On effective field theory from string compactification

Tatsuo Kobayashi

1. Introduction
2. Couplings in LEEFT
3. Modular symmetries
4. Comment on kinetic terms
5. Summary

1. Introduction
toward the unified theory

string open string closed string

Superstring theory : the promising candidate for
the unified theory of our world, that is,
all of interactions including gravity
and matter such as quarks and leptons, higgs,
and all of cosmological aspects.

Theory of Everything

Introduction

We have already obtained lots of
heterotic string compactifications
and D-brane models, which lead to
SM gauge groups SU(3)xSU(2)xU(1)
as well as its extensions
and three generations of quarks and leptons.

We have already lots of realistic massless spectra
through string compactifcations.

Introduction

We have already lots of realistic massless spectra through string compactifcations.

What we need are

realistic Yukawa couplings $\Rightarrow$ quark, lepton masses, mixing, CP

other terms in Lagrangian of low-energy effective field theory

symmetries to control LEEFT

2. Couplings in LEEFT

string (massless) modes

CFT operators

OPE

$$V_i(z)V_j(0) \sim \sum_k \frac{y_{ijk}}{z^{h_k-h_i-h_j}} V_k(0),$$

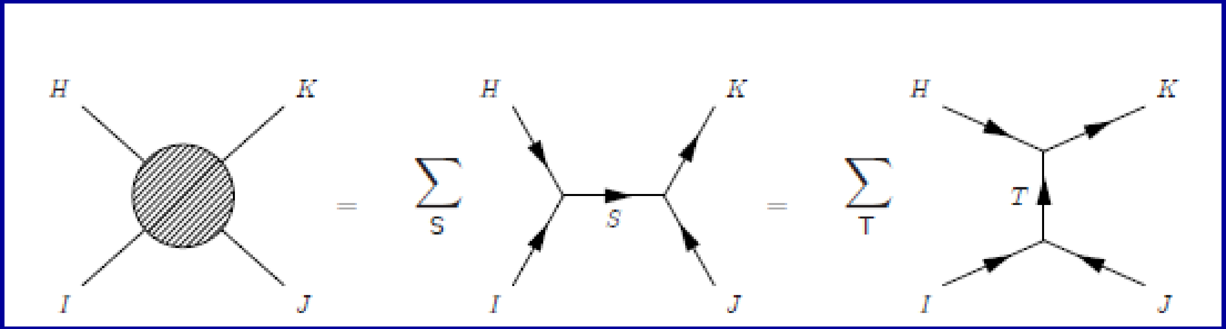
y_{ijk}

3-point coupling

Yukawa coupling

2. Couplings

4-point coupling



$$y_{ijk\ell} \sim y_{ijm} y_{mk\ell}$$

higher order couplings

$$\text{n-point coupling} \quad \sim (y_{ijk})^{n-2}$$

2. Couplings

explicit computations

$$y_{ijk\ell} \sim y_{ijm} y_{mk\ell}$$

$$\text{n-point coupling} \quad \sim (y_{ijk})^{n-2}$$

mode m may correspond to massless

or massive modes

intersecting D-brane models

Abel, Owen '04

heterotic string theory on orbifolds

Choi, T.K. '08

2. Couplings

explicit computations

$$y_{ijk\ell} \sim y_{ijm} y_{mk\ell}$$

n-point coupling $\sim (y_{ijk})^{n-2}$

mode m may correspond to massless

or massive modes

This structure has been well-known for a long time,
and many people have known already

Anyway, this structure is a typical character
in string-derived low-energy effective field theory.
We would like to (re)study its implications.

2. Couplings

Structure in couplings

$$y_{ijk\ell} \sim y_{ijm} y_{mk\ell}$$

$$\text{n-point coupling} \quad \sim (y_{ijk})^{n-2}$$

Another derivation from field thoery

2. Couplings

string modes $\Rightarrow$ fields in 4+6 dimensions

field theory in 4 + 6 dimensions

KK decomposition

$$\Psi = \sum_n \chi_n(x)\psi_n(y)$$

For example,

massless modes on magnetized D-brane

with torus, orbifold background

$$i\gamma^i(\partial_i - iA_i)\psi(y) = 0$$

2. Couplings

several magnetized D-branes

two types of modes with different magnetic fluxes

ψ^i, ψ^j

w.f. product expansion

Their product can be expanded by other modes

$\psi^i \psi^j = \sum_k c_{ijk} \psi^k$

because

$$i\gamma^i(\partial_i - iA_i^{(i)})\psi^i = 0$$

$$i\gamma^i(\partial_i - iA_i^{(j)})\psi^j = 0$$

$$i\gamma^i(\partial_i - i(A_i^{(i)} + A_i^{(j)}))\psi^k = 0$$

$$i\gamma^i(\partial_i - i(A_i^{(i)} + A_i^{(j)}))\psi^i\psi^j = 0$$

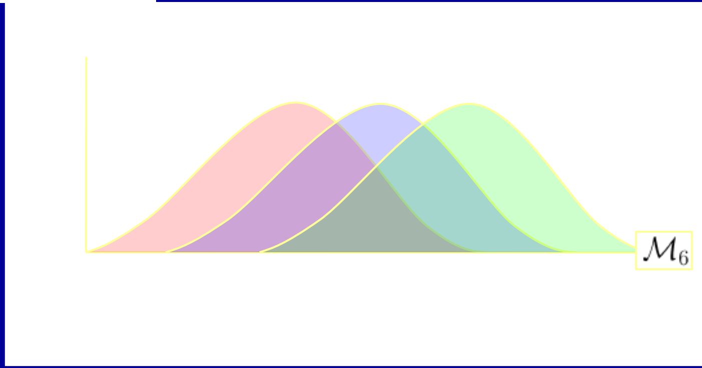
2. couplings

3-point couplings

The 3-point couplings are obtained by overlap integral of three zero-mode w.f.'s.

