5}},\mathbf{1})$	$\overline{F}$
1	$(\mathbf{5},\mathbf{2})$	F'
1	$(\mathbf{1},\mathbf{2})$	ϕ

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1	$(\mathbf{5},\mathbf{2})$	F'
1	$(\mathbf{1},\mathbf{2})$	ϕ

☞ Confined spectrum

#	irrep	label
3	$(\mathbf{10},\mathbf{1})$	$T,\mathcal{T}$
3	$(\overline{\mathbf{5}},\mathbf{1})$	$\overline{F}$
1	$(\overline{\mathbf{5}},\mathbf{1})$	$\overline{F}$
1	$(\mathbf{5},\mathbf{1})$	$\mathcal{F}$

Comment on the representation content and symmetries

☞ Is $(\overline{\mathbf{6}},\mathbf{2})\oplus(\mathbf{15},\mathbf{1})$ under $\mathrm{SU}(6)\times\mathrm{SU}(2)_s$ contrived?

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Ramos-Sánchez, M.R., Shirman, Shukla & Waterbury [2021]

$$\begin{aligned}\mathsf{E}_6 &\to \mathrm{SU}(6)\times\mathrm{SU}(2)_s\\ &\to \mathrm{SU}(5)\times\mathrm{SU}(2)_s\times\mathrm{U}(1)\\ \mathbf{27} &\to (\overline{\mathbf{6}},\mathbf{2})\oplus(\mathbf{15},\mathbf{1})\\ &\to (\overline{\mathbf{5}},\mathbf{2})_1\oplus(\mathbf{1},\mathbf{2})_{-5}\oplus(\mathbf{10},\mathbf{1})_{-2}\oplus(\mathbf{5},\mathbf{1})_4\end{aligned}$$

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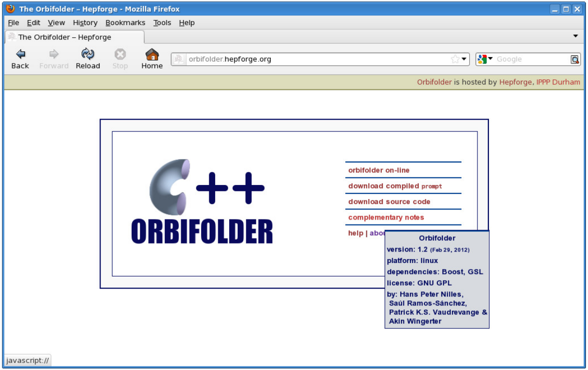
- ☞ So it is not surprising that anomalies cancel
- ☞ However, are there reasons why $\mathrm{SU}(2)_s$ can be more strongly coupled than $\mathrm{SU}(6)$?

Stringy realization

Ramos-Sánchez, M.R., Shirman, Shukla & Waterbury [2021]

☞ Scan $\mathrm{E}_8\times\mathrm{E}_8$ heterotic orbifolds for models in which $\mathrm{SU}(2)_s$ and $\mathrm{SU}(6)$ come from different E_8 factors

Nilles, Ramos-Sánchez, Vaudrevange & Wingerter [2012]



Orbifolder is hosted by Hepforge, IPPP Durham

orbifolder on-line

download compiled prompt

download source code

complementary notes

help | abo

Orbifolder
version: 1.2 (Feb 29, 2012)
platform: linux
dependencies: Boost, GSL
license: GNU GPL
by: Hans Peter Nilles,
Saúl Ramos-Sánchez,
Patrick K.S. Vaudrevange &
Akin Wingerter

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bottom–line:

String theory appears to host models exhibiting generation flow $\curvearrowright$ it is not enough to count the generations at the tree level

Implications for string model building

☞ Chiral spectrum may change in the low–energy effective theory

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- ☞ Many more models

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- ☞ Better understanding of the QFT dynamics desirable

R symmetries
for the
MSSM

R symmetries in heterotic orbifolds

- ☞ R symmetries can be derived from the so–called H–momentum selection rule

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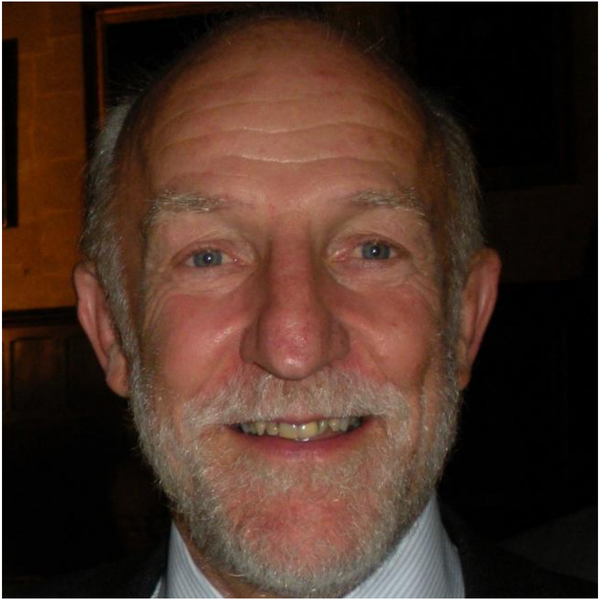
☞ This talk: focus on implications of $\mathbb{Z}_4^R$ for MSSM model building



www.physics.ox.ac.uk/

$\mathbb{Z}_4^R$ features

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- ☞ $\mathbb{Z}_2 \subset \mathbb{Z}_4^R$ is anomaly–free without GS axion and exact, and is in fact nothing but the well–known R parity or matter parity

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  Witten [1981] ,. . . ,Shadmi & Shirman [2000] ,. . . ,Intriligator, Seiberg & Shih [2006]

$\mathbb{Z}_4^R$ summarized

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effective neutrino mass operator

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Summary

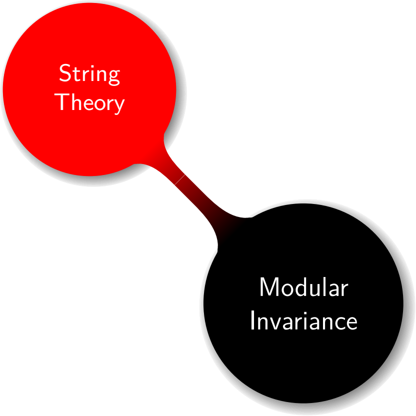
and

outlook

Ever–growing importance of modular invariance

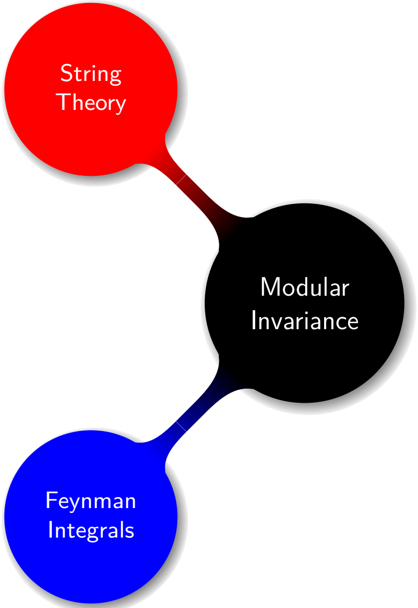
Modular
Invariance

Ever–growing importance of modular invariance


String Theory

Modular Invariance

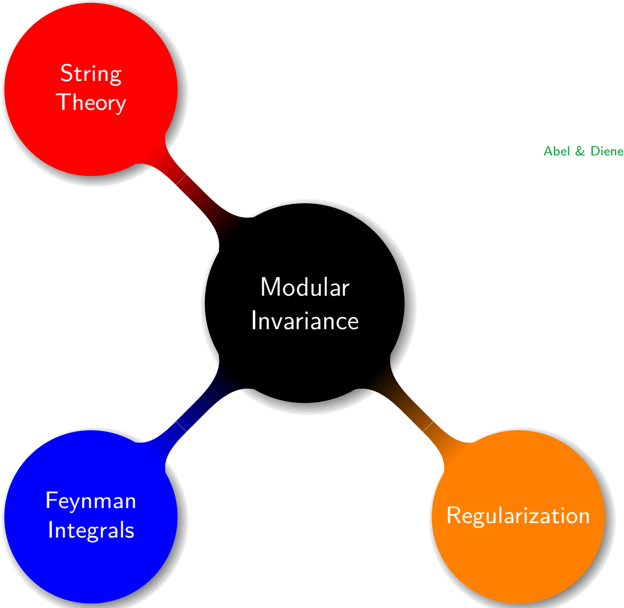
Ever–growing importance of modular invariance


String Theory

Modular Invariance

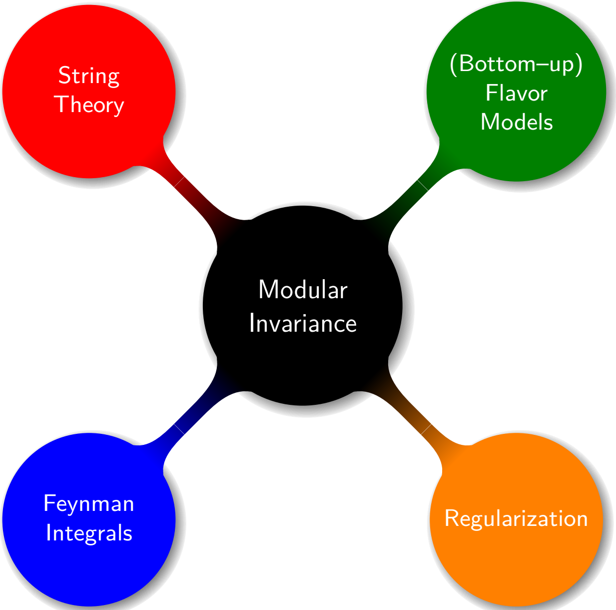
Feynman Integrals

Ever–growing importance of modular invariance



Abel & Dienes [2021]

Ever–growing importance of modular invariance



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- ☞ So–called eclectic symmetries contain all the above, and appear in explicit string models

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- ☞ Our past model scans appear even more incomplete than previously appreciated due to the possibility of phenomena such as generation flow
- ☞ Composites such as those appearing in generation flow scenarios may come with modular weights of the type used in bottom–up models

Outlook

- ☞ Nonsupersymmetric model building (e.g. modular flavor symmetries do not seem to necessarily require supersymmetry)
- ☞ Other new ideas

UNIVERSITÄT BONN

bctp Bethe Center for Theoretical Physics

Bethe Forum

Modular Flavor Symmetries

May 2 - 6, 2022

Bonn, Germany

Speakers include

Mu-Chun Chen
Gui-Jun Ding
Claudia Hagedorn*
Stephen King
Hajime Otsuka
Sergey Petcov
Pierre Ramond
Saúl Ramos-Sánchez
Andrea Romanino*
Morimitsu Tanimoto
Andreas Trautner
Hikaru Uchida*

*to be confirmed

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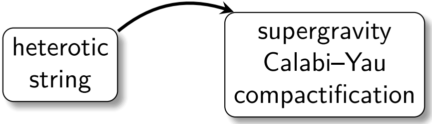
Possible discussion topic: smooth compactifications

- ☞ Famous result: one can obtain Calabi–Yau compactifications from string theory

Candelas, Horowitz, Strominger & Witten [1985]



heterotic string

supergravity Calabi–Yau compactification

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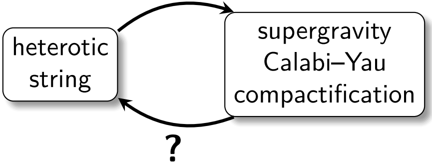

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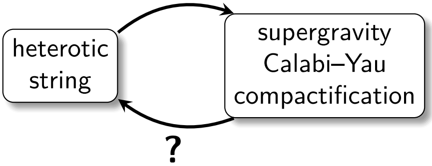

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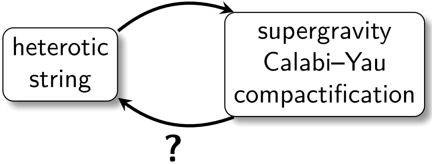

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provocative question:

Is it clear that all Calabi–Yau models are string compacifications? If not, how can one tell which of them are?

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provocative question:

Is the absence of chiral vacua just a feature of the simple model considered by Buchmüller et al., or is it a more general problem?

Thank you very much!

GUT scale

vs.

string scale

GUT vs. string scale

e.g. Witten [1996]

- Supergravity description of heterotic string

$$\mathscr{L} \;=\; -\int \mathsf{d}^{10}x\,\sqrt{g}\,g_s\left(\frac{4}{(\alpha')^4}\,R+\frac{1}{(\alpha')^3}\,\mathrm{tr}\,F^2+\dots\right)$$

$$g_s = \mathrm{e}^{-\phi}$$

$$\alpha' = \frac{1}{2\pi\,M_{\text{string}}^2}$$

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curvature ↷ gravity

field strength ↷ gauge interactions

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➥ Effective 4D action after compactification

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- Relation between Newton's constant and gauge coupling

$$G_{\mathrm{N}} = \frac{\alpha_{\mathrm{GUT}}\,\alpha'}{4} = \frac{\alpha_{\mathrm{GUT}}}{8\pi\,M_{\mathrm{string}}^2} \simeq \frac{1}{(24\,M_{\mathrm{string}})^2} \overset{!}{=} \frac{1}{M_{\mathrm{P}}^2}$$

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$$G_{\mathrm{N}} = \frac{\mathrm{e}^{2\phi}\,(\alpha')^4}{64\pi\,V} \quad \text{and} \quad \alpha_{\mathrm{GUT}} = \frac{\mathrm{e}^{2\phi}\,(\alpha')^3}{16\pi\,V}$$

- ➥ Relation between Newton's constant and gauge coupling

$$G_{\mathrm{N}} = \frac{\alpha_{\mathrm{GUT}}\,\alpha'}{4} = \frac{\alpha_{\mathrm{GUT}}}{8\pi\,M_{\mathrm{string}}^2} \simeq \frac{1}{(24\,M_{\mathrm{string}})^2} \overset{!}{=} \frac{1}{M_{\mathrm{P}}^2}$$

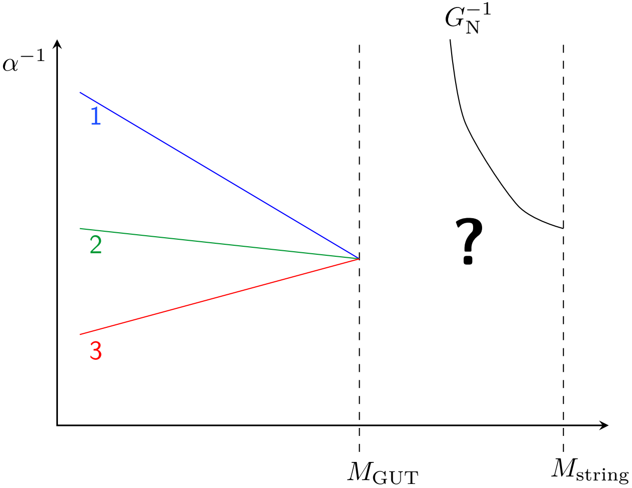
- ➥ Well–known problem: using $\alpha_{\mathrm{GUT}} = g_{\mathrm{GUT}}^2/4\pi \simeq 1/25$

$$M_{\mathrm{string}} \simeq 9\cdot 10^{17}\,\mathrm{GeV} \quad \text{and} \quad M_{\mathrm{GUT}} \simeq (2-3)\cdot 10^{16}\,\mathrm{GeV}$$

$$\curvearrowright \frac{M_{\mathrm{string}}}{M_{\mathrm{GUT}}} \sim 30\dots 40$$

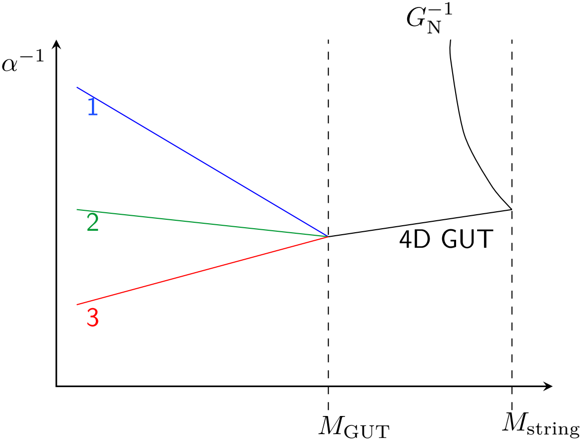
Gauge unification: GUT vs. string scale

cf. Dienes [1997]



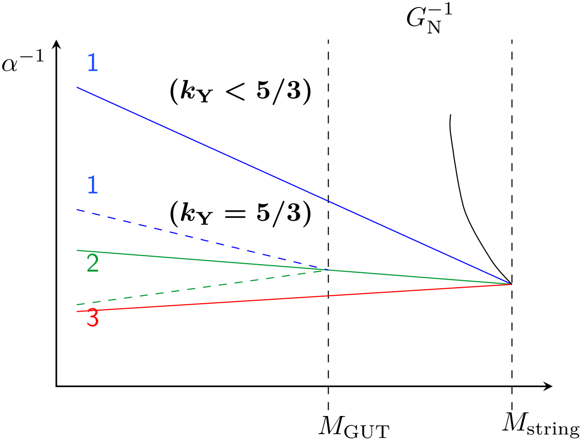
Gauge unification: 4D GUT picture

cf. Dienes [1997]



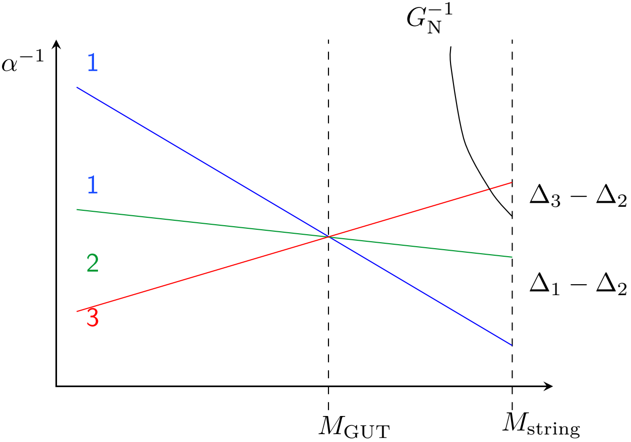
Gauge unification: changing hypercharge normalization

cf. Ibáñez [1993]



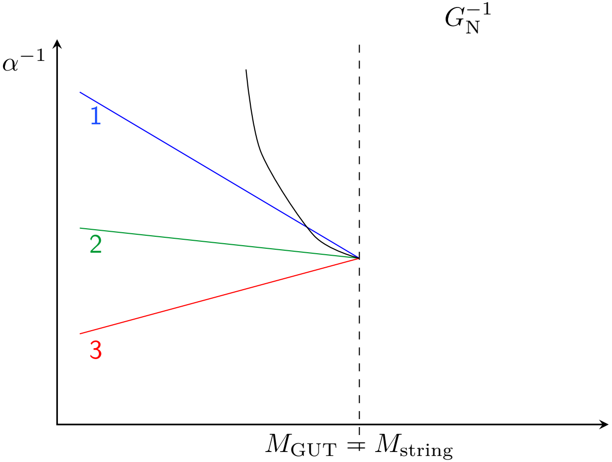
Gauge unification: string thresholds

cf. Nilles & Stieberger [1997]



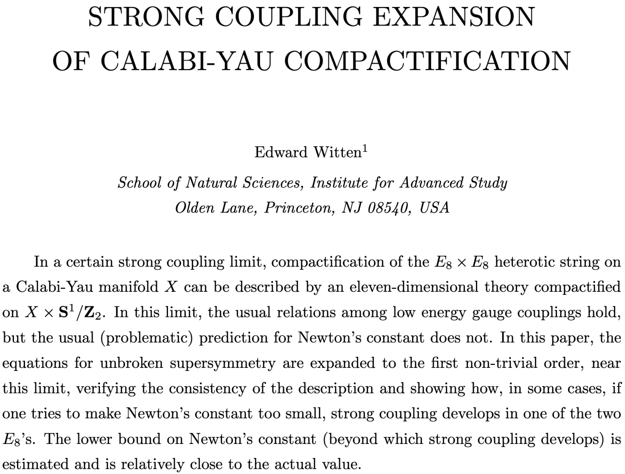
Gauge unification: M–theory or type I string

Witten [1996]



GUT vs. string scale: M–theory

Witten [1996]



STRONG COUPLING EXPANSION OF CALABI-YAU COMPACTIFICATION

Edward Witten[1]

School of Natural Sciences, Institute for Advanced Study
Olden Lane, Princeton, NJ 08540, USA

In a certain strong coupling limit, compactification of the $E_8 \times E_8$ heterotic string on a Calabi-Yau manifold X can be described by an eleven-dimensional theory compactified on $X \times \mathbf{S}^1/\mathbf{Z}_2$. In this limit, the usual relations among low energy gauge couplings hold, but the usual (problematic) prediction for Newton's constant does not. In this paper, the equations for unbroken supersymmetry are expanded to the first non-trivial order, near this limit, verifying the consistency of the description and showing how, in some cases, if one tries to make Newton's constant too small, strong coupling develops in one of the two E_8's. The lower bound on Newton's constant (beyond which strong coupling develops) is estimated and is relatively close to the actual value.

