The Mathematics of Layers and Interfaces

November 9th-13th,
Casa Matematica Oaxaca
Oaxaca, Mexico.

SCHEDULE AND ABSTRACTS

SCHEDULE

Talks are 30 minutes long, with 15 minutes of questions.

Monday morning:

- 9:00-9:45: **Ray Schmitt**. *Double-diffusive layers in the Ocean*
- 9:45-10:30: **Mary-Louise Timmermans**. *Layers in the Arctic Ocean*
- Break
- 11:30-12:15: **Alfred Wuest**. *Insights from investigating double-diffusive convection in deep and strongly stratified lakes*
- 12:15-1:00: **Bruce Sutherland**. *Internal Wave Transmission through a Staircase Density Profile*

Monday afternoon:

- 3:00-3:45: **Francesco Paparella**. *A simple model of staircase formation in fingering convection*
- 3:45-4:30: **Grae Worster**. *On the evolution of under-ice melt ponds*
- Break
- 5:30-6:15: **Jo Fawna Reali**. *Layer formation in sedimentary fingering convection*
- 6:15-7:00: **David Hughes**. *Magnetic Layering*

Tuesday morning:

- 9:00-9:45: **Nicholas Brummell**. *2D or not 2D*
- 9:45-10:30: **Neil Balmforth**. *Layering in slots and staircases*
- Break
- 11:30-12:15: **Bill Young**. *Layering and mixing in stratified flows*.
- 12:15-1:00: **Dick Peltier**. *Stratified turbulent layers, diapycnal diffusivity, and the low frequency variability of the MOC*

Tuesday afternoon:

- 3:00-3:45: **Karan Venayagamoorthy**. *On the prediction of turbulent diapycnal mixing in stably stratified geophysical flows*
- 3:45-4:30: **Paul Linden**. *Interface dynamics in stratified shear flows*
- Break
- 5:30-6:15: **Cristobal Arratia**. *Transient mechanisms of vertical scale selection for the layering process in unstationary stratified flows*
- 6:15-7:00: **Colm-cille Caulfield**. *Spontaneous layer formation and interface dynamics in stratified Taylor-Couette flow*

Wednesday morning:

- 9:00-9:45: **Edgar Knobloch**. *Large scale structure formation in geostrophic turbulence*
- 9:45-10:30: **Pascale Lelong**. *Near-inertial energy propagation inside a Mediterranean anticyclonic eddy*
- Break
- 11:30-12:15: **Claudia Cenedese**. *Mixing and entraining at the interface of lock-release gravity currents over a sparse and dense rough bottom.*
- 12:15-1:00: **Eckart Meiburg**. *Vorticity-based Models of Gravity Currents Propagating into Ambients with Arbitrary Shear and Density Stratification*

Wednesday afternoon: Free time

Thurday morning:

- 9:00-9:45: **Eliot Fried**. *Sharp-interface limits of diffuse-interface theories for phase transitions*
- 9:45-10:30: **Ryan Moll**. *A review of layer formation in oscillatory double-diffusive convection in astrophysics*
- Break
- 11:30-12:15: **Gerardo Hernandez-Duena**. *Dissection of Boussinesq non-linear interactions using intermediate models.*
- 12:15-1:00: **Brad Marston.** *Direct Statistical Simulation of Anisotropic and Inhomogeneous Flows*

Thursday afternoon:

- 3:00-3:45: **Guilhelm Dif-Pradalier**. *Plasma ExB Staircase*
- 3:45-4:30: **Pat Diamond**. *On What We Can Learn From Reduced Models of Staircase Formation in QG Fluids and Magnetized Plasmas*
- Break
- 5:30-6:15: **James Cho.** *PV staircases and jet formation on giant planets*
- 6:15-7:00: **Stephan Stellmach**. *Spontaneous Formation of Buoyancy and Potential Vorticity Staircases in Rotating, Compressible Convection*