$$Y_{ijk} = \int d^2 z \, \psi_M^i(z) \psi_N^j(z) \left(\psi_{M+N}^k(z) \right)^*$$

$$\psi^i \psi^j = \sum_k c_{ijk} \psi^k$$



$$\int d^2 z \;\; \psi_M^i(z) \left(\psi_M^k(z) \right)^* = \delta^{ik}$$

Yukawa couplings= expansion coefficients $y_{ijk} = c_{ijk}$

See for explicit form,

Cremades, Ibanez, Marchesano, '04

Magnetized D-brane models

4-point coupling Abe, Choi, T.K., Ohki, 0903.3800

$$y_{ijk\ell} = \int d^2z \; \underline{\psi^{i,M_1}(z)\psi^{j,M_2}(z)}\psi^{k,M_3}(z)(\psi^{\ell,M_4}(z))^*$$

$$\psi^{i,M}(z)\psi^{j,N}(z) = \sum_k c_{ijk}\psi^{k,M+N}(z)$$

w.f. product expansions

$$y_{ijk\ell} = \int d^2z \; \sum_s c_{ijs}\psi^{s,M_1+M_2}(z)\psi^{k,M_3}(z)(\psi^{\ell,M_4}(z))^*$$

$$y_{ijk\ell} = \sum_s c_{ijs}c_{sk\ell}$$

Magnetized D-brane models

4-point coupling

$$y_{ijk\ell} = \int d^2z \, \psi^{i,M_1}(z)\underline{\psi^{j,M_2}(z)\psi^{k,M_3}(z)}(\psi^{\ell,M_4}(z))^*$$

$$\psi^{i,M}(z)\psi^{j,N}(z) = \sum_k c_{ijk}\psi^{k,M+N}(z)$$

w.f product expansions

$$y_{ijk\ell} = \int d^2z \, \sum_t c_{jkt}\psi^{t,M_2+M_3}(z)\psi^{i,M_1}(z)(\psi^{\ell,M_4}(z))^*$$

$$y_{ijk\ell} = \sum_t c_{jkt}c_{ti\ell}$$

Magnetized D-brane models

4-point coupling

$$y_{ijk\ell} = \sum_s c_{ijs} c_{sk\ell}$$

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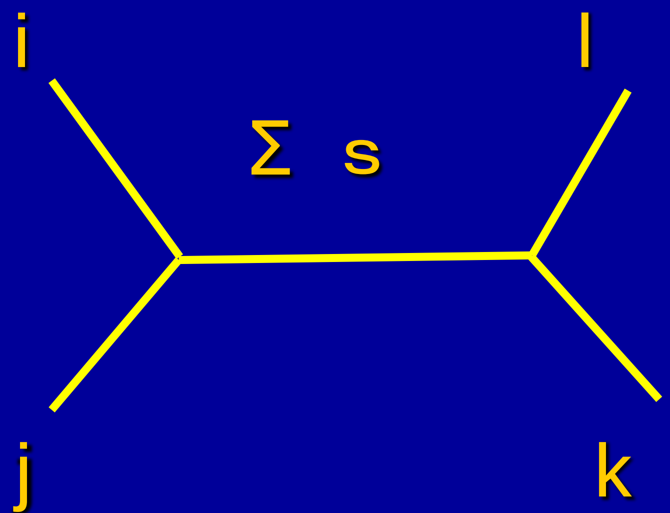

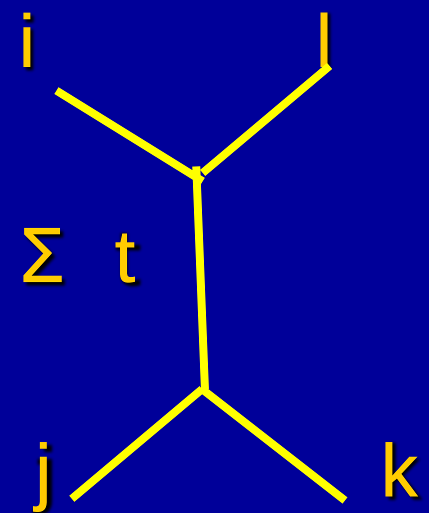

Magnetized D-brane models

Similar computation for higher order couplings

$$y_{ij\cdots} = \int d^2z \; \psi^{i,M_1}(z)\psi^{j,M_2}(z)\cdots$$

$$\psi^{i,M}(z)\psi^{j,N}(z) = \sum_k c_{ijk}\psi^{k,M+N}(z)$$

$$y_{ij\cdots} = \sum c_{ijn}c_{nk\ell}\cdots$$

possibility for compact space

other than torus orbifold

Honda, T.K.,Otsuka, arXiv:1812.03357

2. Couplings

Many string compactifications lead to the following structure

4-point coupling

$$y_{ijk\ell} \sim y_{ijm} y_{mk\ell}$$

higher order couplings

$$\text{n-point coupling} \quad \sim (y_{ijk})^{n-2}$$

symmetries of 3-point couplings

$\Rightarrow$ symmetries of all higher order couplings

2. Couplings

Many string compactifications lead to the following structure

4-point coupling

$$y_{ijk\ell} \sim y_{ijm} y_{mk\ell}$$

higher order couplings

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This structure appears for string perturbation.

What about non-perturbative effects ?

We have examples to satisfy this rule in D-brane instanton effects for unbroken symmetries.

We need more studies on non-perturbative effects.

2. Couplings

Many string compactifications lead to the
following structure

$$y_{ijk\ell} \sim y_{ijm} y_{mk\ell}$$

4-point coupling

higher order couplings

$$\text{n-point coupling} \quad \sim (y_{ijk})^{n-2}$$

This structure has been well-known for a long time.