Anisotropic compactifications

- ☞ However, Witten also mentions in a footnote

---

[3] Note that the problem might be ameliorated by considering an anisotropic Calabi-Yau, for instance one with a scale $\sqrt{\alpha'}$ in d directions and $1/M_{GUT}$ in $6-d$ directions (with some fairly severe restrictions on d and the Calabi-Yau manifold X to ensure that it is the large dimensions in X that control the GUT breaking), so that $V \sim (\alpha')^{d/2}/M_{GUT}^{6-d}$. The amelioration obtained this way, if too small, could possibly be combined with the strong coupling effect considered below.

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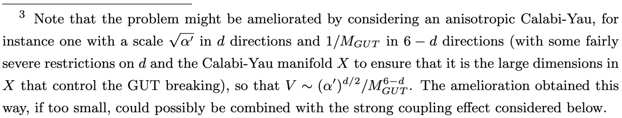

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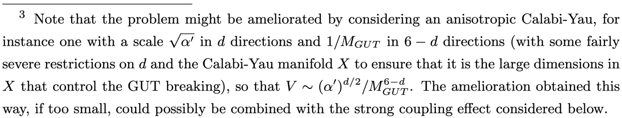

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- Anisotropic compactification
- Need string (rather than supergravity) description!

Orbifold GUT limits

- ☞ Anisotropic compactification may mitigate the discrepancy between M_{GUT} and M_{string}

Witten [1996]
...
Hebecker & Trapletti [2005]

Orbifold GUT limits

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☞ For example

$$\frac{\alpha_{\mathrm{GUT}}}{2} = \frac{g_{\mathrm{het}}^2}{(R\,m_{\mathrm{het}})^6} \to \frac{g_{\mathrm{het}}^2}{(R_{\mathrm{large}}\,m_{\mathrm{het}})\,(R_{\mathrm{small}}\,m_{\mathrm{het}})^5}$$

. . . works if $R_{\mathrm{large}}\,m_{\mathrm{het}} \sim 50$ or $R_{\mathrm{large}}^{-1} \sim 3 \cdot 10^{16}\,\mathrm{GeV}$

suspiciously close to M_{GUT}

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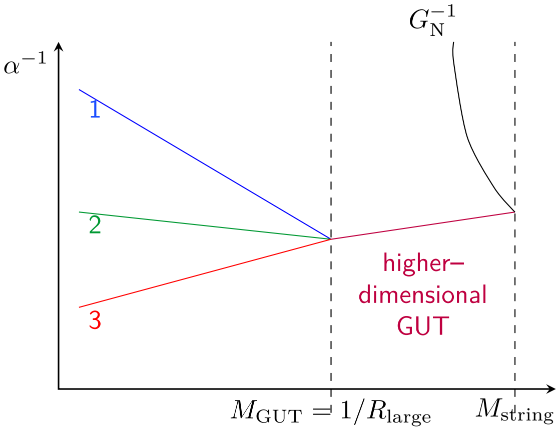
- ☞ Hebecker & Trapletti are a bit sceptical but their bound may be a bit too conservative (volume of orbifold ≠ volume of torus)
- ☞ In any case we need a complete string model in order to deal with the smaller directions

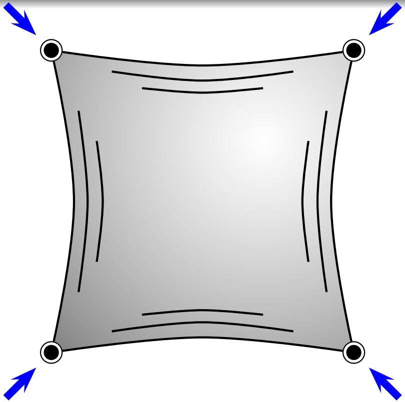
Gauge unification: orbifold GUT picture



Orbifold

basics

What is an orbifold?



☞ an orbifold is a space which is smooth/flat everywhere except for special (orbifold fixed) points

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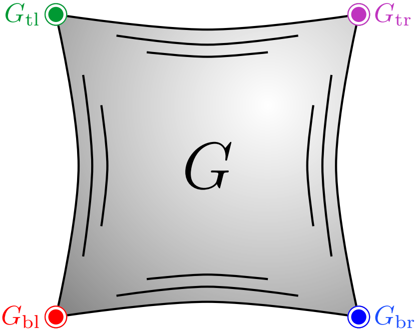

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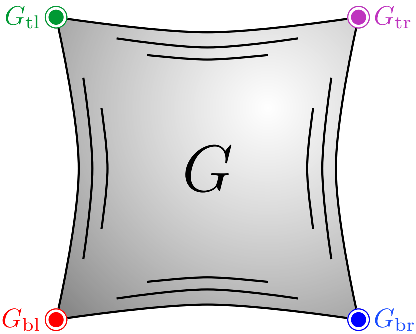

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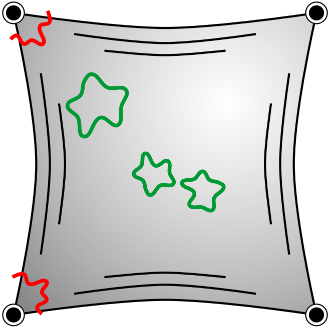
- ☞ 'bulk' gauge symmetry G is broken to (different) subgroups (local GUTs) at the fixed points
- ☞ low–energy gauge group : $G_{\text{low}-\text{energy}} = G_{\text{bl}} \cap G_{\text{br}} \cap G_{\text{tl}} \cap G_{\text{tr}}$

Strings on orbifolds

heterotic string	field theory
untwisted sector = strings closed on the torus	extra components of gauge fields
‘twisted’ sectors = strings which are only closed on the orbifold	‘brane fields’ (hard to understand in field–theoretical framework)



☞ (‘brane’) Fields living at a fixed point with a certain symmetry appear as complete multiplet of that symmetry

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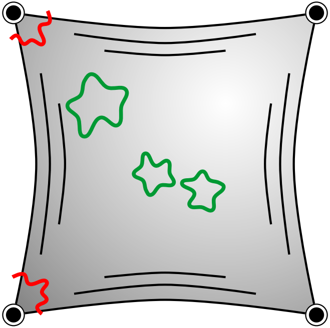

- ☞ ('brane') Fields living at a fixed point with a certain symmetry appear as complete multiplet of that symmetry
- ➥ e.g. if the electron lives at a point with SO(10) symmetry also u and d quarks live there

Some comments on orbifold history

- ☞ Very first stringy model of particle physics based on $\mathbb{Z}_3$ orbifold

  Ibáñez, Kim, Nilles & Quevedo [1987]

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☞ A rather common concern in many models: fractionally charged vector–like exotics

Non–local
GUT breaking

Orbifolds & Wilson lines

Ibáñez, Nilles & Quevedo [1987], Hall, Murayama & Nomura [2002a]

skip

☞ Local gauge embedding at fixed point f

$$V_f^I \;=\; k\,V_N^I + m_\alpha\,W_{n\alpha}^I$$

Orbifolds & Wilson lines

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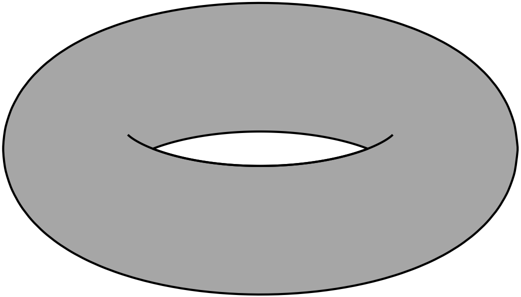

- Upshot: so–called discrete Wilson lines are differences between local shifts (and *not* Wilson lines in the usual sense)

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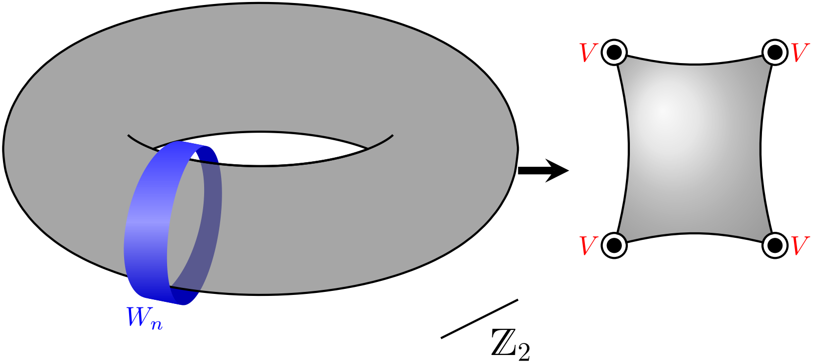

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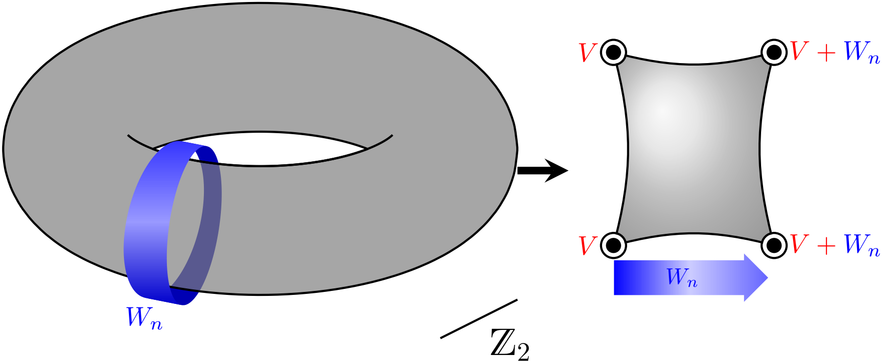

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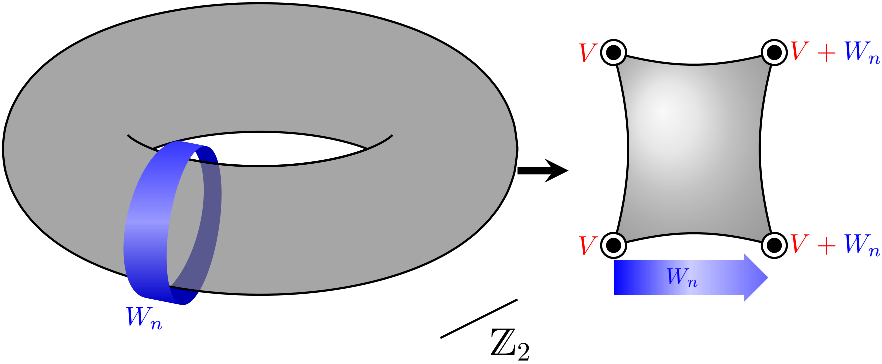

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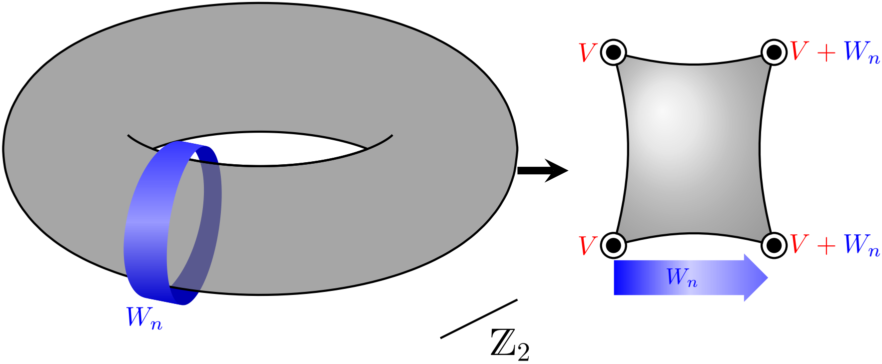

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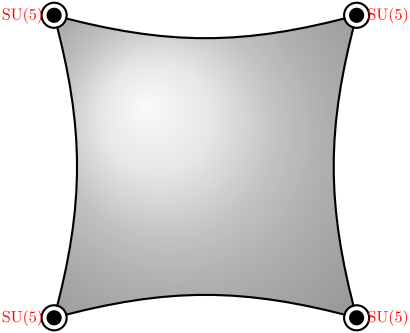
Local vs. non–local GUT breaking

Hall, Murayama & Nomura [2002a], Hebecker [2004]



❶ step: construct $\mathbb{T}^2/\mathbb{Z}_2$ orbifold which breaks $\mathrm{SU}(6)$ locally to $\mathrm{SU}(5)$

$$\mathbb{Z}_2 \quad : \quad (x_5, x_6) \;\to\; (-x_5, -x_6)$$

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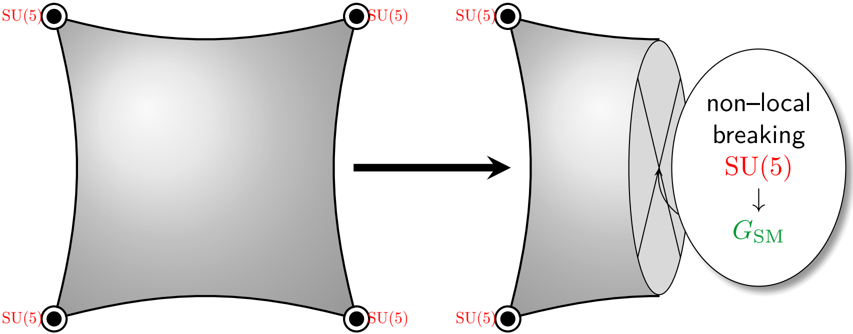


❶ step: construct $\mathbb{T}^2/\mathbb{Z}_2$ orbifold which breaks $\mathrm{SU}(6)$ locally to $\mathrm{SU}(5)$

❷ step: mod out a freely acting $\mathbb{Z}_2'$ symmetry which breaks $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$

$$\mathbb{Z}_2' \quad : \quad (x_5, x_6) \;\to\; (-x_5 + \pi\, R_5, -x_6 + \pi\, R_6)$$

Orbifold

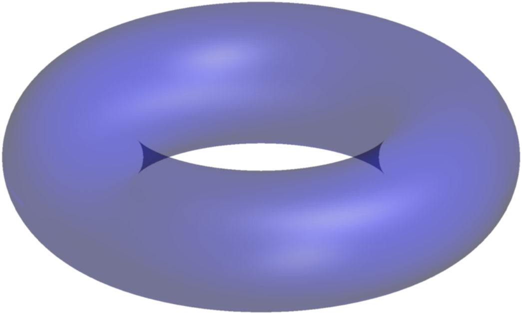
compactifications

of the

heterotic string

$\mathbb{Z}_2$ orbifold pillow

- ☞ Starting point: torus



$\mathbb{Z}_2$ orbifold pillow

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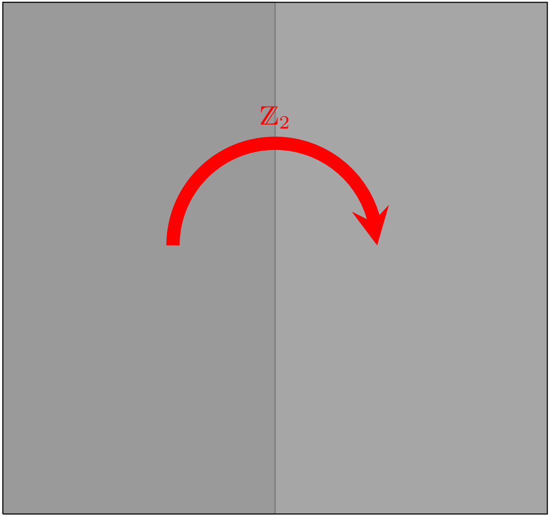

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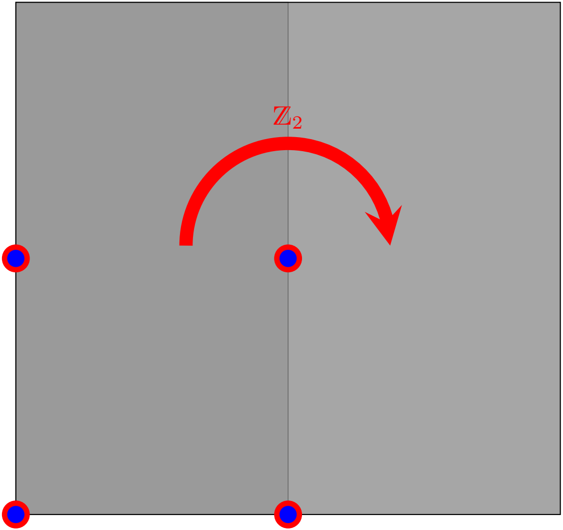

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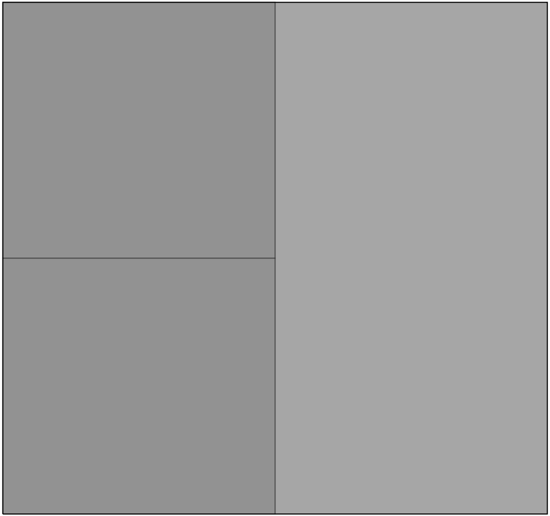

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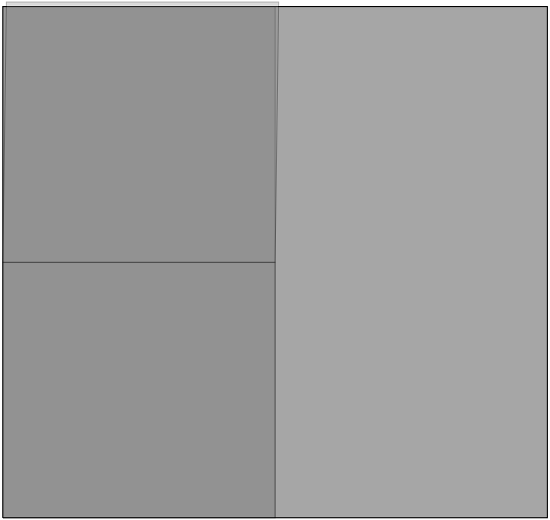