Monday

Ray Schmitt
Woods Hole Oceanographic Institute

Double-diffusive layers in the ocean

Quite prominent thermohaline staircases can be found in several regions of the global ocean. These have been observed to persist for decades and seem robust against the active internal wave and meso-scale eddy fields. One salt fingering staircase in the western tropical Atlantic has been particularly well studied, and vertical mixing rates quantified by both tracer release experiments and microstructure observations. Up to 15 mixed layers in temperature and salinity, 5-40 m thick, are separated by ~1 m thick high gradient interfaces with net temperature jumps of 0.1 to 1oC. There are also strong lateral variations in layer properties over scales of hundreds of kilometers. The temperature, salinity and density vary horizontally in a manner that confirms the double-diffusive nature of the staircase. A time series of repeat temperature and salinity profiles from a moored profiler provides insight into variations of the flux convergence ratio and the density ratio that are consistent with layer formation theories of Radko (2003, 2005). Some of the challenges of relating such one-D models to the real 3-D structure of the ocean are discussed.

Mary-Louise Timmermans
Yale

Layers in the Arctic Ocean

A range of Arctic Ocean layers that are of relevance to sea ice and climate will be examined. Locally absorbed summer solar heat is stored in a near-surface ocean layer. It is shown that the heat in this layer is released to the surface in the fall and winter by shear-driven mixing, and convective mixing during sea-ice growth. This results in significantly reduced sea-ice thickness at the end of the growth season. A strengthened Arctic Ocean stratification limits the vertical flux of heat from warm ocean layers deeper than the near-surface layer. The physics of these deeper ocean layers will also be discussed.

Alfred Wüest (with T. Sommer, M. Schmid, J.R. Carpenter, B. Scheifele, R. Pawlowicz, and M. Toffolon)
EAWAG

Insights from investigating double-diffusive convection in deep and strongly stratified lakes

The three lakes Nyos, Kivu and Powell Lake have in common that their water bodies are permanently stratified due to increasing salinity and gas contents with depth. Although all three lakes have a different history, which led to individual stratification structures, the temperature increase with depth favours double-diffusive convection in each of the three lakes. The vertical structures of the spectacular staircase-like layering are distinctively different, but the physical dimensions of the high-gradient interface and mixed layer thicknesses as well as the steps of salinity and temperature between the homogeneous mixed layers are astonishingly similar and even close to the staircase properties in the Arctic Ocean. All of these systems have different heat fluxes and different density ratios (stabilizing N^2 of the dissolved components divided by the destabilising N^2 of temperature) and therefore the available data allow for an evaluation of double-diffusive signatures and relationships (layering, fluxes) by comparing their physical properties. The double-diffusive layering in Kivu, Powell Lake and the Arctic Ocean show identical quasi-stationary behaviour whereas non-stationary Nyos deviates remarkably. The goal of the talk is to explain this difference.

Bruce R. Sutherland
University of Alberta, Edmonton

Internal Wave Transmission through a Staircase Density Profile

Whether due to double diffusion or mixing, several regions in the oceans have density profiles exhibiting a staircase structure in which a sequence of finite-depth layers of near-uniform density fluid are separated by sharp density gradients. The transmission of vertically propagating internal waves through a single step was previously considered by Sutherland & Yewchuk (JFM 2004) who showed in particular that near perfect transmission occurs if the frequency and horizontal wavenumber of the incident waves satisfy a resonance condition with interfacial waves residing at the density jumps on either side of the well-mixed region. Here this work is extended to consider the transmission of internal and inertia gravity waves through an arbitrary number of steps each with equal thickness and with the same magnitude of density jump on either flank of each step. As anticipated, waves are able to pass through the staircase with near-perfect transmission if their horizontal wavelength is sufficiently long. Resonance peaks in transmission are also evident for relatively short wavelength internal waves. For a prescribed frequency of the incident wave upon a staircase with N steps, transmission spikes occur at N different values of horizontal wavenumber.

Francesco Paparella (with Jost von Hardenberg)
Università del Salento

A simple model of staircase formation in fingering convection.