Although many people have known this,
this is a typical character in string-derived LEEFT.

Maybe it is important to (re)study
the implications of this structure among couplings.

I would like discuss them, and please tell me
if you have any opinions about its implications.

One example is as follows.

2. Couplings

T.K.,Otsuka, 2108.02700

4-point coupling $y_{ijk\ell} \sim y_{ijm} y_{mk\ell}$

phenomenological implications

$$\frac{c_{\bar{i}j\bar{k}l}}{\Lambda^2}(\bar{Q}_i \gamma_\mu P_{L,R} Q_j)(\bar{Q}_k \gamma^\mu P_{L,R} Q_l),$$

$$\frac{d_{\bar{i}j\bar{k}l}}{\Lambda^2}(\bar{Q}_i \gamma_\mu P_{L,R} Q_j)(\bar{L}_k \gamma^\mu P_{L,R} L_l),$$

can be obtained by $\sum_m y_{\bar{i}jm} y_{\bar{m}\bar{k}l}$

Cf. minimal flavor violation scenario in SMEFT

D'Ambrosio,Giudice,Isidori,Strumia, '02

Standar Model Effective Field Theory (SMEFT)

Renomalizable SM Lagrangian

+ higher dimensional operators

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \frac{c^{(5)}}{\Lambda_{\mathrm{NP}}}\mathcal{O}_5 + \frac{c^{(6)}}{\Lambda_{\mathrm{NP}}}\mathcal{O}_6 + \cdots$$

SM-gauge group invariant

Minimal flavor violation hypothesis in SMEFT

D'Ambrosio, Giudice, Ishidori, Strumia, '02

In the limit that all of yukawa couplings vanish, there are flavor symmetries

$$G_F = SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_{L_L} \times SU(3)_{e_R}$$

Higgs field is singlet

Yukawa couplings are spurions

$$Y_u \ : \ (3, \bar{3}, 1, 1, 1), \qquad Y_d \ : \ (3, 1, \bar{3}, 1, 1)$$
$$Y_e \ : \ (, 1, 1, 1, 3, \bar{3})$$

MFV hypothesis in SMEFT

D'Ambrosio, Giudice, Ishidori, Strumia, '02

$$G_F = SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_{L_L} \times SU(3)_{e_R}$$

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$$Y_e \ : \ (, 1, 1, 1, 3, \bar{3})$$

Ordinary Yukawa couplings are written in terms of these spurions.

Also higher order couplings are GF-invariant and can be written in terms of these spurions.

$$\frac{c_{\bar{i}j\bar{k}l}}{\Lambda^2} (\bar{Q}_i \gamma_\mu P_{L,R} Q_j)(\bar{Q}_k \gamma^\mu P_{L,R} Q_l),$$

$$\frac{d_{\bar{i}j\bar{k}l}}{\Lambda^2} (\bar{Q}_i \gamma_\mu P_{L,R} Q_j)(\bar{L}_k \gamma^\mu P_{L,R} L_l),$$

MFV hypothesis in SMEFT

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$$Y_e \; : \; (, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$$

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$$\frac{c_{\bar{i}j\bar{k}l}}{\Lambda^2}(\bar{Q}_i\gamma_\mu P_{L,R} Q_j)(\bar{Q}_k\gamma^\mu P_{L,R} Q_l),$$

$$\frac{d_{\bar{i}j\bar{k}l}}{\Lambda^2}(\bar{Q}_i\gamma_\mu P_{L,R} Q_j)(\bar{L}_k\gamma^\mu P_{L,R} L_l),$$

Flavor and CP violations are controlled by Yukawa couplings.

Stringy Couplings

4-point coupling $y_{ijk\ell} \sim y_{ijm} y_{mk\ell}$

higher order couplings

$$\text{n-point coupling} \quad \sim (y_{ijk})^{n-2}$$

These behaviors looks like
MFV hypothesis.

symmetries of 3-point couplings
$\Rightarrow$ symmetries of all higher order couplings

Below compatification scale

4-point coupling $y_{ijk\ell} \sim y_{ijm} y_{mk\ell}$

higher order couplings n-point coupling $\sim (y_{ijk})^{n-2}$

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \frac{c^{(5)}}{\Lambda_{\mathrm{NP}}} \mathcal{O}_5 + \frac{c^{(6)}}{\Lambda_{\mathrm{NP}}} \mathcal{O}_6 + \cdots$$

In stringy computation,
the cutoff scale is the compactification scale.

Below the compactification,
(i) some modes become massive
(ii) some fields develop VEVs

Below compatification scale

(i) some massive modes

we integrate such modes

we still obtain the structure

$$y_{ijk\ell} \sim y_{ijm} y_{mk\ell}$$

$$\text{n-point coupling} \quad \sim (y_{ijk})^{n-2}$$

The effective cutoff scale is written by masses of such massive modes, and would be different from the compactification scale.

Below compatification scale

(ii) some fields develop VEVS

we integrate such modes

$$y_{ijk\ell}\phi^i\phi^j\phi^k\phi^\ell \rightarrow y_{ijk\ell} < \phi^i > \phi^j\phi^k\phi^\ell$$

for unbroken symmetries

we still obtain the structure

$$y_{ijk\ell} \sim y_{ijm}y_{mk\ell}$$

$$\text{n-point coupling} \quad \sim (y_{ijk})^{n-2}$$

The effective cutoff scale would be different from the compactification scale.

Couplings in string theory

MFV

$$G_F = SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_{L_L} \times SU(3)_{e_R}$$

Yukawa couplings are spurions

$$Y_u \ : \ (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1}), \qquad Y_d \ : \ (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1})$$
$$Y_e \ : \ (, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$$

In string theory, couplings depend on moduli.