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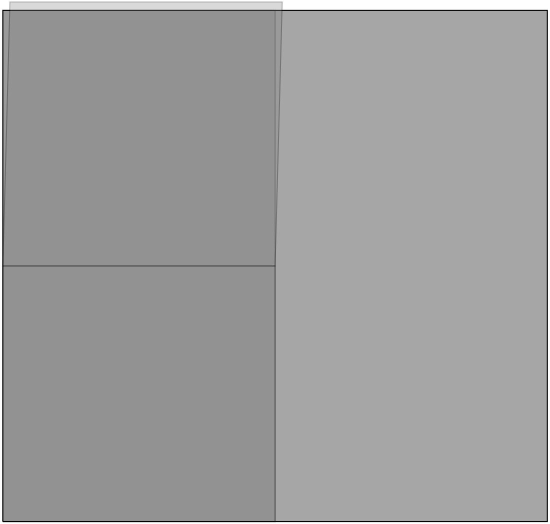

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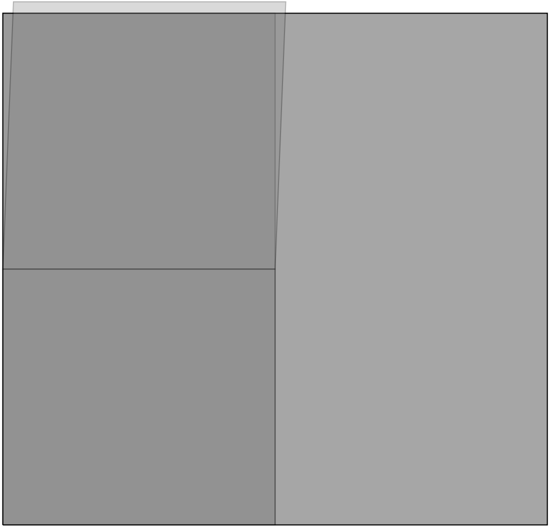

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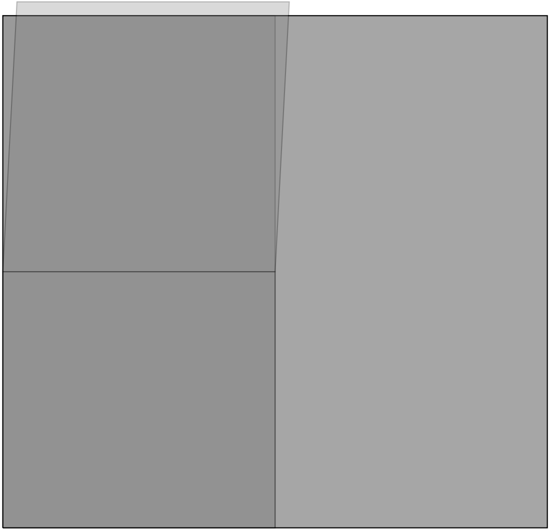

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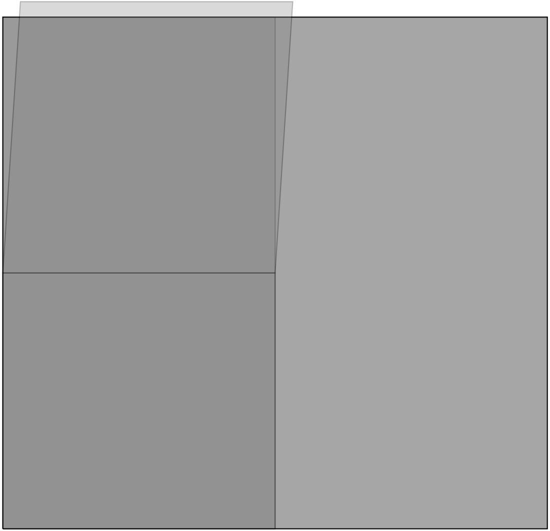

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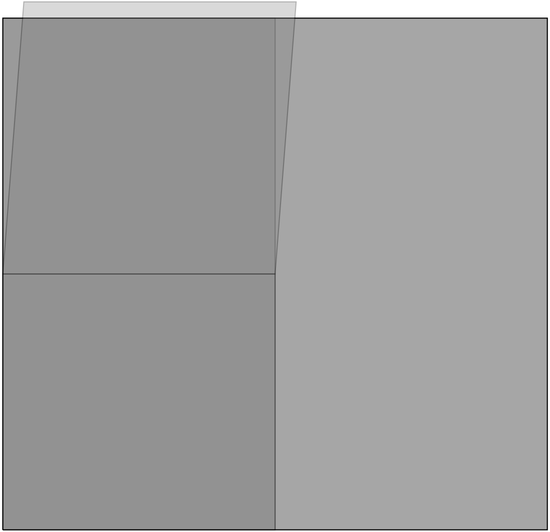

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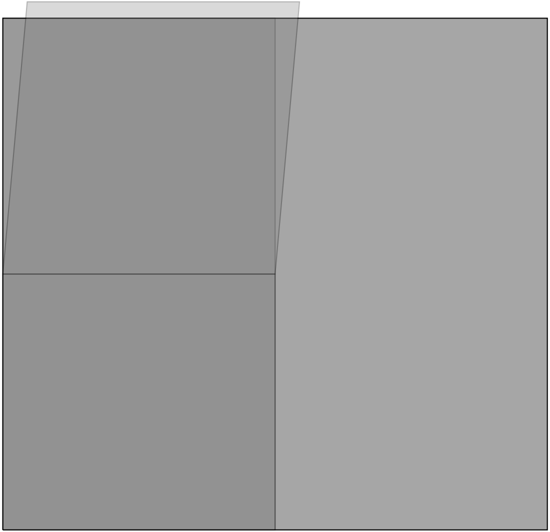

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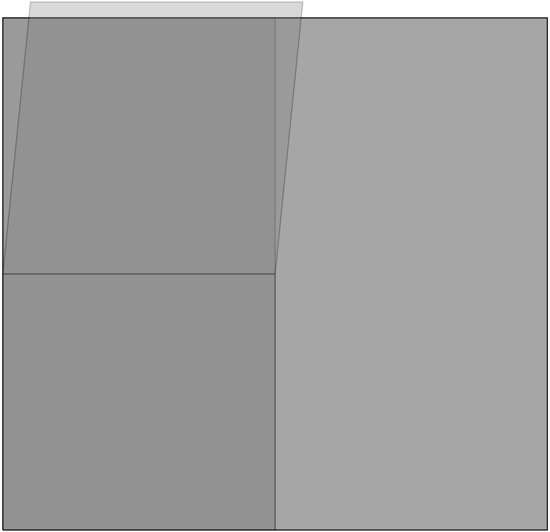

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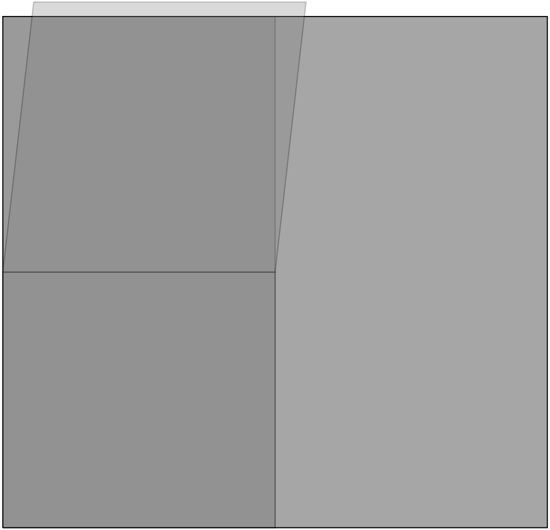

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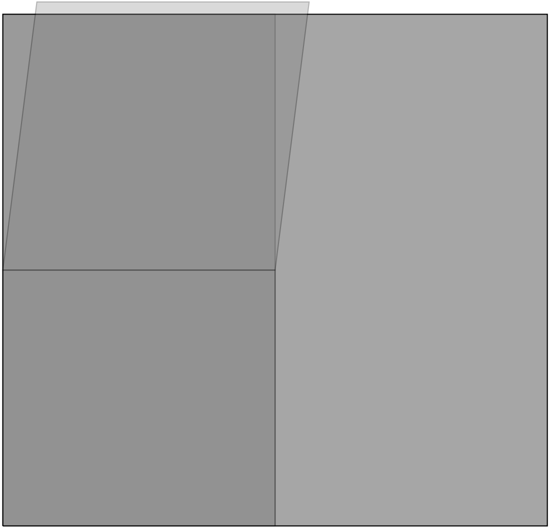
$\mathbb{Z}_2$ orbifold pillow



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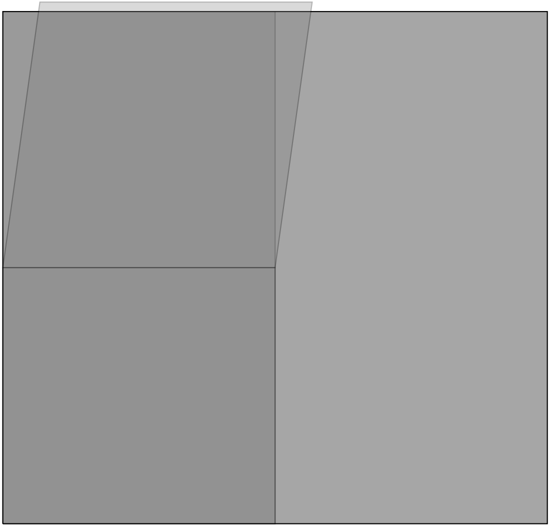

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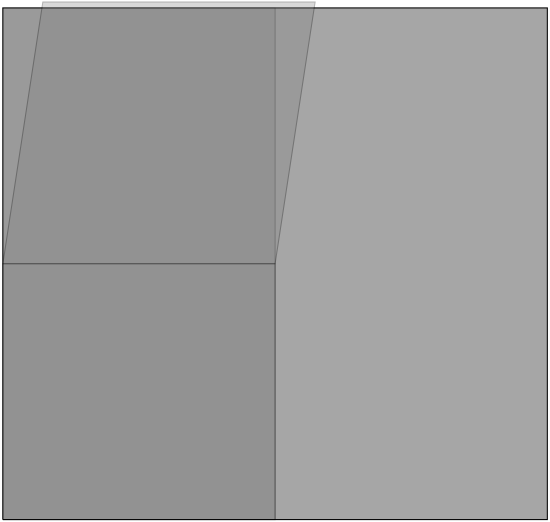

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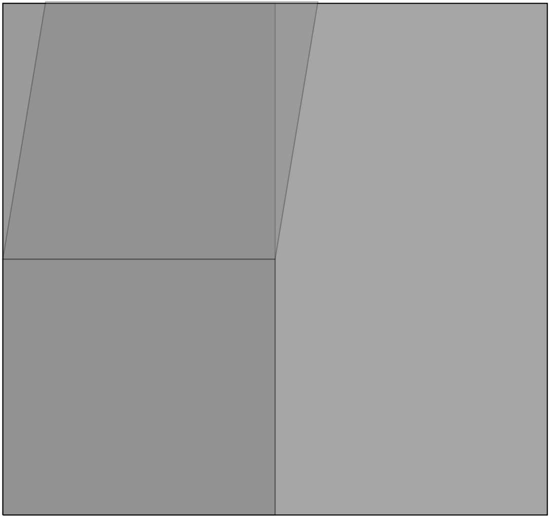

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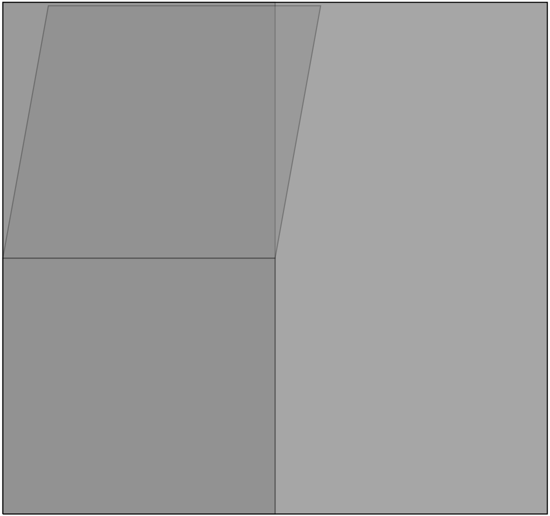

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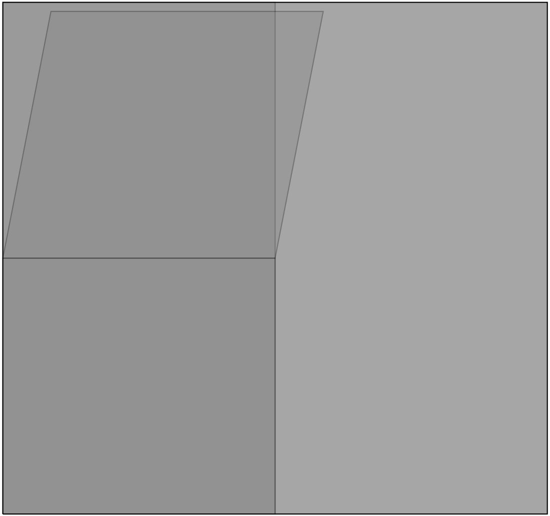

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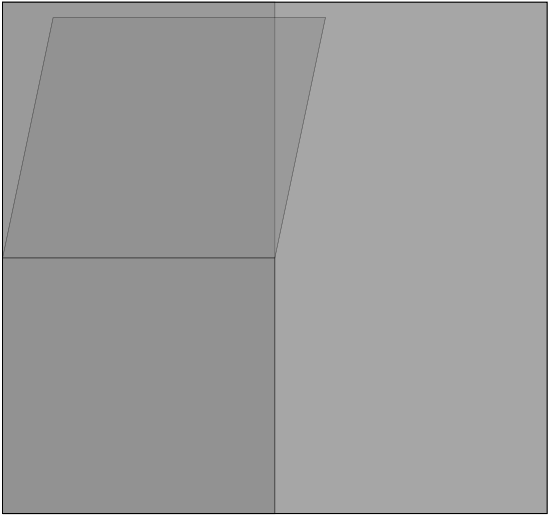

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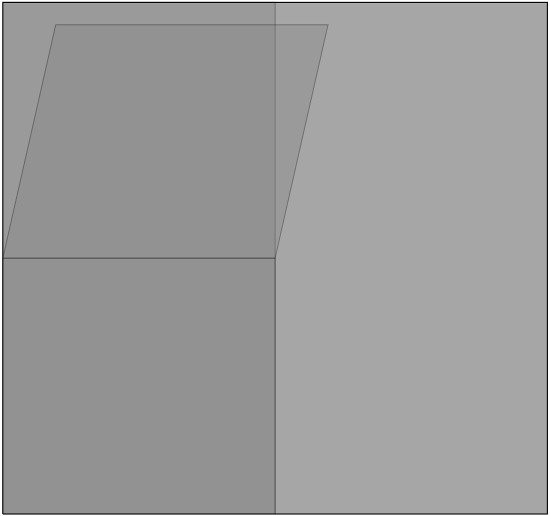

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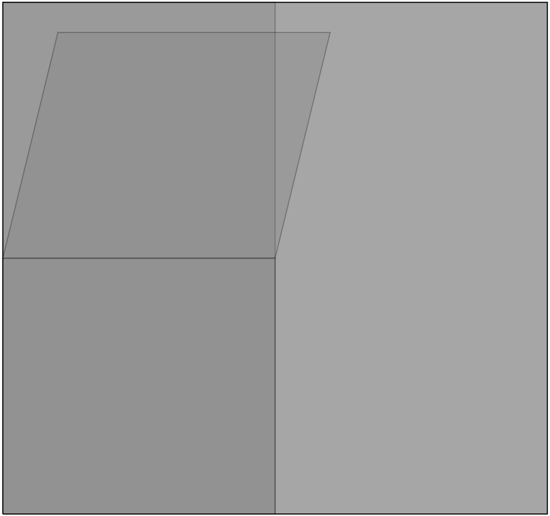

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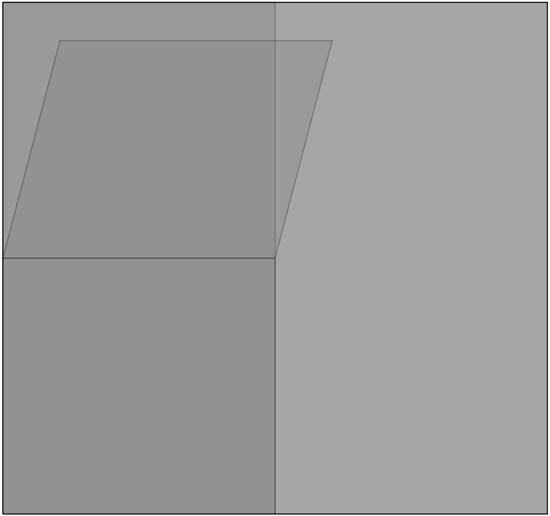

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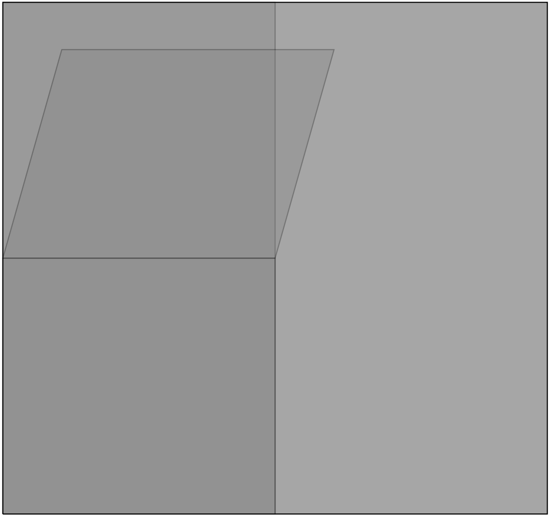

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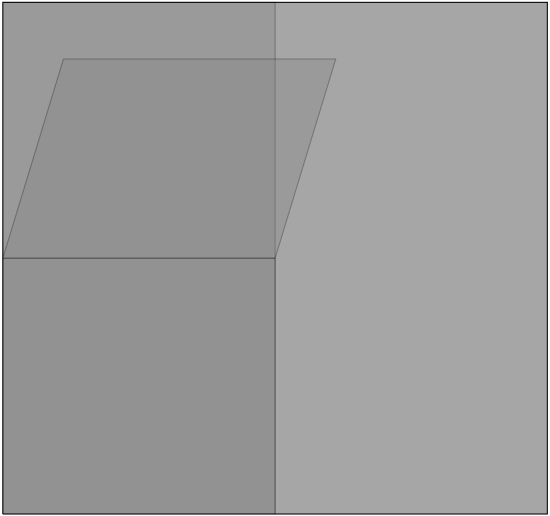

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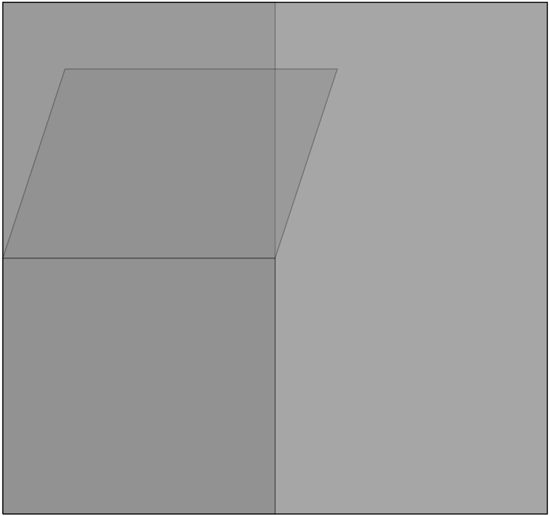