In a sense, couplings are spurions

if we ignore dynamics of moduli.

The symmetries of moduli control SMEFT.

Couplings in string theory

MFV

$$G_F = SU(3)_{Q_L} \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_{L_L} \times SU(3)_{e_R}$$

Yukawa couplings are spurions

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$$Y_e \ : \ (, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$$

The symmetries of moduli control SMEFT.

The symmetries of moduli, under which couplings transform non-trivially, would be important.

3. Modular symmetry

The symmetries of moduli, under which couplings non-trivially transform, would be important.

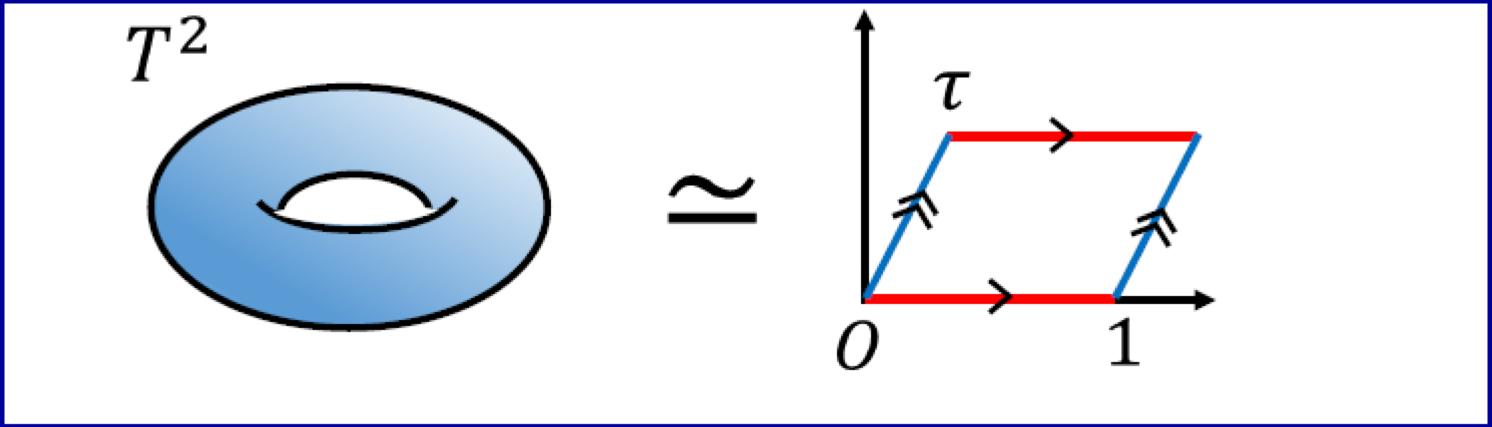
One example is the modular symmetry.

Recently, lots of studies have been done in the top-down (stringy) approach

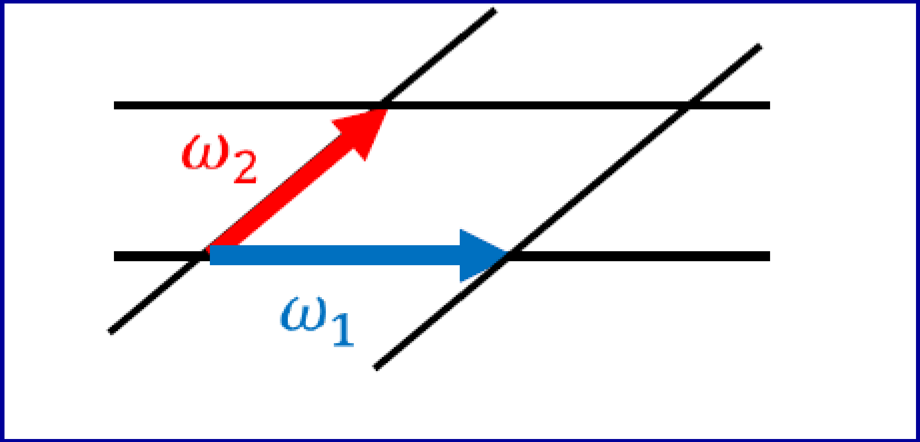
bottom-up approach
field-theoretical model building

3. Modular symmetry

torus compactification



modulus



Lattice vectors

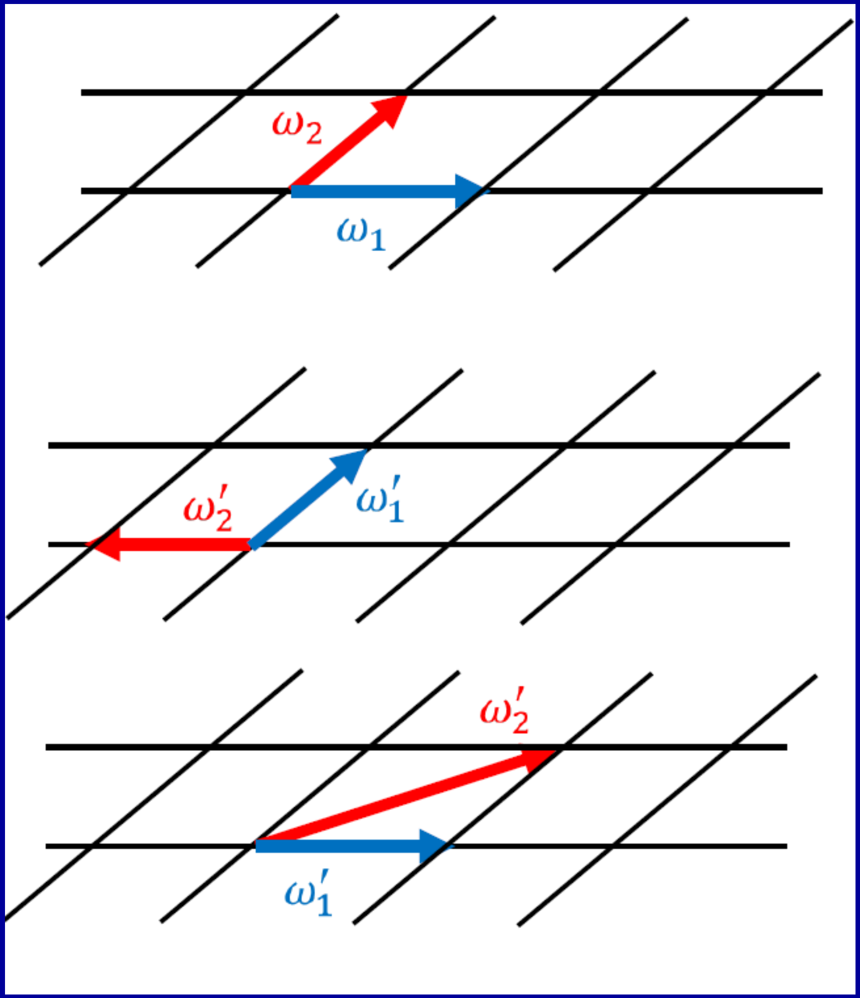
copyright by my student (Tatsuishi)

Modular symmetry

change of

lattice vectors

(cycle basis)



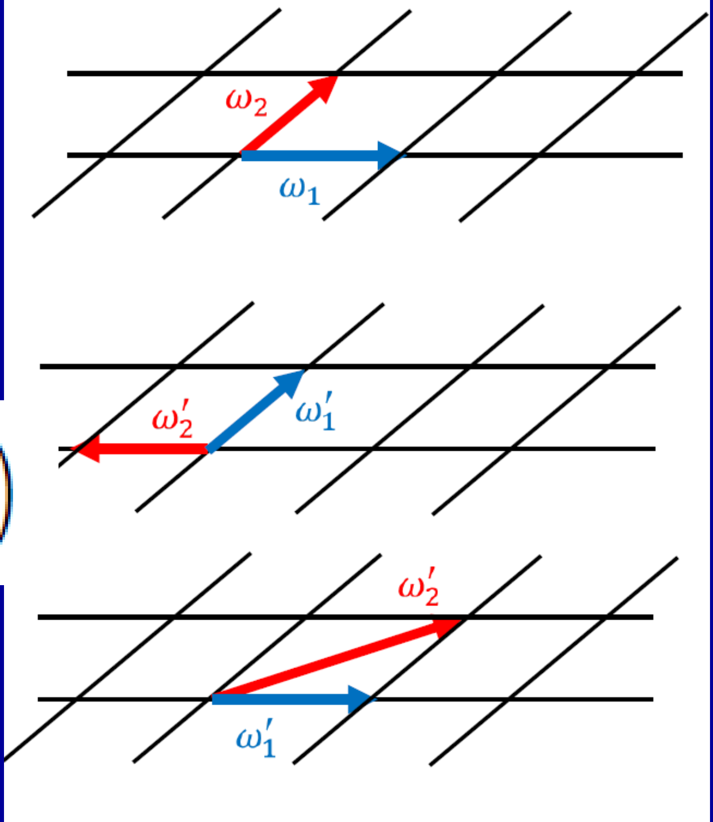
Modular symmetry

Change of lattice vectors

of cycle basis

SL(2,Z)

$$\begin{pmatrix} \omega_2' \\ \omega_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \omega_2 \\ \omega_1 \end{pmatrix}$$



(homogeneous)

modular symmetry

$$SL(2,\mathbb{Z}) \equiv \Gamma,$$

Modulus

$$\tau = \frac{\omega_2}{\omega_1}$$

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d}$$

Modular symmetry

When

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

we obtain

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d} = \frac{-\tau}{-1} = \tau$$

(imhomogeneous) Modular symmetry
= SL(2,Z)/{I,-I}

$$\bar{\Gamma} \equiv \Gamma / \{\pm \mathbb{I}\}$$

remark tau is the ratio of two basis vectors

Modular symmetry

generators of modular group S and T

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

for tau

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d}$$

$$S : \tau \longrightarrow -\frac{1}{\tau},$$

$$T : \tau \longrightarrow \tau + 1.$$

Modular symmetry

Generator S and T

$$S : \tau \longrightarrow -\frac{1}{\tau},$$
$$T : \tau \longrightarrow \tau + 1.$$

algebraic relations

$$S^2 = 1, \qquad (ST)^3 = 1.$$

infinite number of elements

Modular symmetry

Generator S and T

for SL(2,Z) $S^2 = -1$

it is identified as I=-I in modular group

algebraic relations of modular group

$$S^2 = 1, \qquad (ST)^3 = 1.$$

in finite number of elements

SL(2,Z) double covering of modular group

$$S^2 = -1 \qquad (ST)^3 = 1.$$

Congruence subgroup

$$\Gamma(N) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in SL(2,\mathbb{Z}) \middle| \begin{bmatrix} a & b \\ c & d \end{bmatrix} \equiv \begin{bmatrix} 1 \pmod N & 0 \pmod N \\ 0 \pmod N & 1 \pmod N \end{bmatrix} \right\}$$