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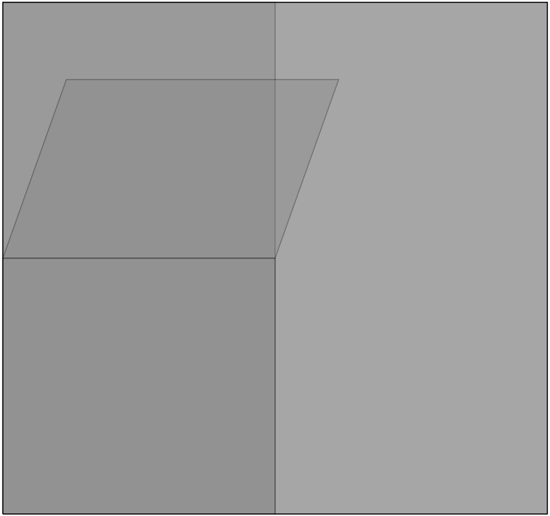

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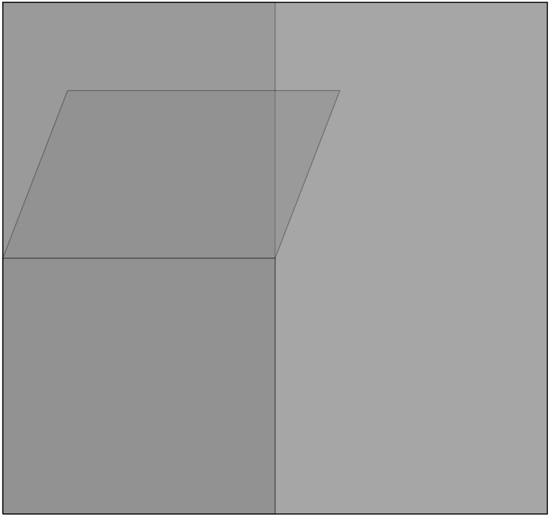

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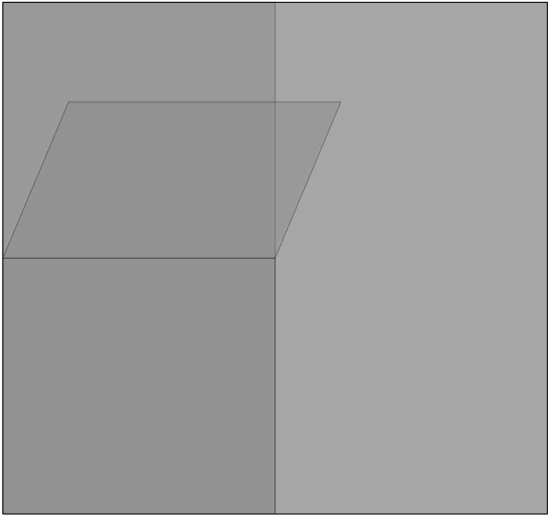

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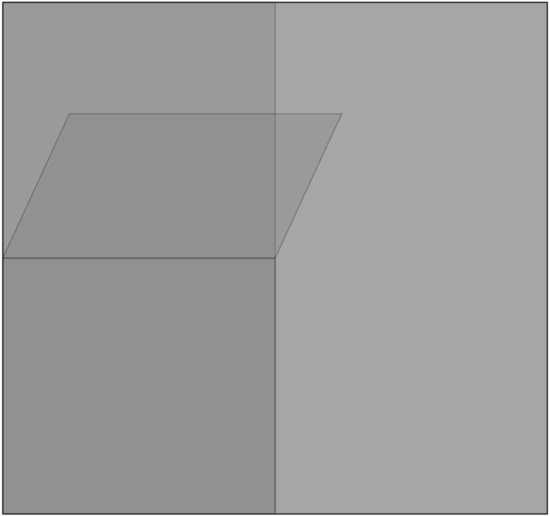

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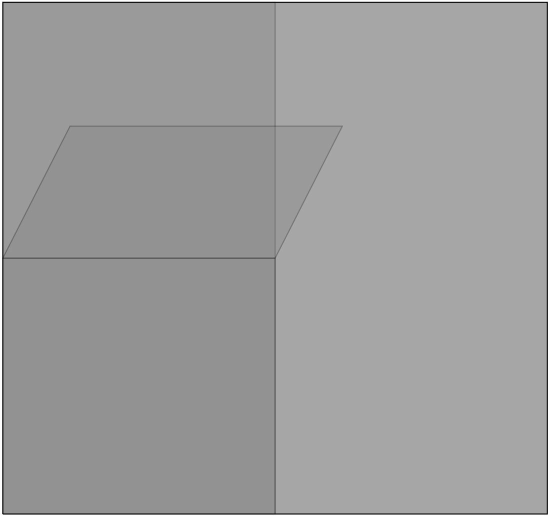

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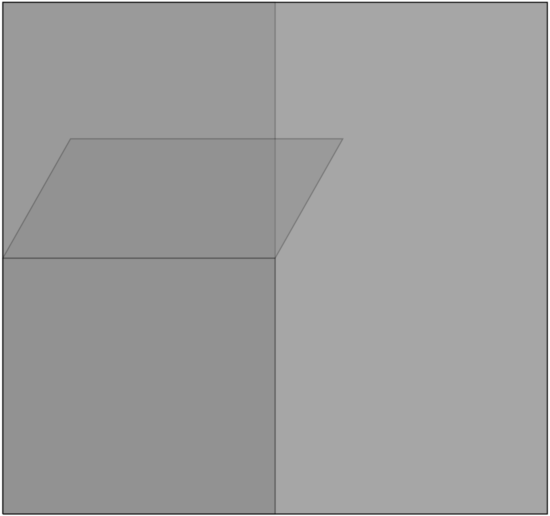

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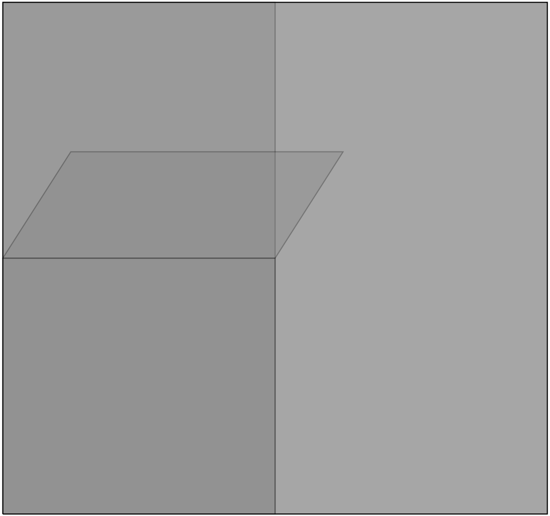

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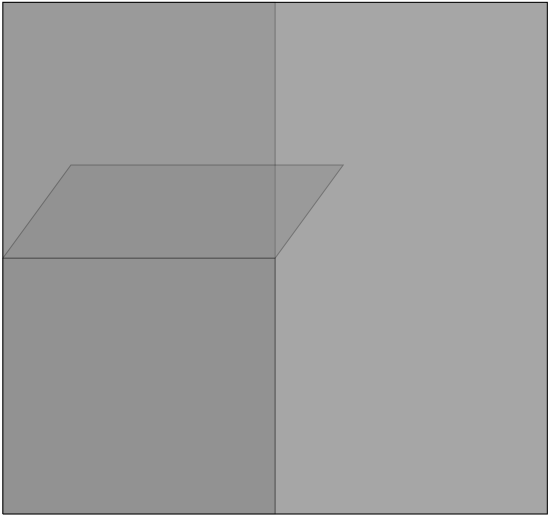

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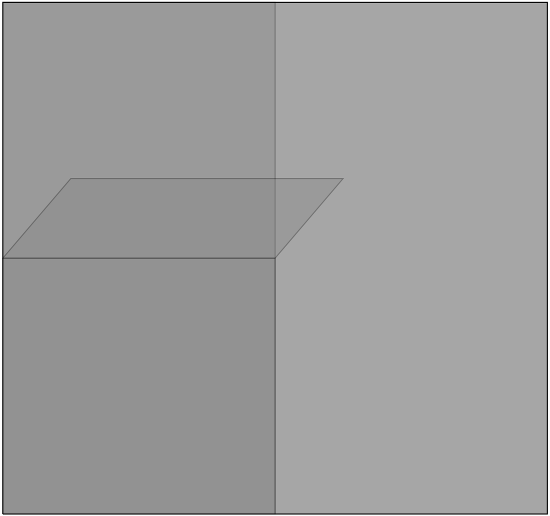

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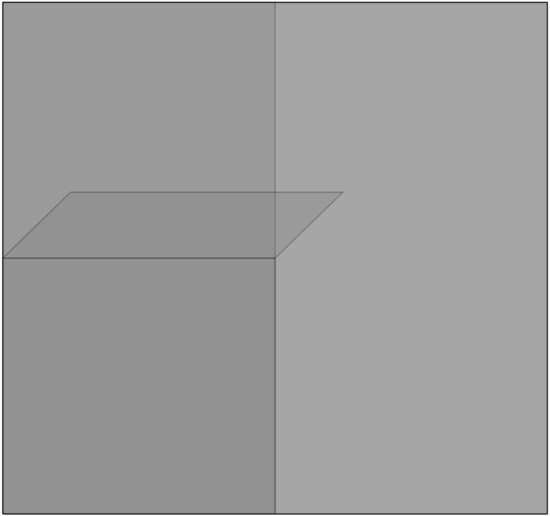

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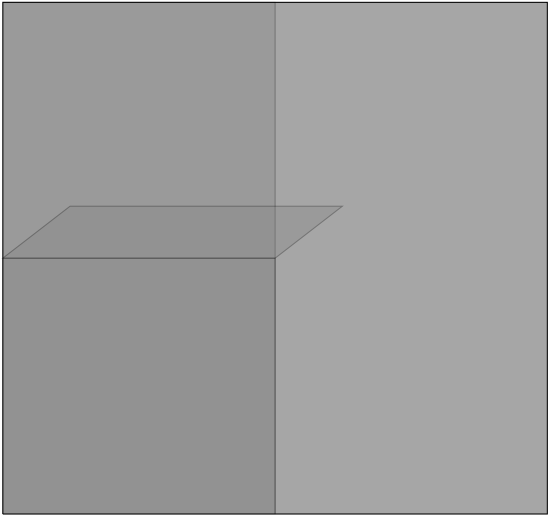

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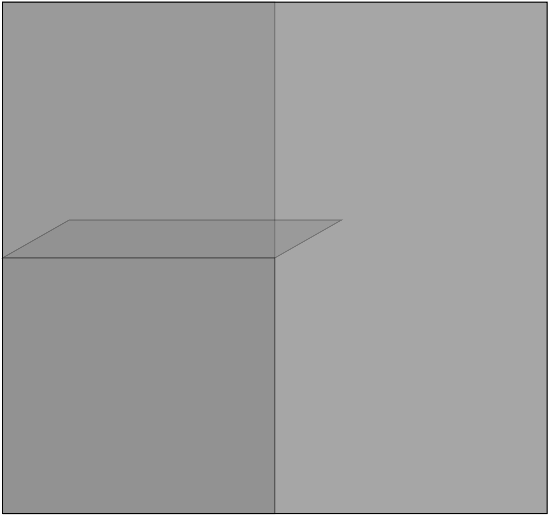

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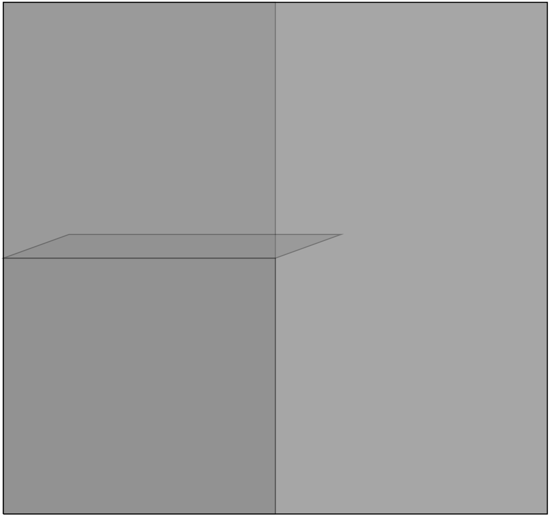

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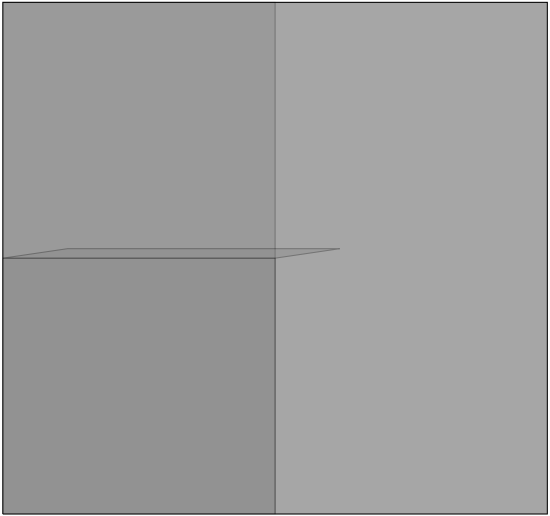

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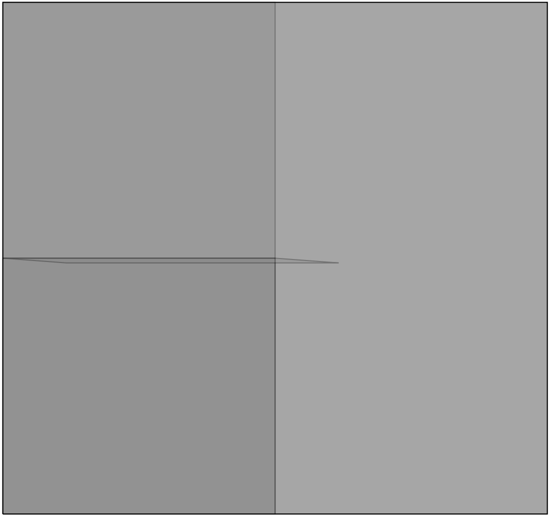

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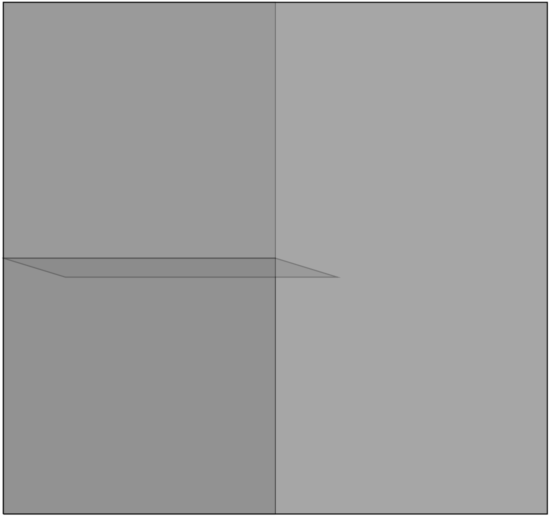

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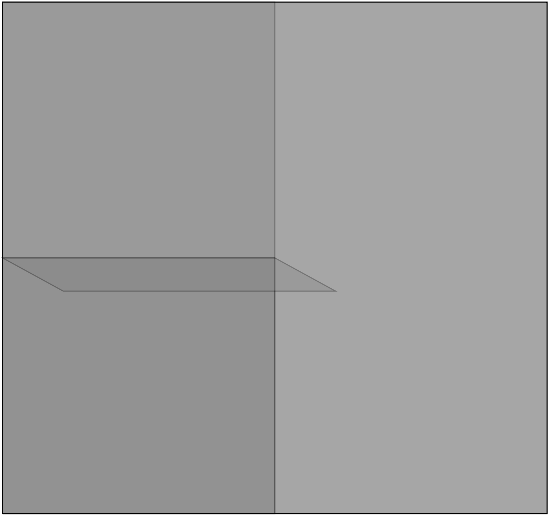

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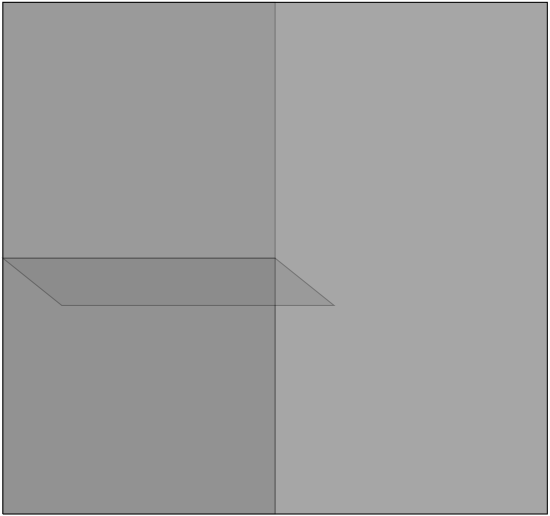

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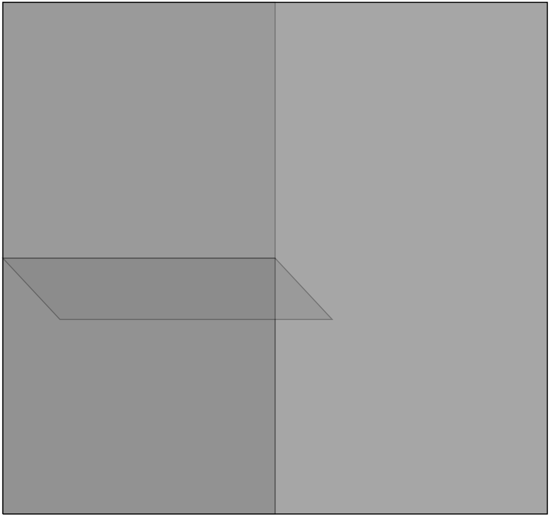

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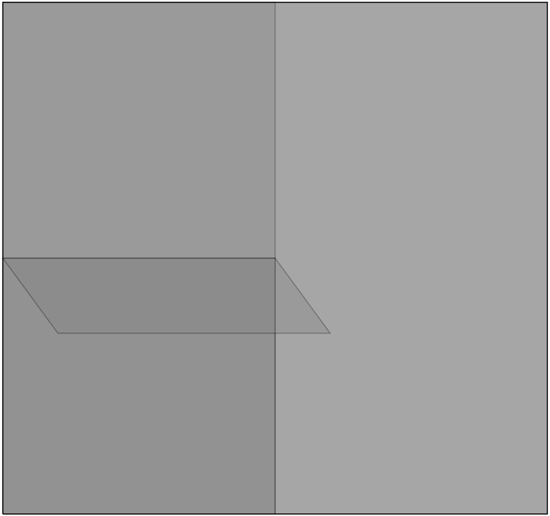

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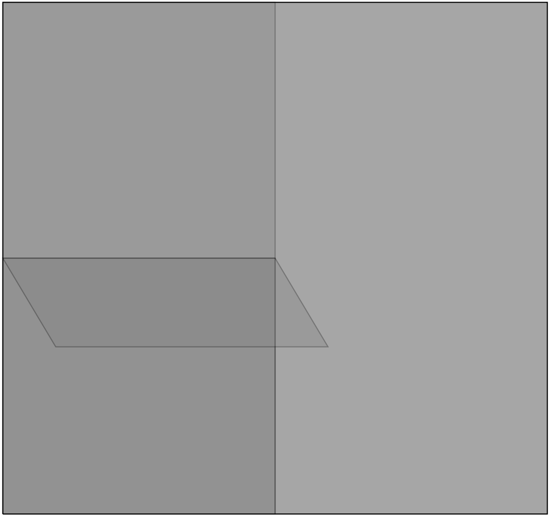

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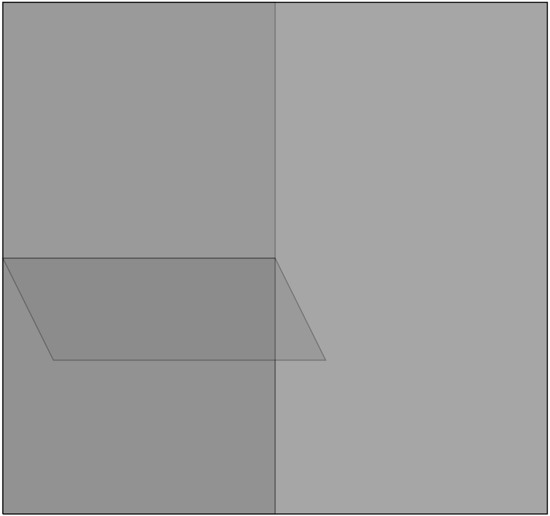