S, T are not included

Generator S and T

T^N is included

Quotients

$$\Gamma_N = \Gamma/\Gamma(N)$$

$$\Gamma_N \equiv \bar{\Gamma}/\bar{\Gamma}(N), \text{ where } \bar{\Gamma}(N) \equiv \Gamma(N)/\{\pm\mathbb{I}\} \text{ for } N = 1, 2$$

$$S^2 = 1, \qquad (ST)^3 = 1.$$

$$T^N = 1,$$

Γ_N = S3, A4, S4, A5 for N=2,3,4,5

Δ(96), Δ(384) are also included in Γn

N =8, 16

Modular forms

$$\tau \longrightarrow \gamma(\tau) = \frac{a\tau + b}{c\tau + d}$$

$$f(\gamma\tau)_i \;=\; (c\tau + d)^w \rho(\gamma)_{ij} f_j(\tau)$$

w: modular weight

Γ(N) modular form

$$f(\gamma\tau)_i = (c\tau + d)^w f_i(\tau)$$

$$\rho(\gamma)_{ij} \;:\; \text{representaion of } \Gamma_N$$

S3,A4,S4,A5,

Δ(96), Δ(384), ...

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S^2 = 1$$

$$f(S^2\tau)_i \;=\; (-1)^w \rho(S^2)_{ij} f_j(\tau)$$

weight w must be even

Modular forms

$$\tau \longrightarrow \gamma(\tau) = \frac{a\tau + b}{c\tau + d}$$

$$f(\gamma\tau)_i \;=\; (c\tau + d)^w \rho(\gamma)_{ij} f_j(\tau)$$

$$f(S^2\tau)_i \;=\; (-1)^w \rho(S^2)_{ij} f_j(\tau)$$

modular group $\bar{\Gamma} \equiv \Gamma/\{\pm\mathbb{I}\}$ $S^2 = 1$

weight w must be even

SL(2,Z) double covering of modular group $S^2 = -1$

weight can be inter including odd

SL(2,Z) transformation of basis vector

Double covering of SL(2,Z) weight can be half-integer spinor in the complex plane

Example: magnetized D-brane

Wave function and Yukawa couplings

on T2

$$\psi_0^{j,|M|}(z,\tau) = \left(\frac{|M|}{\mathcal{A}^2}\right)^{1/4} e^{i\pi|M|(z+\zeta)\frac{\mathrm{Im}(z+\zeta)}{\mathrm{Im}\tau}} \vartheta\begin{bmatrix} \frac{j}{|M|} \\ 0 \end{bmatrix}(|M|z,|M|\tau),$$

$$(j = 0, 1, \cdots, |M|-1)$$

on T2/Z2

$$\psi^{j,M}_{T^2/\mathbb{Z}_2^{(t)m}}(z,\tau) = \mathcal{N}^j_{(t)}\left(\psi^{j,M}_{T^2}(z,\tau) + (-1)^m \psi^{j,M}_{T^2}(-z,\tau)\right),$$

Both wave functions have weight 1/2

couplings

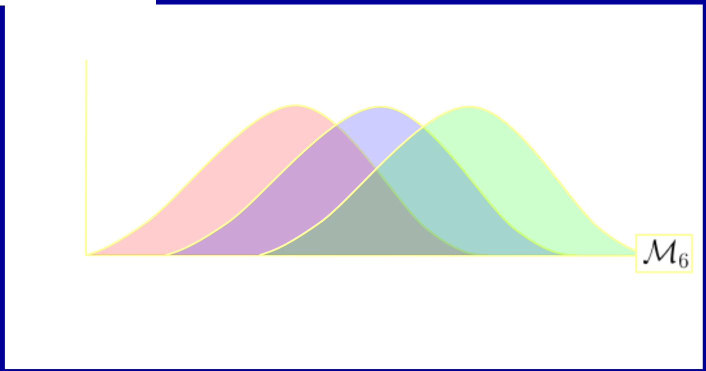
3-point couplings

The 3-point couplings are obtained by overlap integral of three zero-mode w.f.'s.

$$Y_{ijk} = \int d^2 z \, \psi_M^i(z) \psi_N^j(z) \left(\psi_{M+N}^k(z) \right)^*$$

$$\psi^i \psi^j = \sum_k c_{ijk} \psi^k$$

$$\int d^2 z \; \psi_M^i(z) \left(\psi_M^k(z) \right)^* = \delta^{ik}$$



Yukawa couplings= expansion coefficients $y_{ijk} = c_{ijk}$

See for explicit form,

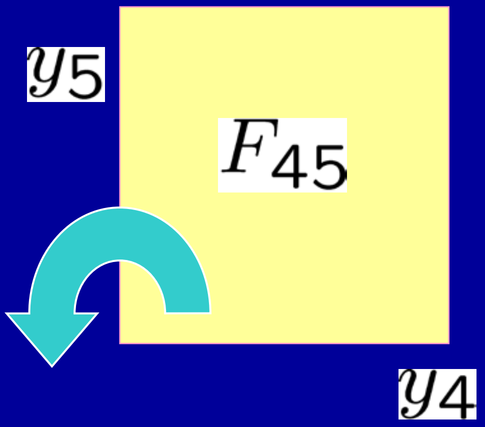
Cremades, Ibanez, Marchesano, '04

Orbifold models without SS phase(=WL)

Abe, T.K., Ohki, '08

The number of even and odd zero-modes

$M = I^{ab}$	0	1	2	3	4	5	6	7	8	9	10
even	1	1	2	2	3	3	4	4	5	5	6
odd	0	0	0	1	1	2	2	3	3	4	4



We can also embed Z2 into the gauge space.

$$Z_2 : \psi(y_4, y_5) \to \psi(-y_4, -y_5) = (-i)\Gamma^4\Gamma^5 P\psi(-y_4, -y_5)$$

$$(P^2 = 1)$$

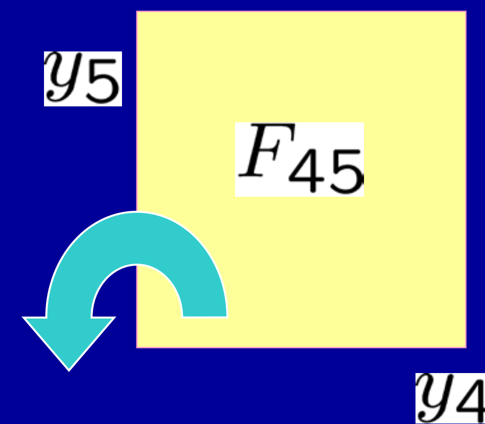
Orbifolding projects out adjoint matter fields.

Orbifold models with SS phase(=WL)

Abe,Fujimoto, T.K.,Miura, Nishiwaki, '13

The number of even and odd zero-modes

(α_1, α_τ)	M	$\psi^{(j+\alpha_1,\alpha_\tau)}_{T^2/Z_2\pm,0}(z)_{+1}$	$\psi^{(j+\alpha_1,\alpha_\tau)}_{T^2/Z_2\pm,0}(z)_{-1}$
$(0,0)$	even	$\frac{\lvert M\rvert}{2}+1$	$\frac{\lvert M\rvert}{2}-1$
	odd	$\frac{\lvert M\rvert+1}{2}$	$\frac{\lvert M\rvert-1}{2}$
$(\frac{1}{2},0)$	even	$\frac{\lvert M\rvert}{2}$	$\frac{\lvert M\rvert}{2}$
	odd	$\frac{\lvert M\rvert+1}{2}$	$\frac{\lvert M\rvert-1}{2}$
$(0,\frac{1}{2})$	even	$\frac{\lvert M\rvert}{2}$	$\frac{\lvert M\rvert}{2}$
	odd	$\frac{\lvert M\rvert+1}{2}$	$\frac{\lvert M\rvert-1}{2}$
$(\frac{1}{2},\frac{1}{2})$	even	$\frac{\lvert M\rvert}{2}$	$\frac{\lvert M\rvert}{2}$
	odd	$\frac{\lvert M\rvert-1}{2}$	$\frac{\lvert M\rvert+1}{2}$



Example: magnetized D-brane

Wave functions and Yukawa couplings have the modular weight ½.

$$\psi_0^{j,|M|}(z,\tau) = \left(\frac{|M|}{\mathcal{A}^2}\right)^{1/4} e^{i\pi|M|(z+\zeta)\frac{\mathrm{Im}(z+\zeta)}{\mathrm{Im}\tau}}\vartheta\begin{bmatrix}\frac{j}{|M|}\\ 0\end{bmatrix}(|M|z,|M|\tau),$$

They transform each other under the modular symmetry.
That is the flavor symmetry, which also transform Yukawa couplings non-trivially.

Flavor groups are covering groups of S3, A4, S4, A5, Δ(96), Δ(384), PSL(2,7) with center extensions. Kikuchi, T.K.Uchida, ‘21

Example: magnetized D-brane

Wave functions and Yukawa couplings have the modular weight ½.

$$\psi_0^{j,|M|}(z,\tau) = \left(\frac{|M|}{\mathcal{A}^2}\right)^{1/4} e^{i\pi|M|(z+\zeta)\frac{\mathrm{Im}(z+\zeta)}{\mathrm{Im}\tau}} \vartheta \begin{bmatrix} \frac{j}{|M|} \\ 0 \end{bmatrix} (|M|z, |M|\tau),$$

Why the modular weight ½ ?

maybe spinor representation,

but not sure.

Modular symmetry

Application of modular symmetry
in SMEFT would be interesting.

T.K., Otsuka, 2108.02700
T.K., Otsuka, Tanimoto, Yamamoto, 2111.XXXXX

Generic compact space

Generic 6-D compact space has many moduli,
e.g. Calabi-Yau

For example, holomorphic three-form is expanded by symplectic basis

$$\Omega = \sum_{I=0}^{h^{2,1}} \left(X^I \alpha_I - \mathcal{F}_I \beta^I \right),$$

$$\int_{\mathcal{M}} \alpha_I \wedge \beta^J = \delta^J{}_I, \quad \int_{\mathcal{M}} \beta^J \wedge \alpha_I = -\delta^J{}_I,$$

Generic compact space

CY

For example, holomorphic three-form is expanded by symplectic basis

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$$\int_{\mathcal{M}} \alpha_I \wedge \beta^J = \delta^J{}_I, \quad \int_{\mathcal{M}} \beta^J \wedge \alpha_I = -\delta^J{}_I,$$

We can change the basis,

$$\begin{pmatrix} \alpha_I \\ \beta^I \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_I \\ \beta^I \end{pmatrix},$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in Sp(2h^{2,1}+2, \mathbb{Z}).$$

Symplectic modular symmetry

Strominger, '90

Candelas, de la Ossa '91

Generic compact space

CY

moduli are defined by

$$u^i \equiv \frac{X^i}{X^0}.$$

XI projective coordinate on special geometry

ui inhomogeneous coordinates

Strominger '90, Candelas, de la Ossa '91

Any way, the modular symmetry is quite rich.

Kahler moduli also have symplectic modular symmetry

Simple example

Generic symplectic modular symmetry is complicated.