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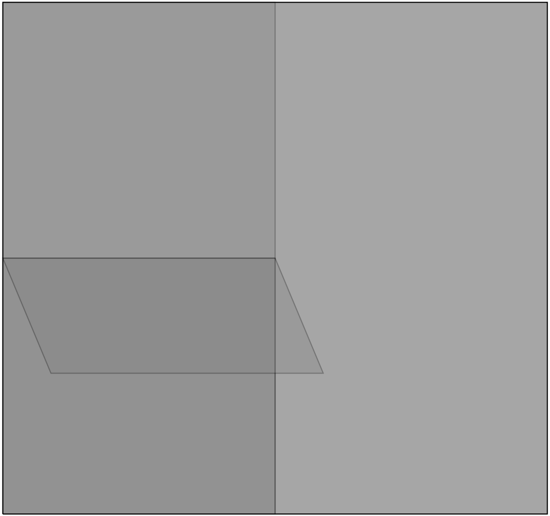

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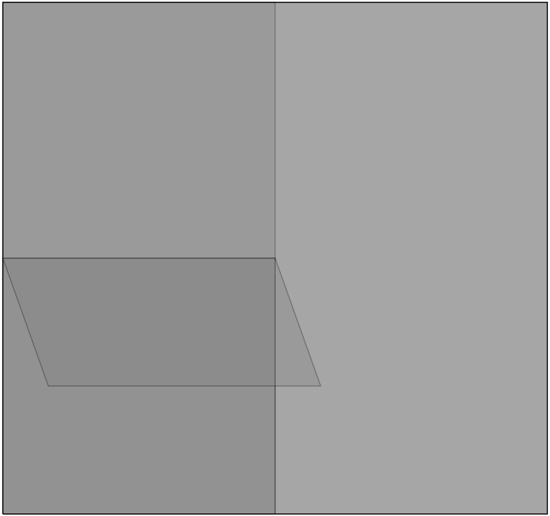

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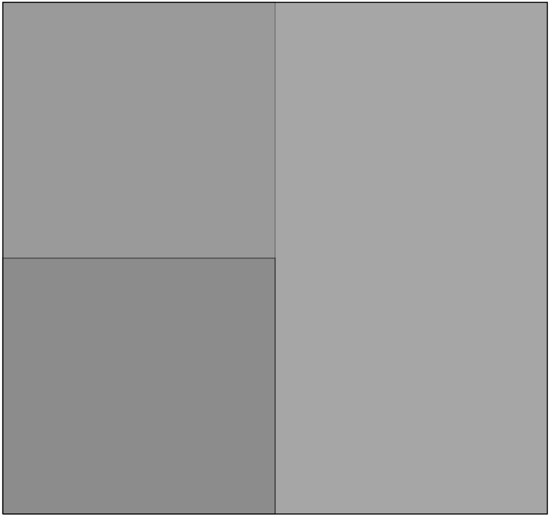

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Orbifold classification in the past and current status

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Bailin & Love [1999]

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➥ The vast majority of heterotic orbifold geometries is known for less than 10 years

Non–local GUT breaking in heterotic orbifolds

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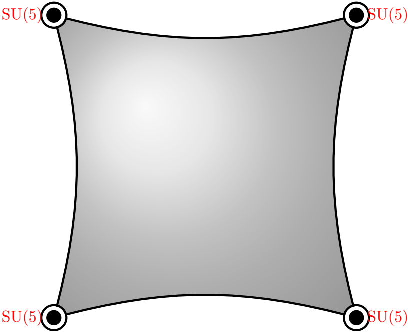
An example

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold example

Blaszczyk, Nibbelink, M.R., Ruehle, Trapletti, et al. [2010] ; Kappl, Petersen, Raby, M.R., Schieren & Vaudrevange [2011]



❶ step: 6 generation $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\mathrm{SU}(5)$ symmetry

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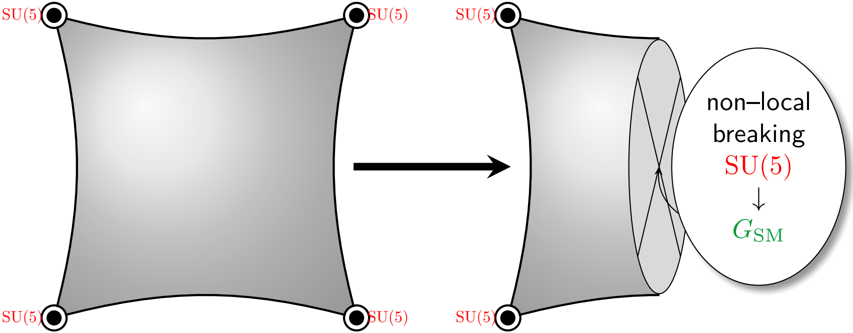

❶ step: 6 generation $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\mathrm{SU}(5)$ symmetry

❷ step: mod out a freely acting $\mathbb{Z}_2$ symmetry which:

- breaks $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$
- reduces the number of generations to 3

analogous mechanism in CY MSSMs Bouchard & Donagi [2006]
Braun, He, Ovrut & Pantev [2005]

Main features

① GUT symmetry breaking non–local
$\curvearrowright$ (almost) no ‘logarithmic running above the GUT scale’

Hebecker & Trapletti [2005] ; Anandakrishnan & Raby [2013]

Main features

❶ GUT symmetry breaking non–local

❷ No localized flux in hypercharge direction
$\curvearrowright$ complete blow–up without breaking SM gauge symmetry in principle possible

Main features

1. GUT symmetry breaking non–local
2. No localized flux in hypercharge direction
3. 4D gauge group:
   $\mathsf{SU}(3)_C \times \mathsf{SU}(2)_\mathrm{L} \times \mathsf{U}(1)_Y \times [\mathsf{SU}(3) \times \mathsf{SU}(2)^2 \times \mathsf{U}(1)^8]$

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4. massless spectrum

#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{D}$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1}$	$\overline{E}$
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	h
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{\delta}$
3	$(\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})_{0}$	x
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{2})_{0}$	y

#	representation	label
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{2}{3}}$	$\overline{U}$
3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{1}{2}}$	L
37	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{0}$	s
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/2}$	$\overline{h}$
3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/3}$	δ
5	$(\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_{0}$	$\overline{x}$
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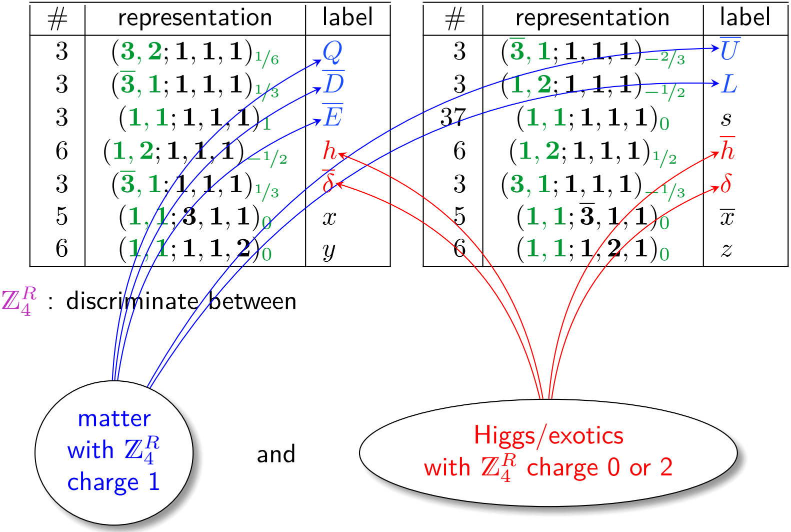
Main features

1. GUT symmetry breaking non–local
2. No localized flux in hypercharge direction
3. 4D gauge group:
   $\mathsf{SU}(3)_C \times \mathsf{SU}(2)_\mathrm{L} \times \mathsf{U}(1)_Y \times [\mathsf{SU}(3) \times \mathsf{SU}(2)^2 \times \mathsf{U}(1)^8]$
4. massless spectrum

   $\text{spectrum} = 3 \times \text{generation} + \text{vector–like}$

Spectrum and $\mathbb{Z}_4^R$



#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{D}$
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3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{\delta}$
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$\mathbb{Z}_4^R$: discriminate between

matter with $\mathbb{Z}_4^R$ charge 1

and

Higgs/exotics with $\mathbb{Z}_4^R$ charge 0 or 2

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☞ Many other good features:

- no fractionally charged exotics (i.e. all SM fields come from $\mathrm{SU}(5)$ representations)
- non–trivial full–rank Yukawa couplings
- gauge–top unification
- SU(5) relation $y_\tau \simeq y_b$ (but also for light generations)
- $\mathbb{Z}_4^R$ symmetry

▸ back

The role of SM singlets

☞ Most orbifolds come with a so–called anomalous $\mathrm{U}(1)$ and a Fayet–Iliopoulos (FI) term

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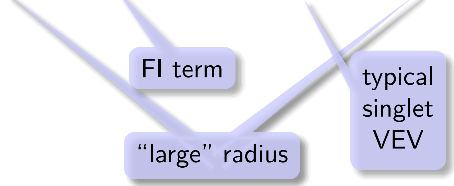

FI term

“large” radius

typical singlet VEV

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Open (?) question:

How can one explain R_{large} and obtain a reliable effective 4D description?

Anisotropic compactifications

☞ There are some ideas to explain R_{large}

Buchmüller, Catena & Schmidt-Hoberg [2008]

Anisotropic compactifications

- ☞ There are some ideas to explain R_{large} …but it is probably fair to say that more research is needed to obtain a complete picture

  Buchmüller, Catena & Schmidt-Hoberg [2008]

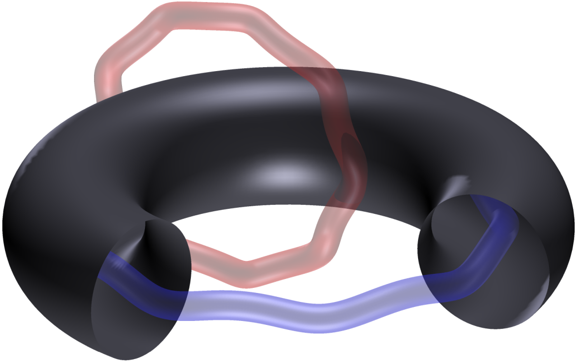
- ☞ It appears much more straightforward to explain the small radii

  Font, Ibáñez, Lüst & Quevedo [1990], Nilles & Olechowski [1990] ,…

Phsysic of the winding modes

- ☞ Winding modes have also been used to stabilize compact directions

e.g. Danos, Frey & Brandenberger [2008], Easther, Greene & Jackson [2002]



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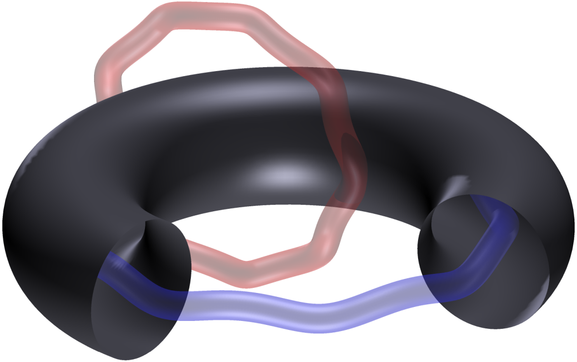

- ☞ Winding modes may even be dark matter

Mütter & Vaudrevange [2020]

Brief recap

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- ☞ Modular invariance seems to play a crucial role. . . also for the following

R Symmetries

for the

MSSM

Claim 1: Non–R symmetries cannot forbid μ

Hall, Nomura & Pierce [2002b], Lee, Raby, M.R., Ross, Schieren, Schmidt-Hoberg & Vaudrevange [2011a]

☞ Anomaly coefficients for non–R symmetry with SU(5) relations for matter charges

Ibáñez & Ross [1991] ,Banks & Dine [1992] ,. . .
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$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \sum_{g=1}^{3}\left[\frac{3}{2}q_{\mathbf{10}}^{g} + \frac{1}{2}q_{\overline{\mathbf{5}}}^{g}\right]$$

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sum over matter charges

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☞ Anomaly universality: $A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} - A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} \;=\; 0$

$$\curvearrowright \quad \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) \;=\; 0 \mod \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even}\end{cases}$$

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bottom-line:

non–R $\mathbb{Z}_N$ symmetry cannot forbid μ term

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- ☞ Notice that there are also non–Abelian discrete R symmetries in $\mathcal{N} = 1$ SUSY

  Chen, M.R. & Trautner [2013b]

Claim 2: $\mathrm{SO}(10)$ implies unique symmetry

Lee, Raby, M.R., Ross, Schieren, Schmidt-Hoberg & Vaudrevange [2011a]

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$$2q + q_{H_u} = 2 \mod N \quad \text{and} \quad 2q + q_{H_d} = 2 \mod N$$

by convention $\mathscr{W}$ has R charge 2

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- ➥ first conclusion:

$$q_{H_u} = q_{H_d} = 0 \mod N$$

Claim 2: SO(10) implies unique symmetry (cont'd)

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$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N^R} = 6q + \frac{1}{2}\left(q_{H_u}+q_{H_d}\right) - 5$$

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bottom-line:

$N=2$ or $N=4$

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- Anomaly universality

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N^R}$$

however: there is no meaningful $\mathbb{Z}_2^R$ symmetry

cf. e.g. Dine & Kehayias [2010]

$$\curvearrowright \quad q_{H_u} + q_{H_d} = 4 \mod \left\{ \begin{array}{ll} & \\ N & \text{for } N \text{ even} \end{array} \right.$$

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$N=4$ unique

Unique $\mathbb{Z}_4^R$ symmetry

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e.g. $q_{H_u} = q_{H_d} = 16$

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bottom-line:

- $\mathbb{Z}_4^R$ is anomaly free via GS mechanism
- GS axion contributes to gravitational anomaly

't Hooft anomaly matching for R symmetries

- ☞ Powerful tool: anomaly matching

't Hooft [1980], Csáki & Murayama [1998]

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extra

matter

gauginos

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SM gauginos

universal

extra gauginos from X, Y bosons

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➥ Extra stuff must be non–universal (split multiplets) with the simplest option being the pair of Higgs doublets!

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bottom–line:

't Hooft anomaly matching for (discrete) R symmetries implies the presence of split multiplets below the GUT scale!

Claim 3: only 5 symmetries obey SU(5) relations

Lee, Raby, M.R., Ross, Schieren, Schmidt-Hoberg & Vaudrevange [2011b]

☞ Demanding SU(5) rather than SO(10) relations we find that the order N of possible $\mathbb{Z}_N^R$ symmetries has to divide 24

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- ☞ There are only five viable charge assignments

N	$q_{\mathbf{10}}$	$q_{\overline{\mathbf{5}}}$	q_{H_u}	q_{H_d}	ρ	$A_0^R(\mathrm{MSSM})$
4	1	1	0	0	1	1
6	5	3	4	0	0	1
8	1	5	0	4	1	3
12	5	9	4	0	3	1
24	5	9	16	12	9	7

Recall

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} q^{(f)} \stackrel{!}{=} \rho \mod \eta$$

$$A_{\mathsf{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)} \stackrel{!}{=} \rho \mod \eta$$

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- $\mathbb{Z}_6^R$ is anomaly–free without Green–Schwarz axion and requires 3 generations

Evans, Ibe, Kehayias & Yanagida [2012]

back

Large Hierarchies in Nature

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$\mathcal{O}(10)$

RG invariant scale

$\mathcal{O}(1)$ coupling

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Witten [1981]

$$\Lambda \sim M_{\mathrm{P}} \exp\left(-b/g^2\right)$$

➥ hierarchically small gravitino mass (‘gaugino condensation’)

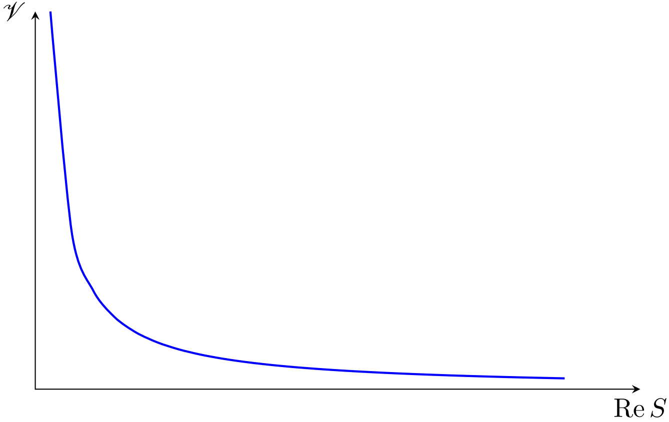
Nilles [1982]

$$m_W \sim m_{3/2} \sim \frac{\Lambda^3}{{M_{\mathrm{P}}}^2}$$

Problem with string theory realization

☞ **However**: embedding into string theory $\curvearrowright$ run–away problem

Dine & Seiberg [1985]



Moduli fixing and non–perturbative terms

There exist various possibilities to fix the gauge coupling/stabilize the dilaton:

Moduli fixing and non–perturbative terms

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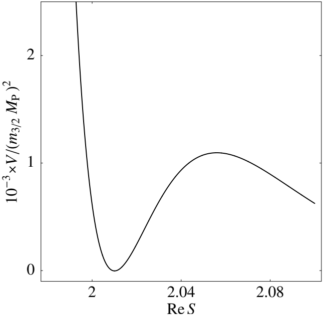
- Race–track
  Krasnikov [1987] ; . . .

use several gaugino condensates



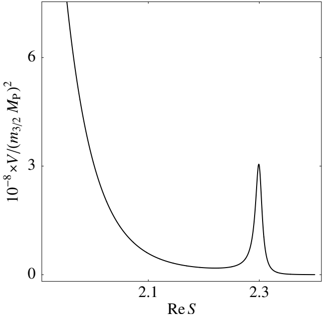
Moduli fixing and non–perturbative terms

There exist various possibilities to fix the gauge coupling/stabilize the dilaton:

- Race–track
- Kähler stabilization

Casas [1996] ; Binétruy, Gaillard & Wu [1997] ; . . .

non–perturbative corrections to the Kähler potential



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There exist various possibilities to fix the gauge coupling/stabilize the dilaton:

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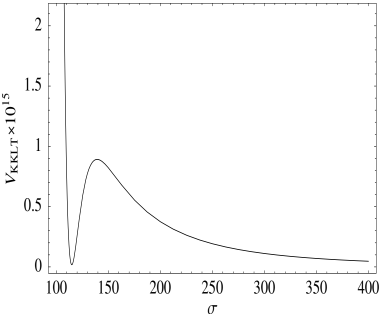
e.g. Kachru, Kallosh, Linde & Trivedi [2003]

e.g. KKLT proposal



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- etc. . . .

?

Constant + exponential scheme

☞ KKLT type proposal: $\mathscr{W}_{\mathrm{eff}} = c + A\,\mathrm{e}^{-a\,S}$

constant

non–perturbative

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- Alternative proposal: hierarchically small expectation of the perturbative superpotential due to approximate $\mathrm{U}(1)_R$ symmetry

$$c \to \langle \mathscr{W}_{\mathrm{pert}} \rangle \sim \langle \phi \rangle^N \quad \text{with} \quad N = \mathcal{O}(10)$$

typical VEV < 1

order of $\mathrm{U}(1)_R$ breaking

Hierarchically small $\langle \mathscr{W} \rangle$

Two observations:

1. in the presence of an exact $\mathrm{U}(1)_R$ symmetry

$$\frac{\partial \mathscr{W}}{\partial \phi_i} = 0 \quad \curvearrowright \quad \langle \mathscr{W} \rangle = 0$$

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fields

superpotential

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2. for approximate R symmetries

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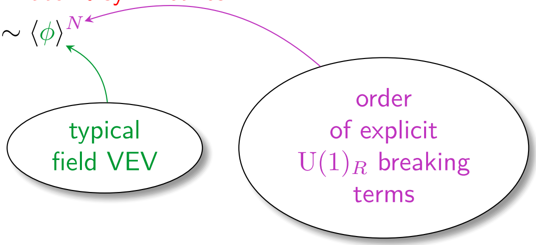

typical field VEV

order of explicit $\mathrm{U}(1)_R$ breaking terms

$\langle\mathscr{W}\rangle = 0$ because of $\mathrm{U}(1)_R$ (I)

aim: show that

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Consider a superpotential

$$\mathscr{W} = \sum c_{n_1 \cdots n_M} \phi_1^{n_1} \cdots \phi_M^{n_M}$$

with an exact R symmetry

$$\mathscr{W} \to \mathrm{e}^{2\mathrm{i}\,\alpha}\,\mathscr{W} \;, \quad \phi_j \to \phi_j' = \mathrm{e}^{\mathrm{i}\,r_j\,\alpha}\,\phi_j$$

where each monomial in $\mathscr{W}$ has total R charge 2

$\langle \mathscr{W} \rangle = 0$ because of $\mathrm{U}(1)_R$ (II)

Consider a field configuration $\langle \phi_i \rangle$ with

$$F_i = \frac{\partial \mathscr{W}}{\partial \phi_i} = 0 \quad \text{at } \phi_j = \langle \phi_j \rangle$$

Under an infinitesimal $\mathrm{U}(1)_R$ transformation, the superpotential transforms nontrivially

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This is only possible if $\langle \mathscr{W} \rangle = 0$!

bottom–line:

$$\frac{\partial \mathscr{W}}{\partial \phi_i} = 0 \qquad \curvearrowright \qquad \langle \mathscr{W} \rangle = 0$$

Comments

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Nelson & Seiberg [1994]

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❹ in 'no-scale' type settings

Weinberg [1989]

$$\boxed{\begin{array}{c}\text{solutions of}\\ \text{global SUSY}\\ F\ \text{term eq.'s}\end{array}} = \boxed{\begin{array}{c}\text{stationary points}\\ \text{of supergravity}\\ \text{scalar potential}\end{array}}$$

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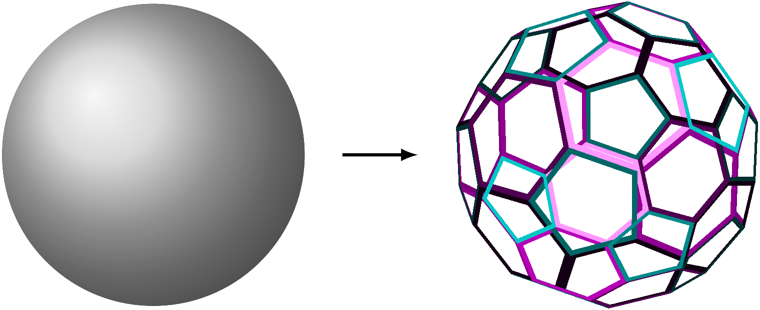
- ☞ Confirmed in various field–theoretic examples

back

Explicit
string theory
realization

Origin of high–power discrete R symmetries

- ☞ Discrete R symmetries arise as remnants of Lorentz symmetries of compact space



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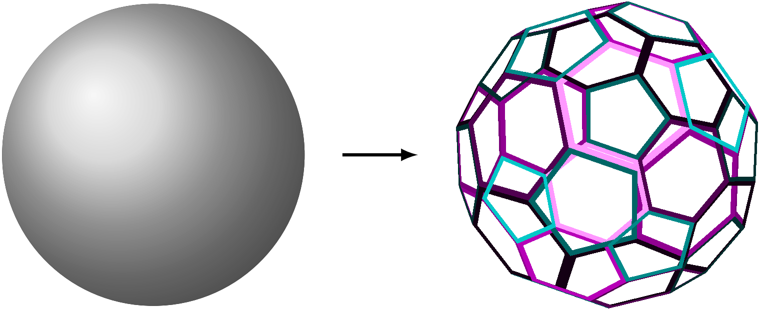
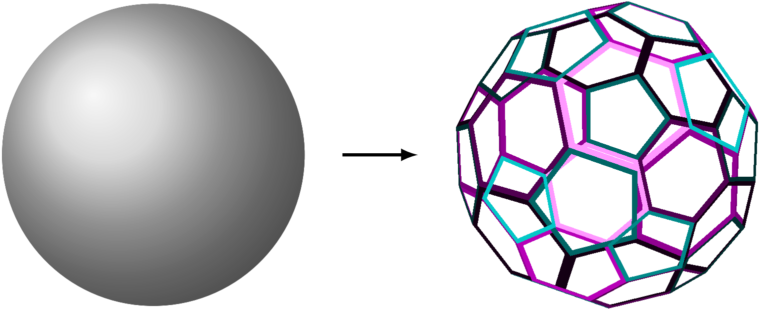


- ☞ Orbifolds break $\mathrm{SO}(6) \simeq \mathrm{SU}(4)$ Lorentz symmetry of compact space to discrete subgroups

Origin of high–power discrete R symmetries

- Discrete R symmetries arise as remnants of Lorentz symmetries of compact space



- Orbifolds break $\mathrm{SO}(6) \simeq \mathrm{SU}(4)$ Lorentz symmetry of compact space to discrete subgroups
- For example: a $\mathbb{Z}_2$ orbifold plane leads to $\mathbb{Z}_4^R$ symmetry

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Note: in order
to prove the existence
a full understanding
of coupling coefficients
is required

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bottom–line:

straightforward embedding in heterotic orbifolds

General picture

☞ The more fields are switched on, the lower N we obtain examples:

- model with 23 fields $\curvearrowright$ $N = 9$
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- ☞ Minima survive supergravity corrections

back

A_4 models

from the

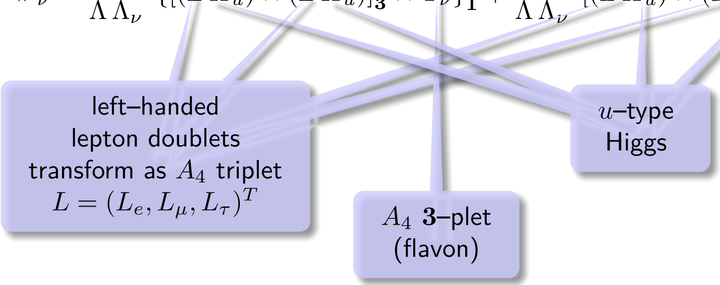
bottom–up

A popular example: A_4

Ma & Rajasekaran [2001], Babu, Ma & Valle [2003b], Hirsch, Romao, Skadhauge, Valle & Villanova del Moral [2004]
Altarelli & Feruglio [2005] ,. . .

☞ Superpotential couplings

$$\mathscr{W}_\nu = \frac{\lambda_1}{\Lambda\,\Lambda_\nu}\,\{[(L\,H_u)\times(L\,H_u)]_{\mathbf{3}}\times\Phi_\nu\}_{\mathbf{1}}+\frac{\lambda_2}{\Lambda\,\Lambda_\nu}\,[(L\,H_u)\times(L\,H_u)]_{\mathbf{1}}\;\xi$$



left–handed
lepton doublets
transform as A_4 triplet
$L = (L_e, L_\mu, L_\tau)^T$

u–type
Higgs

A_4 $\mathbf{3}$–plet
(flavon)

A popular example: A_4

Altarelli & Feruglio [2005] ,...

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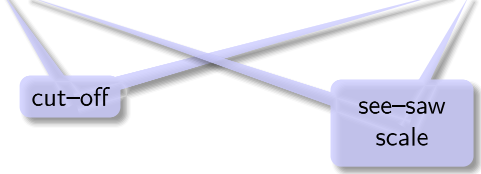

cut–off

see–saw scale

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triplet contraction

singlet contraction

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another triplet flavon

singlet″ contraction

singlet′ contraction

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\boxed{G_{\mathrm{e}} = \mathbb{Z}_2} & & \boxed{G_\nu = \mathbb{Z}_3}
\end{tikzcd}$$

- A_4 symmetry broken by VEVs of flavons

$$\langle\Phi_\nu\rangle = (v,v,v)$$
$$\langle\Phi_e\rangle = (v',0,0)$$
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- ➥ Tri–bi–maximal mixing (TBM)

Structure lepton masses

Altarelli & Feruglio [2005]

- ☞ After inserting the flavon VEVs

$$\mathscr{W}_\nu = (L_e\,H_u, L_\mu\,H_u, L_\tau\,H_u) \begin{pmatrix} a+2d & -d & -d \\ -d & 2d & a-d \\ -d & a-d & 2d \end{pmatrix} \begin{pmatrix} L_e\,H_u \\ L_\mu\,H_u \\ L_\tau\,H_u \end{pmatrix}$$

$$a = 2\lambda_1\,\lambda_2\,\frac{w}{\Lambda}\,\frac{1}{\Lambda_\nu}$$

$$d = \frac{\lambda_1}{3}\,\frac{v}{\Lambda}\,\frac{1}{\Lambda_\nu}$$

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$$\mathscr{W}_e = (L_e, L_\mu, L_\tau) \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \begin{pmatrix} e_\mathrm{R} \\ \mu_\mathrm{R} \\ \tau_\mathrm{R} \end{pmatrix} H_d$$

$$y_{e,\mu,\tau} = h_{e,\mu,\tau}\, v'/\Lambda$$

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- After inserting the electroweak VEVs

$$\mathscr{W}_\nu \xrightarrow{H_u \to (0, v_u)^T} \frac{v_u^2}{2}\,(\nu_e, \nu_\mu, \nu_\tau) \begin{pmatrix} a+2d & -d & -d \\ -d & 2d & a-d \\ -d & a-d & 2d \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

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Harrison, Perkins & Scott [2002]

☞ Structure of neutrino masses (in the basis in which the charged lepton masses are diagonal)

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$$U_{\mathrm{(P)MNS}}^{\mathrm{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- Mixing angles: $\begin{cases} \theta_{12} \simeq 35^\circ \\ \theta_{13} = 0 \\ \theta_{23} = 45^\circ \end{cases}$
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- ☞ Unrealistic but close to the actual values

back

“Corrections”

to

model

predictions

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? How predictive are such models?

Kähler corrections

e.g. Leurer, Nir & Seiberg [1994]

- ☞ Superpotential: holomorphic, e.g.

$$\mathscr{W}_\nu = \frac{1}{2}\left(L\,H_u\right)^T\,\kappa_\nu\,L\,H_u$$

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- Canonical Kähler potential

$$K_{\text{canonical}} \;\supset\; \sum_f \left[(L_f)^\dagger\;L_f + (R_f)^\dagger\;R_f\right]$$

charged
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- Correction

$$\Delta K \;=\; \sum_{f,g}\left[L_f^\dagger\,P_{f\,g}\,L_g + R_f^\dagger\,Q_{f\,g}\,R_g\right]$$

Hermitean matrices composed of e.g. flavons

Back to the A_4 example

- ☞ Kähler potential may contain

$$\Delta K_{\Phi}^{\text{linear}} \supset \sum_{i=1}^{2} \frac{1}{\Lambda}\,\kappa_{\Phi,\text{linear}}^{(i)}\,L^{\dagger}\,(L\Phi)_{\mathbf{3}_i} + \text{h.c.}$$

one of
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- ☞ However, such terms may be forbidden by additional symmetries

A_4 example continued

☞ 'Quadratic' Kähler corrections

$$\Delta K_{\Phi}^{\mathrm{quadratic}} \supset \frac{1}{\Lambda^2}\sum_{\boldsymbol{X}}^{6} \kappa_{\Phi,\mathrm{quadratic}}^{\boldsymbol{X}} \left(L\Phi\right)^{\dagger}_{\boldsymbol{X}} \left(L\Phi\right)_{\boldsymbol{X}} + \text{h.c.}$$

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- ☞ Such terms cannot be forbidden by any (conventional) symmetry
- ➥ Kähler corrections when flavon fields attain their VEVs
- ☞ Additional parameters $\kappa_{\Phi}^{\boldsymbol{X}}$ reduce the predictivity of the scheme

Linear independent flavon corrections

- ☞ From $\langle\Phi_e\rangle$

$$P_{\mathrm{I}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{\mathrm{II}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$P_{\mathrm{III}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- ☞ From $\langle\Phi_\nu\rangle$

$$P_{\mathrm{IV}} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad P_{\mathrm{V}} = \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix}$$

Change of θ_{13} in the A_4 model

Chen, Fallbacher, M.R. & Staudt [2012], Chen, Fallbacher, Omura, M.R. & Staudt [2013a]

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- ☞ The analytic formula evaluated at tri–bi–maximal mixing reads $(m_e \ll m_\mu \ll m_\tau)$

$$\Delta\theta_{13} = \kappa_\mathrm{V} \cdot \frac{v^2}{\Lambda^2} \cdot 3\sqrt{\frac{3}{2}}\left(\frac{2m_1}{m_1 + m_3} + \frac{m_e^2}{m_\mu^2 - m_e^2} + \frac{m_e^2}{m_\tau^2 - m_e^2}\right)$$

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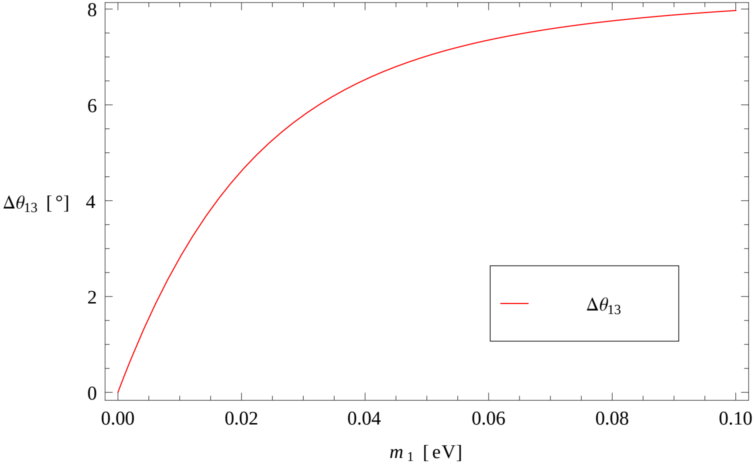
- ☞ Complex P matrix $\curvearrowright$ $\cancel{\mathcal{CP}}$ is induced: $\delta \;\approx\; \pi/2$

Change of θ_{13}

☞ $\Delta\theta_{13}$ for Kähler coefficient $\kappa_{\mathrm{V}} = 1$, $v/\Lambda = 0.2$



Impact of “corrections”

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- ☞ A UV completion, such as the one that may be provided by string theory, can possibly help us to make the models more predictive
- ☞ There are works which compute some of the relevant terms

e.g. Antoniadis, Gava, Narain & Taylor [1994], Olguín-Trejo & Ramos-Sánchez [2017]

$\mathcal{CP}$ violation

from

finite groups

(Details)

The canonical $\mathcal{CP}$ transformation

- ☞ Scalar field operator

$$\phi(x) = \int \mathrm{d}^3 p \, \frac{1}{2E_{\vec{p}}} \, \left[a(\vec{p}) \, \mathrm{e}^{-\mathrm{i}\, p\cdot x} + b^\dagger(\vec{p}) \, \mathrm{e}^{\mathrm{i}\, p\cdot x} \right]$$

annihilates particle

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creates anti–particle

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- $\mathcal{CP}$ exchanges particles & anti–particles

$$\begin{aligned} (\boldsymbol{\mathcal{CP}})^{-1}\, a(\vec{p}) \boldsymbol{\mathcal{CP}} &= \eta_{\mathcal{CP}}\, b(-\vec{p}) \quad & \quad (\boldsymbol{\mathcal{CP}})^{-1}\, a^\dagger(\vec{p})\, \boldsymbol{\mathcal{CP}} &= \eta^*_{\mathcal{CP}}\, b^\dagger(-\vec{p}) \\ (\boldsymbol{\mathcal{CP}})^{-1}\, b(\vec{p})\, \boldsymbol{\mathcal{CP}} &= \eta^*_{\mathcal{CP}}\, a(-\vec{p}) \quad & \quad (\boldsymbol{\mathcal{CP}})^{-1}\, b^\dagger(\vec{p})\, \boldsymbol{\mathcal{CP}} &= \eta_{\mathcal{CP}}\, a^\dagger(-\vec{p}) \end{aligned}$$

phase factor

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- $\mathcal{CP}$ transformation of (scalar) fields

freedom of re–phasing fields

$$\phi(x) \xmapsto{\boldsymbol{\mathcal{CP}}} \eta_{\mathcal{CP}} \, \phi^*(\mathcal{P}x)$$

Generalized $\mathcal{CP}$ transformations

☞ Setting w/ discrete symmetry G

Generalized $\mathcal{CP}$ transformations

vector of creation operators

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$$(\boldsymbol{\mathcal{CP}})^{-1}\,\boldsymbol{b}(\vec{p})\,\boldsymbol{\mathcal{CP}} = \boldsymbol{a}(-\vec{p})\,U_{\mathcal{CP}}{}^{\dagger} \quad \& \quad (\boldsymbol{\mathcal{CP}})^{-1}\,\boldsymbol{b}^{\dagger}(\vec{p})\,\boldsymbol{\mathcal{CP}} = U_{\mathcal{CP}}\,\boldsymbol{a}^{\dagger}(-\vec{p})$$

vector of annihilation operators

unitary matrix

Generalized $\mathcal{CP}$ transformations

- Setting w/ discrete symmetry G
- Generalized $\mathcal{CP}$ transformation
- Invariant contraction/coupling in A_4 or T'

Holthausen, Lindner & Schmidt [2013]

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi\,\left(x_1\,y_1 + \omega^2\,x_2\,y_2 + \omega^2\,x_3\,y_3\right)$$

$\omega = \mathrm{e}^{2\pi\,\mathrm{i}/3}$

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➥ Need generalized $\mathcal{CP}$ transformation $\widetilde{\mathcal{CP}}$: $\phi \xmapsto{\widetilde{\mathcal{CP}}} \phi^*$ as usual but

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} x_1^* \\ x_3^* \\ x_2^* \end{pmatrix} \quad \& \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} y_1^* \\ y_3^* \\ y_2^* \end{pmatrix}$$

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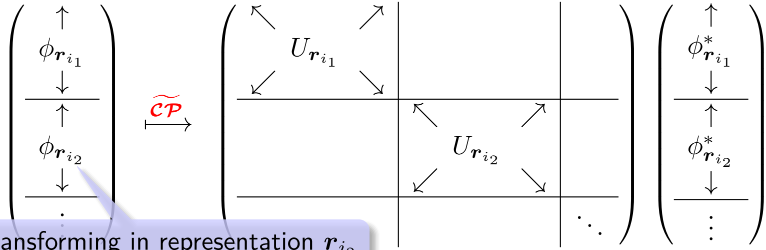

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automorphism $u\ :\ G \to G$

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Bickerstaff & Damhus [1985]

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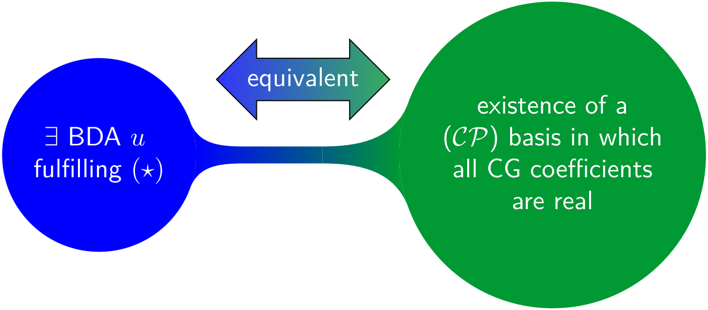

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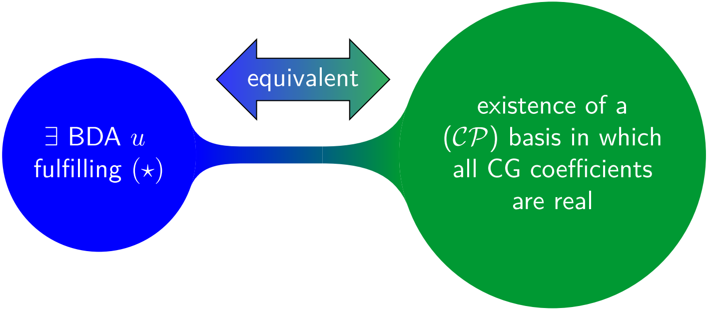

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The twisted Frobenius–Schur indicator

☞ how can one tell whether or not a given automorphism u is a BDA?