T6/ZN

3 diagonal Kahler moduli + dilaton

$$K = \ln(S + \bar{S}) - \sum_{a=1}^{3} \ln(T_a + \bar{T}_a - |A_a|^2)$$

A1, A2, A3 : untwisted matter fields corresponding to Kahler moduli, T1, T2, T3

Simple example

T6/ZN

3 diagonal Kahler moduli + dilaton

$$K = \ln(S + \bar{S}) - \sum_{a=1}^{3} \ln(T_a + \bar{T}_a - |A_a|^2)$$

there are SL(2,Z)a a=1,2,3 corresponding to each Ta

superpotential

$$W = yA_1A_2A_3$$

there is a permutation symmetry of Ta, Aa, i.e. S3

Simple example

T6/ZN

3 diagonal Kahler moduli + dilaton

$$K = \ln(S + \bar{S}) - \sum_{a=1}^{3} \ln(T_a + \bar{T}_a - |A_a|^2)$$

superpotential $W = y A_1 A_2 A_3$

there is a permutation symmetry of Ta, Aa, i.e. S3

Moreover, this system has the S4 symmetry,

Ta, Aa are S4 triplet, S: S4 singlet for y = const

Ishiguro, T.K.,Otsuka, 2107.00487

Simple example

T6/ZN

3 diagonal Kahler moduli + dilaton

$$K = \ln(S + \bar{S}) - \sum_{a=1}^{3} \ln(T_a + \bar{T}_a - |A_a|^2)$$

superpotential $W = yA_1A_2A_3$

Moreover, this system has the S4 symmetry,

T2, Aa is S4 triplet, S: S4 singlet for y=const

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},$$

Ishiguro, T.K.,Otsuka, 2107.00487

Simple example

T6/ZN

3 diagonal Kahler moduli + dilaton

$$K = \ln(S + \bar{S}) - \sum_{a=1}^{3} \ln(T_a + \bar{T}_a - |A_a|^2)$$

superpotential

$$W = y A_1 A_2 A_3$$

A1, A2, A3 S4 triplet

10D N=1 SUSY → 4D N=4 SUSY

SU(4) R symmetry

4= 1+3 4D gauge multiplet + A1, A2, A3

Ishiguro, T.K.,Otsuka, 2107.00487

Simple example

T6/ZN

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$$K = \ln(S+\bar{S}) - \sum_{a=1}^{3} \ln(T_a + \bar{T}_a - |A_a|^2)$$

there are SL(2,Z)a a=1,2,3

superpotential $$W = yA_1A_2A_3$$

Symmetry

SL(2,Z)xSL(2,Z)xSL(2,Z)xS4 for Y = constant

Sp(8,Z) symmetry is broken by yukawa coupling intersection,

Yukawa is spurion -> Sp(8,Z) remains

Generic compactifictiaon

Full Sp modular symmetry

sub-symmetries G remain for y=constant.

Yukawa couplings are spurions under Sp/G,

and transform non-trivially under Sp/G.

We would like to study its implications

Generic symplectic modular symmetry is

complicated and involves more rich structure.

That is interesting.

4. Comment on kinetic terms

The simplest computation of kinetic terms for matter fields is the dimensional reduction from higher dimensional super YM theory.

For example, torus background wit magnetic fluxes

Kahler metric for Ai

$$Z^i_{ab} = \frac{1}{T_i + \overline{T_i}} \left(\prod_{k=1}^{3} \frac{1}{\sqrt{(U_k + \overline{U_k})}} \right) \sqrt{\frac{|I^{(i)}_{ab}|}{\prod_{j \neq i} |I^{(j)}_{ab}|}}.$$

4. Comment on kinetic terms

super YM theory.

DBI action describes the dynamics of open string on the D-brane

$$S_{\rm NDBI} = -T_p \int d^{p+1}\xi \, e^{-\varphi} \, {\rm str} \sqrt{-\det_{p+1}(g_{MN} + 2\pi\alpha' F_{MN})}.$$

For example, torus background with magnetic fluxes

$$Z^i_{ab} = \frac{1}{T_i + \overline{T_i}} \left(\prod_{k=1}^{3} \frac{1}{\sqrt{(U_k + \overline{U_k})}} \right) \sqrt{\frac{|I^{(i)}_{ab}|}{\prod_{j \neq i} |I^{(j)}_{ab}|}}.$$

Corrections to Kahler metric

$$\mathcal{Z}^i_{ab} = Z^i_{ab} \times \left[1 + \frac{(T_i + \overline{T_i})}{6(S + \overline{S})} \left(I^{(j)}_{ab} I^{(k)}_{ab} - 3M^{(j)}_a M^{(k)}_a - 3M^{(j)}_b M^{(k)}_b \right) \right]$$

in the small flux expansion

Abe, Higaki, T.K., Takada, Takahashi, 2107.11961

4. Comment on kinetic terms

DBI action

$$S_{\rm NDBI} = -T_p \int d^{p+1}\xi\, e^{-\varphi}\, {\rm str} \sqrt{-\det_{p+1}(g_{MN} + 2\pi\alpha' F_{MN})}.$$

Corrections to Kahler metric

$$Z^i_{ab} = Z^i_{ab} \times \left[1 + \frac{(T_i + \overline{T}_i)}{6(S + \overline{S})}\left(I^{(j)}_{ab} I^{(k)}_{ab} - 3M^{(j)}_a M^{(k)}_a - 3M^{(j)}_b M^{(k)}_b\right)\right]$$

in the small flux expansion

Abe, Higaki, T.K., Takada, Takahashi, 2107.11961

This result is not exact, we have more corrections

including higher orders of M, O(M4), O(M6), ...

Full results can be written by a simple function

or not ?

Summary

We have studied the low energy effective field theory derived from string compactification.

Many compactifications lead the rule

$$y_{ijk\ell} \sim y_{ijm} y_{mk\ell}$$

$$\text{n-point coupling} \quad \sim (y_{ijk})^{n-2}$$

This structure would be important,
e.g. from the viewpoint of MFV scenario in SMEFT.

Yukawa couplings are spurions.

The symmetries, which transform non-trivially Yukawa couplings, would be important,
e.g. modular symmetries including Sp symmetries.