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$$\mathrm{FS}(\boldsymbol{r}_i) = \begin{cases} +1, & \text{if } \boldsymbol{r}_i \text{ is a real representation,}\\ 0, & \text{if } \boldsymbol{r}_i \text{ is a complex representation,}\\ -1, & \text{if } \boldsymbol{r}_i \text{ is a pseudo–real representation.}\end{cases}$$

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- ☞ crucial property

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \begin{cases} +1 \quad \forall\; i, & \text{if } u \text{ is a BDA,} \\ +1 \text{ or } -1 \quad \forall\; i, & \text{if } u \text{ is class–inverting \& involutory,} \\ \text{different from } \pm 1, & \text{otherwise.} \end{cases}$$

Extended twisted Frobenius–Schur indicator

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$$\mathrm{FS}_{u}^{(n)}(\boldsymbol{r}_i) \;:=\; \frac{(\dim \boldsymbol{r}_i)^{n-1}}{|G|^n} \sum_{g_1,\dots,g_n\in G} \chi_{\boldsymbol{r}_i}(g_1\,u(g_1)\cdots g_n\,u(g_n))$$

$$n = \begin{cases} \mathrm{ord}(u)/2 & \text{if } \mathrm{ord}(u) \text{ is even} \\ \mathrm{ord}(u) & \text{if } \mathrm{ord}(u) \text{ is odd} \end{cases}$$

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▸ back

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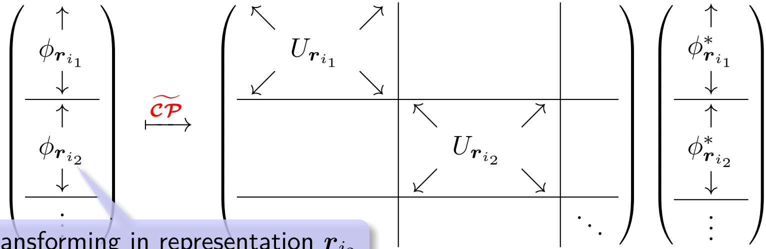

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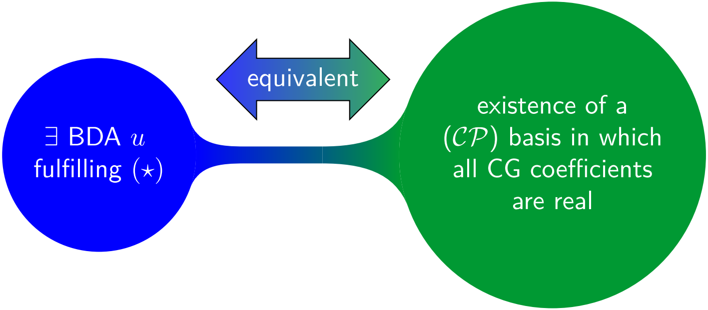

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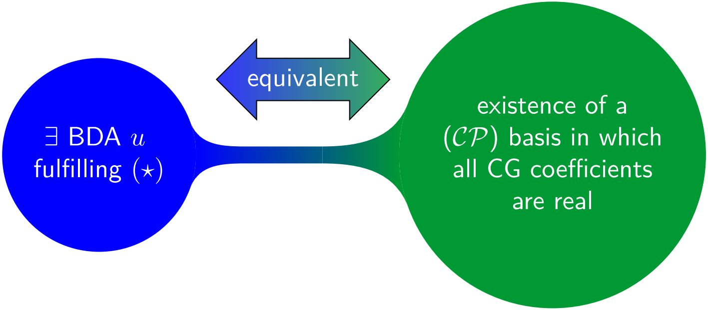

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$$\mathrm{FS}(\boldsymbol{r}_i) \;:=\; \frac{1}{|G|}\sum_{g\in G}\chi_{\boldsymbol{r}_i}(g^2)=\frac{1}{|G|}\sum_{g\in G}\mathrm{tr}\left[\rho_{\boldsymbol{r}_i}(g)^2\right]$$

Bickerstaff & Damhus [1985], Kawanaka & Matsuyama [1990]

- twisted Frobenius–Schur indicator

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;=\; \frac{1}{|G|}\sum_{g\in G}\left[\rho_{\boldsymbol{r}_i}(g)\right]_{\alpha\beta}\left[\rho_{\boldsymbol{r}_i}(u(g))\right]_{\beta\alpha}$$

The twisted Frobenius–Schur indicator

- ☞ how can one tell whether or not a given automorphism u is a BDA?
- ☞ Frobenius–Schur indicator

$$\mathrm{FS}(\boldsymbol{r}_i) \;:=\; \frac{1}{|G|}\sum_{g\in G}\chi_{\boldsymbol{r}_i}(g^2) = \frac{1}{|G|}\sum_{g\in G}\mathrm{tr}\left[\rho_{\boldsymbol{r}_i}(g)^2\right]$$

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- ☞ twisted Frobenius–Schur indicator

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- ☞ crucial property

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \begin{cases} +1 \quad \forall\; i, & \text{if } u \text{ is a BDA,}\\ +1 \text{ or } -1 \quad \forall\; i, & \text{if } u \text{ is class–inverting \& involutory,}\\ \text{different from } \pm 1, & \text{otherwise.}\end{cases}$$

Extended twisted Frobenius–Schur indicator

- ☞ extended twisted Frobenius–Schur indicator

$$\mathrm{FS}_{u}^{(n)}(\boldsymbol{r}_i) \;:=\; \frac{(\dim \boldsymbol{r}_i)^{n-1}}{|G|^n} \sum_{g_1,\dots,g_n\in G} \chi_{\boldsymbol{r}_i}(g_1\,u(g_1)\cdots g_n\,u(g_n))$$

$$n = \begin{cases} \mathrm{ord}(u)/2 & \text{if } \mathrm{ord}(u) \text{ is even} \\ \mathrm{ord}(u) & \text{if } \mathrm{ord}(u) \text{ is odd} \end{cases}$$

Extended twisted Frobenius–Schur indicator

☞ extended twisted Frobenius–Schur indicator

$$\mathrm{FS}_u^{(n)}(\boldsymbol{r}_i) \;:=\; \frac{(\dim \boldsymbol{r}_i)^{n-1}}{|G|^n} \sum_{g_1,\dots,g_n\in G} \chi_{\boldsymbol{r}_i}(g_1\,u(g_1)\cdots g_n\,u(g_n))$$

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$$\mathrm{FS}_u^{(n)}(\boldsymbol{r}_i) = \begin{cases} \pm1 \quad \forall\ i, & \text{if } u \text{ is class–inverting,} \\ \text{different from } \pm1 & \\ \text{for at least one } \boldsymbol{r}_i, & \text{otherwise.} \end{cases}$$

▸ back

$\mathcal{CP}$ violation with an unbroken $\mathcal{CP}$ transformation

Example: $\mathrm{SU}(3) \to \mathrm{T}_7$

☞ Starting point: $\mathrm{SU}(3)$ gauge theory with

$$\mathscr{L} \;=\; (D_\mu\,\phi)^\dagger\,(D^\mu\,\phi) - \frac{1}{4}\,G_{\mu\nu}^a\,G^{\mu\nu,a} - \mathscr{V}(\phi)$$

$D_\mu = \partial_\mu - \mathrm{i}g\,A_\mu$

field strength

Example: $\mathrm{SU}(3) \to \mathrm{T}_7$

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$\mathbf{15}$–plet

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- ☞ Potential: $\mathscr{V}(\phi) \;=\; -\,\mu^2\,\phi^\dagger\phi + \sum\limits_{i=1}^{5}\lambda_i\,\mathcal{I}^{(4)}{}_i(\phi)$

quartic $\mathrm{SU}(3)$ invariants

Example: $\mathrm{SU}(3) \to \mathrm{T}_7$

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- Potential: $\mathscr{V}(\phi) \;=\; -\,\mu^2\,\phi^\dagger\phi + \sum\limits_{i=1}^{5}\lambda_i\,\mathcal{I}^{(4)}{}_i(\phi)$

- Action invariant under $\mathcal{CP}$ transformation

$$\begin{aligned} A^a_\mu(x) &\xmapsto{\mathrm{SU}(3)-\mathcal{CP}} R^{ab}\,\mathcal{P}_\mu^{\;\nu}\,A^b_\nu(\mathcal{P}\,x)\\ \phi_i(x) &\xmapsto{\mathrm{SU}(3)-\mathcal{CP}} U_{ij}\,\phi^*_j(\mathcal{P}\,x)\end{aligned}$$

$\mathcal{P} = \mathrm{diag}(1,-1,-1,-1)$

Example: $\mathrm{SU}(3) \to \mathrm{T}_7$

- Starting point: $\mathrm{SU}(3)$ gauge theory with

$$\mathscr{L} \;=\; (D_\mu\,\phi)^\dagger\,(D^\mu\,\phi) - \frac{1}{4}\,G^a_{\mu\nu}\,G^{\mu\nu,a} - \mathscr{V}(\phi)$$

- Potential: $\mathscr{V}(\phi) \;=\; -\,\mu^2\,\phi^\dagger\phi + \sum\limits_{i=1}^{5}\lambda_i\,\mathcal{I}^{(4)}{}_i(\phi)$

- Action invariant under $\mathcal{CP}$ transformation

$$U\;\left(-\mathsf{t}_a^{\mathrm{T}}\right)\;U^{-1}\;=\;R_{ab}\,\mathsf{t}_b$$

$$A^a_\mu(x) \xmapsto{\mathrm{SU}(3)-\mathcal{CP}} R^{ab}\,\mathcal{P}_\mu^{\;\nu}\,A^b_\nu(\mathcal{P}\,x)$$

$$\phi_i(x) \xmapsto{\mathrm{SU}(3)-\mathcal{CP}} U_{ij}\,\phi^*_j(\mathcal{P}\,x)$$

$\mathrm{SU}(3) \to \mathsf{T}_7$

see e.g. Luhn [2011], Merle & Zwicky [2012]

☞ $\langle\phi\rangle$ breaks $\mathrm{SU}(3)$ to T_7

$$\mathrm{SU}(3) \rtimes \mathbb{Z}_2 \xrightarrow{\langle\phi\rangle} \mathsf{T}_7 \rtimes \mathbb{Z}_2$$

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$$\mathrm{SU}(3) \rtimes \mathbb{Z}_2 \xrightarrow{\langle\phi\rangle} \mathsf{T}_7 \rtimes \mathbb{Z}_2$$

☞ Physical fields before and after symmetry breaking

name	$\mathrm{SU}(3) \xrightarrow{\langle\phi\rangle}$	name	T_7
A_μ	$\mathbf{8}$	Z_μ	$\mathbf{1_1}$
		W_μ	$\mathbf{3}$
ϕ	$\mathbf{15}$	$\mathrm{Re}\,\sigma_0, \mathrm{Im}\,\sigma_0$	$\mathbf{1_0}$
		σ_1	$\mathbf{1_1}$
		τ_1	$\mathbf{3}$
		τ_2	$\mathbf{3}$
		τ_3	$\mathbf{3}$

$\mathrm{SU}(3)-\mathcal{CP}$ vs. $\mathrm{Out}(\mathsf{T}_7)$

☞ $\mathrm{SU}(3)-\mathcal{CP}$ breaks to unique $\mathbb{Z}_2$ outer automorphism of T_7

$$\mathrm{Out}(\mathsf{T}_7)\ :\qquad \boldsymbol{1_1} \longleftrightarrow \boldsymbol{1_1}\;,\quad \overline{\boldsymbol{1}}_{\boldsymbol{1}} \longleftrightarrow \overline{\boldsymbol{1}}_{\boldsymbol{1}}\;,\quad \boldsymbol{3} \longleftrightarrow \overline{\boldsymbol{3}}$$

$\mathrm{SU}(3)-\mathcal{CP}$ vs. $\mathrm{Out}(\mathrm{T}_7)$

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- ☞ T_7 character table

T_7	C_{1a}	C_{3a}	C_{3b}	C_{7a}	C_{7b}
	e	b	b^2	a	a^3
$\mathbf{1_0}$	1	1	1	1	1
$\mathbf{1_1}$	1	ω	ω^2	1	1
$\overline{\mathbf{1}}_\mathbf{1}$	1	ω^2	ω	1	1
$\mathbf{3}$	3	0	0	η	η^*
$\overline{\mathbf{3}}$	3	0	0	η^*	η

$\mathrm{SU}(3) - \mathcal{CP}$ vs. $\mathrm{Out}(\mathrm{T}_7)$

- $\mathrm{SU}(3) - \mathcal{CP}$ breaks to unique $\mathbb{Z}_2$ outer automorphism of T_7

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- T_7 character table

T_7	C_{1a} e	C_{3a} b	C_{3b} b^2	C_{7a} a	C_{7b} a^3
$\mathbf{1_0}$	1	1	1	1	1
$\mathbf{1_1}$	1	ω	ω^2	1	1
$\overline{\mathbf{1}}_\mathbf{1}$	1	ω^2	ω	1	1
$\mathbf{3}$	3	0	0	η	η^*
$\overline{\mathbf{3}}$	3	0	0	η^*	η

$\omega := \mathrm{e}^{2\pi\mathrm{i}/3}$

$\eta = \rho + \rho^2 + \rho^4$ with $\rho := \mathrm{e}^{2\pi\mathrm{i}/7}$

$\mathrm{SU}(3)-\mathcal{CP}$ vs. $\mathrm{Out}(\mathrm{T}_7)$

- $\mathrm{SU}(3)-\mathcal{CP}$ breaks to unique $\mathbb{Z}_2$ outer automorphism of T_7

$$\mathrm{Out}(\mathrm{T}_7)\quad:\qquad \mathbf{1_1}\longleftrightarrow\mathbf{1_1}\;,\quad \overline{\mathbf{1}}_\mathbf{1}\longleftrightarrow\overline{\mathbf{1}}_\mathbf{1}\;,\quad \mathbf{3}\longleftrightarrow\overline{\mathbf{3}}$$

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$\mathbf{3}$	3	0	0	η	η^*
$\overline{\mathbf{3}}$	3	0	0	η^*	η

- $\mathbf{1_1}$ and $\overline{\mathbf{1_1}}$ do **not** get swapped!

T_7

☞ T_7 can be generated by two elements with the presentation

$$\left\langle \mathsf{a}, \mathsf{b} \;\middle|\; \mathsf{a}^7 \;=\; \mathsf{b}^3 \;=\; \mathsf{e}\,, \mathsf{b}^{-1}\,\mathsf{a}\,\mathsf{b} \;=\; \mathsf{a}^4 \right\rangle$$

T_7

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- ☞ Triplet representation

$$A \;=\; \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix} \quad \text{and} \quad B \;=\; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

T_7

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- ☞ Embedding into $\mathrm{SU}(3)$

$$X^{(\boldsymbol{r})} \;=\; \exp\left(\mathrm{i}\,\alpha_a\,\mathsf{t}_a^{(\boldsymbol{r})}\right)$$

$$\vec{\alpha}^{(A)} \;=\; \frac{2\pi}{7}\,(0,0,0,0,0,0,\,\sqrt{3},5)$$

T_7

- ☞ T_7 can be generated by two elements with the presentation

$$\left\langle \mathsf{a}, \mathsf{b} \;\middle|\; \mathsf{a}^7 ~=~ \mathsf{b}^3 ~=~ \mathsf{e}\,, \mathsf{b}^{-1}\,\mathsf{a}\,\mathsf{b} ~=~ \mathsf{a}^4 \right\rangle$$

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$$A ~=~ \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix} \quad \text{and} \quad B ~=~ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

- ☞ Embedding into $\mathrm{SU}(3)$

$$X^{(\boldsymbol{r})} ~=~ \exp\left(\mathrm{i}\,\alpha_a\,\mathsf{t}_a^{(\boldsymbol{r})}\right)$$

$$\vec{\alpha}^{(B)} ~=~ \frac{4\pi}{3\,\sqrt{3}}\,(0,0,1,1,1,0,0,0)$$

T$_7$

- T$_7$ can be generated by two elements with the presentation

$$\langle \mathsf{a},\mathsf{b} \mid \mathsf{a}^7 = \mathsf{b}^3 = \mathsf{e}\,,\mathsf{b}^{-1}\,\mathsf{a}\,\mathsf{b} = \mathsf{a}^4\rangle$$

- Triplet representation

$$A = \begin{pmatrix}\rho & 0 & 0\\ 0 & \rho^2 & 0\\ 0 & 0 & \rho^4\end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{pmatrix}$$

- Embedding into $\mathrm{SU}(3)$

$$X^{(\boldsymbol{r})} = \exp\left(\mathsf{i}\,\alpha_a\,\mathsf{t}_a^{(\boldsymbol{r})}\right)$$

- Work in `SusyNo` basis

Fonseca [2012]

T_7 scalar states

☞ Branchings:

$$\begin{aligned} \mathbf{8} &\to \mathbf{1_1} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \\ \mathbf{15} &\to \mathbf{1_0} \oplus \mathbf{1_1} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}} \end{aligned}$$

T_7 scalar states

- Branchings:

$$\mathbf{8} \;\to\; \mathbf{1_1} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{3} \oplus \overline{\mathbf{3}}$$

$$\mathbf{15} \;\to\; \mathbf{1_0} \oplus \mathbf{1_1} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}}$$

- Physical scalar fields (would–be Goldstone bosons subtracted)

$$\phi \;=\; \left(v + \phi_1,\, \frac{\phi_2}{\sqrt{2}},\, \frac{\phi_2^*}{\sqrt{2}},\, \phi_4,\, \phi_5,\, \phi_6,\, \frac{\phi_7}{\sqrt{2}},\, \frac{\phi_8}{\sqrt{2}},\, \frac{\phi_9}{\sqrt{2}},\, \phi_{10},\, \phi_{11},\, \phi_{12},\, \frac{\phi_7^*}{\sqrt{2}},\, \frac{\phi_8^*}{\sqrt{2}},\, \frac{\phi_9^*}{\sqrt{2}}\right)$$

T_7 scalar states

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$$\mathbf{8} \;\to\; \mathbf{1_1} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{3} \oplus \overline{\mathbf{3}}$$

$$\mathbf{15} \;\to\; \mathbf{1_0} \oplus \mathbf{1_1} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}}$$

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$$\phi \;=\; \left(v+\phi_1,\,\frac{\phi_2}{\sqrt{2}},\,\frac{\phi_2^*}{\sqrt{2}},\,\phi_4,\,\phi_5,\,\phi_6,\,\frac{\phi_7}{\sqrt{2}},\,\frac{\phi_8}{\sqrt{2}},\,\frac{\phi_9}{\sqrt{2}},\,\phi_{10},\,\phi_{11},\,\phi_{12},\,\frac{\phi_7^*}{\sqrt{2}},\,\frac{\phi_8^*}{\sqrt{2}},\,\frac{\phi_9^*}{\sqrt{2}}\right)$$

- ☞ T_7 representations

$$\phi_1 \;\widehat{=}\; \mathbf{1}_0\;, \qquad \phi_2 \;\widehat{=}\; \mathbf{1}_1\;,$$

$$T_1 \;:=\; (\phi_4,\,\phi_5,\,\phi_6) \;\widehat{=}\; \mathbf{3}\;, \qquad T_2 \;:=\; (\phi_7,\,\phi_8,\,\phi_9) \;\widehat{=}\; \mathbf{3}\;,$$

$$\overline{T}_3 \;:=\; (\phi_{10},\,\phi_{11},\,\phi_{12}) \;\widehat{=}\; \overline{\mathbf{3}}$$

T_7 scalar states

- ☞ Branchings:

$$\begin{aligned} \mathbf{8} &\to \mathbf{1}_1 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \\ \mathbf{15} &\to \mathbf{1}_0 \oplus \mathbf{1}_1 \oplus \overline{\mathbf{1}}_1 \oplus \mathbf{3} \oplus \mathbf{3} \oplus \overline{\mathbf{3}} \oplus \overline{\mathbf{3}} \end{aligned}$$

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$$\phi = \left(v + \phi_1, \frac{\phi_2}{\sqrt{2}}, \frac{\phi_2^*}{\sqrt{2}}, \phi_4, \phi_5, \phi_6, \frac{\phi_7}{\sqrt{2}}, \frac{\phi_8}{\sqrt{2}}, \frac{\phi_9}{\sqrt{2}}, \phi_{10}, \phi_{11}, \phi_{12}, \frac{\phi_7^*}{\sqrt{2}}, \frac{\phi_8^*}{\sqrt{2}}, \frac{\phi_9^*}{\sqrt{2}}\right)$$

- ☞ T_7 representations
- ☞ No physical $\mathcal{CP}$ trafo allowed by T_7!

$$\begin{array}{rcccccccccccccccc} & \mathbf{15} & \to & \mathbf{1}_0 & \oplus & \mathbf{1}_1 & \oplus & \overline{\mathbf{1}}_1 & \oplus & \mathbf{3} & \oplus & \mathbf{3} & \oplus & \overline{\mathbf{3}} & \oplus & \overline{\mathbf{3}} \\ \mathbb{Z}_2-\mathrm{Out}: & \big\downarrow & & & & & & & & & & & & & & \\ & \overline{\mathbf{15}} & \to & \mathbf{1}_0 & \oplus & \overline{\mathbf{1}}_1 & \oplus & \mathbf{1}_1 & \oplus & \overline{\mathbf{3}} & \oplus & \overline{\mathbf{3}} & \oplus & \mathbf{3} & \oplus & \mathbf{3} \end{array}$$

T_7 scalar states

- Branchings:

$$\mathbf{8} \;\to\; \mathbf{1_1} \oplus \overline{\mathbf{1}}_\mathbf{1} \oplus \mathbf{3} \oplus \overline{\mathbf{3}}$$

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- Physical scalar fields (would–be Goldstone bosons subtracted)

$$\phi \;=\; \left(v+\phi_1,\, \frac{\phi_2}{\sqrt{2}},\, \frac{\phi_2^*}{\sqrt{2}},\, \phi_4,\, \phi_5,\, \phi_6,\, \frac{\phi_7}{\sqrt{2}},\, \frac{\phi_8}{\sqrt{2}},\, \frac{\phi_9}{\sqrt{2}},\, \phi_{10},\, \phi_{11},\, \phi_{12},\, \frac{\phi_7^*}{\sqrt{2}},\, \frac{\phi_8^*}{\sqrt{2}},\, \frac{\phi_9^*}{\sqrt{2}}\right)$$

- T_7 representations

- No physical $\mathcal{CP}$ trafo allowed by T_7!

$$\begin{tikzcd}
 & \mathbf{15} \arrow[dd] & \rightarrow & \mathbf{1_0} \arrow[dd] & \oplus & \mathbf{1_1} \arrow[ddrr] & \oplus & \overline{\mathbf{1}}_\mathbf{1} \arrow[ddll] & \oplus & \mathbf{3} \arrow[dd] & \oplus & \mathbf{3} \arrow[dd] & \oplus & \overline{\mathbf{3}} \arrow[dd] & \oplus & \overline{\mathbf{3}} \arrow[dd] \\
\mathbb{Z}_2-\mathrm{Out}: & & & & & & & & & & & & & & & \\
 & \overline{\mathbf{15}} & \rightarrow & \mathbf{1_0} & \oplus & \overline{\mathbf{1}}_\mathbf{1} & \oplus & \mathbf{1_1} & \oplus & \overline{\mathbf{3}} & \oplus & \overline{\mathbf{3}} & \oplus & \mathbf{3} & \oplus & \mathbf{3}
\end{tikzcd}$$

Scalar masses

☞ VEV

$$|v| \;=\; \mu \times 3\,\sqrt{\frac{7}{2}}\left(-7\,\sqrt{15}\,\lambda_1+14\,\sqrt{15}\,\lambda_2+20\,\sqrt{6}\,\lambda_4+13\,\sqrt{15}\,\lambda_5\right)^{-1/2}$$

Scalar masses

- ☞ VEV

$$|v| \;=\; \mu \times 3\,\sqrt{\frac{7}{2}}\left(-7\,\sqrt{15}\,\lambda_1 + 14\,\sqrt{15}\,\lambda_2 + 20\,\sqrt{6}\,\lambda_4 + 13\,\sqrt{15}\,\lambda_5\right)^{-1/2}$$

- ☞ T_7 $\mathbf{1}$–plet representations

$$\begin{aligned} \operatorname{Re}\sigma_0 &= \frac{1}{\sqrt{2}}\left(\phi_1+\phi_1^*\right) \qquad \operatorname{Im}\sigma_0 \;=\; -\frac{\mathrm{i}}{\sqrt{2}}\left(\phi_1-\phi_1^*\right)\\ \sigma_1 &= \phi_2 \end{aligned}$$

Scalar masses

- VEV

$$|v| \;=\; \mu \times 3\,\sqrt{\frac{7}{2}}\left(-7\,\sqrt{15}\,\lambda_1 + 14\,\sqrt{15}\,\lambda_2 + 20\,\sqrt{6}\,\lambda_4 + 13\,\sqrt{15}\,\lambda_5\right)^{-1/2}$$

- T$_7$ **1**–plet representations

can be eliminated gauging accidental $\mathrm{U}(1)$

$$\mathrm{Re}\,\sigma_0 \;=\; \frac{1}{\sqrt{2}}\,(\phi_1 + \phi_1^*) \qquad \mathrm{Im}\,\sigma_0 \;=\; -\frac{\mathrm{i}}{\sqrt{2}}\,(\phi_1 - \phi_1^*)$$

$$\sigma_1 \;=\; \phi_2$$

Scalar masses

☞ VEV

$$|v| \;=\; \mu\times 3\,\sqrt{\frac{7}{2}}\left(-7\,\sqrt{15}\,\lambda_1+14\,\sqrt{15}\,\lambda_2+20\,\sqrt{6}\,\lambda_4+13\,\sqrt{15}\,\lambda_5\right)^{-1/2}$$

☞ T_7 $\mathbf{1}$–plet representations

$$\begin{aligned}\mathrm{Re}\,\sigma_0 &= \frac{1}{\sqrt{2}}\,(\phi_1+\phi_1^*) \qquad \mathrm{Im}\,\sigma_0 \;=\; -\frac{\mathrm{i}}{\sqrt{2}}\,(\phi_1-\phi_1^*)\\ \sigma_1 &= \phi_2\end{aligned}$$

☞ Masses

$$\begin{aligned}m^2_{\mathrm{Re}\,\sigma_0} &= 2\,\mu^2\,, \qquad m^2_{\mathrm{Im}\,\sigma_0} \;=\; 0\\ m^2_{\sigma_1} &= -\,\mu^2+\,\sqrt{15}\,\lambda_5\,v^2\end{aligned}$$

Gauge fields

☞ Gauge fields

$$\begin{aligned}
Z^\mu &= \frac{1}{\sqrt{2}}\,(A_7^\mu - \mathsf{i}\,A_8^\mu)\\
W_1^\mu &= \frac{1}{\sqrt{2}}\,(A_4^\mu - \mathsf{i}\,A_1^\mu)\\
W_2^\mu &= \frac{1}{\sqrt{2}}\,(A_5^\mu - \mathsf{i}\,A_2^\mu)\\
W_3^\mu &= \frac{\mathsf{i}}{\sqrt{2}}\,(A_6^\mu - \mathsf{i}\,A_3^\mu)
\end{aligned}$$

Gauge fields

- Gauge fields

$$\begin{aligned}
Z^\mu &= \frac{1}{\sqrt{2}}\,(A_7^\mu - \mathsf{i}\,A_8^\mu)\\
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W_2^\mu &= \frac{1}{\sqrt{2}}\,(A_5^\mu - \mathsf{i}\,A_2^\mu)\\
W_3^\mu &= \frac{\mathsf{i}}{\sqrt{2}}\,(A_6^\mu - \mathsf{i}\,A_3^\mu)
\end{aligned}$$

- Masses

$$m_Z^2 = \frac{7}{3}\,g^2\,v^2 \quad \text{and} \quad m_W^2 = g^2\,v^2$$

Triplet mass eigenstates

- ☞ Mass eigenstates

$$\begin{pmatrix}\tau_1\\ \tau_2\\ \tau_3\end{pmatrix} = \underbrace{\begin{pmatrix}V_{11} & V_{12} & V_{13}\\ V_{21} & V_{22} & V_{23}\\ V_{31} & V_{32} & V_{33}\end{pmatrix}}_{=\,V} \begin{pmatrix}T_2\\ \overline{T}_3^*\\ T_1\end{pmatrix}$$

Triplet mass eigenstates

- ☞ Mass eigenstates

$$\begin{pmatrix}\tau_1\\ \tau_2\\ \tau_3\end{pmatrix} = \underbrace{\begin{pmatrix}V_{11} & V_{12} & V_{13}\\ V_{21} & V_{22} & V_{23}\\ V_{31} & V_{32} & V_{33}\end{pmatrix}}_{=V}\begin{pmatrix}T_2\\ \overline{T}_3^*\\ T_1\end{pmatrix}$$

- ☞ Masses and mixing matrix depend on potential parameters

T_7 outer automorphism vs. $\mathcal{CP}$

☞ $\mathrm{Out}(\mathsf{T}_7)$

$$\begin{array}{lllllll} Z_\mu(x) \mapsto & -\mathcal{P}_\mu^{\ \nu}\, Z_\nu(\mathcal{P}\,x)\;, & \sigma_0(x) \mapsto & \sigma_0(\mathcal{P}\,x)\;, & & \\ W_\mu(x) \mapsto & \mathcal{P}_\mu^{\ \nu}\, W_\nu^*(\mathcal{P}\,x)\;, & \sigma_1(x) \mapsto & \sigma_1(\mathcal{P}\,x)\;, & \tau_i(x) \mapsto & \tau_i^*(\mathcal{P}\,x) \end{array}$$

T_7 outer automorphism vs. $\mathcal{CP}$

- ☞ $\mathrm{Out}(\mathsf{T}_7)$

$$\begin{array}{llll} Z_\mu(x) \;\mapsto\; -\mathcal{P}_\mu^{\;\nu}\,Z_\nu(\mathcal{P}\,x)\;, & \sigma_0(x) \;\mapsto\; \sigma_0(\mathcal{P}\,x)\;, & \\ W_\mu(x) \mapsto \quad \mathcal{P}_\mu^{\;\nu}\,W_\nu^*(\mathcal{P}\,x)\;, & \sigma_1(x) \;\mapsto\; \sigma_1(\mathcal{P}\,x)\;, & \tau_i(x) \;\mapsto\; \tau_i^*(\mathcal{P}\,x) \end{array}$$

- ☞ Mode expansion

$$\boldsymbol{\sigma}_1(x) \;=\; \int\widetilde{\mathrm{d}p}\,\left\{\boldsymbol{a}(\vec{p})\,\mathrm{e}^{-\mathsf{i}\,p\,x}+\boldsymbol{b}^\dagger(\vec{p})\,\mathrm{e}^{\mathsf{i}\,p\,x}\right\}$$

T_7 outer automorphism vs. $\mathcal{CP}$

- ☞ $\mathrm{Out}(\mathsf{T}_7)$

$$\begin{array}{lllllll} Z_\mu(x) & \mapsto & -\mathcal{P}_\mu^{\nu}\, Z_\nu(\mathcal{P}\,x)\;, & \sigma_0(x) & \mapsto & \sigma_0(\mathcal{P}\,x)\;, & \\ W_\mu(x) & \mapsto & \mathcal{P}_\mu^{\nu}\, W_\nu^*(\mathcal{P}\,x)\;, & \sigma_1(x) & \mapsto & \sigma_1(\mathcal{P}\,x)\;, & \tau_i(x) \;\mapsto\; \tau_i^*(\mathcal{P}\,x) \end{array}$$

- ☞ Mode expansion

$$\boldsymbol{\sigma}_1(x) \;=\; \int \widetilde{\mathrm{d}p}\,\left\{\boldsymbol{a}(\vec{p})\,\mathrm{e}^{-\mathsf{i}\,p\,x} + \boldsymbol{b}^\dagger(\vec{p})\,\mathrm{e}^{\mathsf{i}\,p\,x}\right\}$$

- ☞ Outer automorphism of T_7

$$\mathrm{Out}(\mathsf{T}_7)\;:\quad \boldsymbol{a}(\vec{p}) \;\mapsto\; \boldsymbol{a}(-\vec{p}) \qquad \text{and} \qquad \boldsymbol{b}^\dagger(\vec{p}) \;\mapsto\; \boldsymbol{b}^\dagger(-\vec{p})$$

T_7 outer automorphism vs. $\mathcal{CP}$

- ☞ $\mathrm{Out}(\mathsf{T}_7)$

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- ☞ QFT $\mathcal{CP}$ not a symmetry of the action

$$\mathcal{CP}\ :\quad \boldsymbol{a}(\vec{p}) \mapsto \boldsymbol{b}(-\vec{p}) \qquad \text{and} \qquad \boldsymbol{b}^\dagger(\vec{p}) \mapsto \boldsymbol{a}^\dagger(-\vec{p})$$

$\mathcal{CP}$ violation in the T_7 phase

- ☞ Decay asymmetry

$$\varepsilon_{\sigma_1\to W\,W^*} \;:=\; \frac{\left|\mathscr{M}(\sigma_1\to W\,W^*)\right|^2-\left|\mathscr{M}(\sigma_1^*\to W\,W^*)\right|^2}{\left|\mathscr{M}(\sigma_1\to W\,W^*)\right|^2+\left|\mathscr{M}(\sigma_1^*\to W\,W^*)\right|^2}$$

$\mathcal{CP}$ violation in the T_7 phase

- Decay asymmetry

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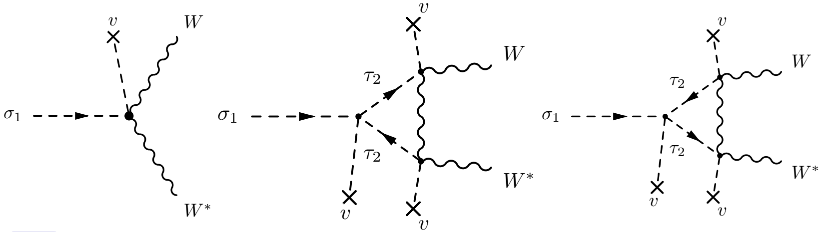
- $\mathcal{CP}$ violation from interference between tree–level and 1–loop



back

Metaplectic flavor symmetries

(Details)

Modular vs. metaplectic flavor symmetries

☞ The zero modes have halfinteger modular weights

$$K_{i\bar{\imath}} \propto \frac{1}{(\operatorname{Im}\tau)^{1/2}}$$

Modular vs. metaplectic flavor symmetries

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internal 4D

object	$\psi^{j,M}$	$\phi^{j,M}$	$\Omega^{j,M}$	Y_{ijk}	$\mathscr{W}$
modular weight k	$^1/_2$	$-^1/_2$	0	$^1/_2$	-1

$$\Omega^{j,M} = \phi^{j,M}(x^\mu) \otimes \psi^{j,M}(z,\tau)$$

Modular vs. metaplectic flavor symmetries

☞ The zero modes have halfinteger modular weights

$$K_{i\bar{\imath}} \propto \frac{1}{(\operatorname{Im}\tau)^{1/2}}$$

object	$\psi^{j,M}$	$\phi^{j,M}$	$\Omega^{j,M}$	Y_{ijk}	$\mathscr{W}$
modular weight k	$1/2$	$-1/2$	0	$1/2$	-1

☞ One has to be careful with signs in modular transformations: metaplectic symmetries

Transformation laws for 4D superfields (for odd M)

$$\psi^{j,M}\,(z,\tau,0) \xmapsto{S} \frac{\mathrm{e}^{\mathrm{i}\,\frac{\pi}{4}}}{\sqrt{M}}\left(-\frac{\tau}{|\tau|}\right)^{1/2}\sum_{k=0}^{M-1}\mathrm{e}^{2\pi\mathrm{i}jk/M}\,\psi^{k,M}\,(z,\tau,0)$$

$$= -\left(-\frac{\tau}{|\tau|}\right)^{1/2}\left[\rho(S)^{\psi}_{M}\right]_{jk}\psi^{k,M}(z,\tau,0)$$

$$\psi^{j,M}\,(z,\tau,0) \xmapsto{T} \mathrm{e}^{\mathrm{i}\pi M\frac{\operatorname{Im} z}{2\operatorname{Im}\tau}}\,\mathrm{e}^{\mathrm{i}\pi j(j/M+1)}\,\psi^{j,M}(z-1/2,\tau,0)$$

$$= \mathrm{e}^{\mathrm{i}\pi M\frac{\operatorname{Im} z}{2\operatorname{Im}\tau}}\left[\rho(T)^{\psi}_{M}\right]_{jk}\psi^{k,M}(z-1/2,\tau,0)$$

☞ Representation matrices of generators

$$\left[\rho(S)^{\psi}_{M}\right]_{jk} = -\frac{\mathrm{e}^{\mathrm{i}\pi/4}}{\sqrt{M}}\exp\left(\frac{2\pi\mathrm{i}\,j\,k}{M}\right)$$

$$\left[\rho(T)^{\psi}_{M}\right]_{jk} = \exp\left[\mathrm{i}\pi\,j\left(\frac{j}{M}+1\right)\right]\delta_{jk}$$

Transformation laws for Yukawa couplings

$$\mathcal{Y}_{\widehat{\alpha}}(\tau) \xmapsto{\widetilde{\gamma}} \mathcal{Y}_{\widehat{\alpha}}(\widetilde{\gamma}\,\tau) = \pm(c\,\tau + d)^{1/2}\,\rho_{\boldsymbol{\lambda}}(\widetilde{\gamma})_{\widehat{\alpha}\widehat{\beta}}\,\mathcal{Y}_{\widehat{\beta}}(\tau)$$

- ☞ Representation matrices of generators

$$\rho_{\boldsymbol{\lambda}}(\widetilde{S})_{\widehat{\alpha}\widehat{\beta}} = -\frac{\mathrm{e}^{\mathrm{i}\pi/4}}{\sqrt{\lambda}}\,\exp\left(\frac{2\pi\mathrm{i}\,\widehat{\alpha}\,\widehat{\beta}}{\lambda}\right)$$

$$\rho_{\boldsymbol{\lambda}}(\widetilde{T})_{\widehat{\alpha}\widehat{\beta}} = \exp\left(\frac{\mathrm{i}\pi\,\widehat{\alpha}^2}{\lambda}\right)\,\delta_{\widehat{\alpha}\widehat{\beta}}$$

bottom–line:

Magnetized tori with $\lambda = \mathrm{lcm}(\#\text{ of flavors})$ exhibit a $\widetilde{\Gamma}_{2\lambda}$ modular flavor symmetry

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