Fingering convection occurs when two buoyancy-changing scalars are stratified in such a way that the least-diffusing one, if taken alone, would produce an upward increase of density, but the most diffusing one reverses this tendency, making the density field increase downward. Although both scalars flow down-gradient, this form of convection transports density up-gradient. The convective motions are sustained by small-scale, blob-like structures that displace vertically temperature and salinity anomalies.

Numerical simulations show that, when the convection is sufficiently vigorous, the blobs self-organize into clusters having enough size and kinetic energy to locally overturn the fluid. I will argue, by means of a simple model involving only buoyancy and kinetic energy, that the appearance of clusters is responsible for the formation of staircases:
kinked, step-like profiles of horizontally averaged density, temperature and salinity, that have long been observed in laboratory experiments and oceanographic measurements.

Grae Worster
University of Cambridge.

On the evolution of under-ice melt ponds

During spring, relatively fresh melt water percolates from superficial melt ponds through sea ice to the ocean beneath, where it can gather in under-ice concavities, floating on the relatively salty, dense ocean beneath. The subsequent evolution of the under-ice melt pond involves three fundamental mechanisms of fluid mechanics, phase change and their interaction. Owing to the density maximum of fresh water at 4 degrees Celsius, the interface between the 'warm' (zero Celsius), fresh water in the pond and the cold (minus two Celsius), salty water below is a unique form of one-sided, double-diffusive 'diffusive' interface in which the fluid above the interface convects while the fluid below the interface is stable to convection. The heat transfer across this interface, which can be quantified mathematically, supercools the melt pond and allows a 'pure' mushy layer to grow downwards within it. Once the mushy layer reaches the interface it initiates the formation of a 'false bottom', a layer of ice separating the cold ocean from the warmer melt pond. The ice grows upwards into the pond while simultaneously being dissolved at its base, giving the appearance of migration. All this was nicely observed in experiments by Martin & Kauffman (1974). This talk will describe mathematical models of each stage of the evolution.

Jo Fawna Reali
University of California Santa Cruz

Layer formation in sedimentary fingering convection

Analogous to salt fingering convection, sedimentary fingering convection occurs in a fluid when temperature is stably stratified but the sediment concentration is not. Just as salt fingers develop thermohaline staircases, sediment fingers may result in density staircases where temperature and sediment vary in a step like fashion. This investigation of layer formation in the presence of sediment fingering instabilities extends the gamma instability of Radko 2003 to the equations for sedimentary fingering convection.

David Hughes (with Nic Brummell)
University of Leeds

Magnetic layering

It is well established that the doubly-diffusive system of thermohaline convection can lead, on a long timescale, to the formation of layers; the density adopts a staircase profile, which influences the turbulent transport of heat and salt. This layering can occur in both the fingering regime (destabilising solutal gradient, stabilising thermal gradient) and the diffusive regime (both gradients reversed). Magnetic buoyancy instabilities may also be regarded as a double-diffusive instability, with competing magnetic field and entropy gradients. Indeed, under certain assumptions there is a non-trivial mapping between the equations of themohaline convection and those of magnetic buoyancy. Here we exploit this analogy to discuss the possibility of magnetic layering and the possible astrophysical implications.

Tuesday

Nicholas Brummell (with Pascale Garaud)
University of California Santa Cruz

2D or not 2D: Fingering convection at low Prandtl number

The fingering version of double-diffusive instabilities is relevant to astrophysical contexts at low Prandtl number. This parameter regime is notoriously difficult to simulate with numerical computations, owing to the high resolution needed. Often two-dimensional (2D) simulations are substituted for fully 3D in order to alleviate this constraint. In the particular context of the fingering instability, we perform comparisons of numerical simulations with different dimensionalities and find that 2D simulations do not represent the full 3D dynamics. However, we discover an empirical rule-of-thumb that suggests that 3D domains that are "slim" (not of square horizontal aspect ratio), yet sufficiently "fat", can indeed recover the correct dynamics, implying that some numerical savings are possible. We speculate about the universality of this rule-of-thumb, and wonder about other parameter regimes and the entertaining dynamics that the 2D cases reveal.

Neil Balmforth
University of British Columbia, Vancouver

Layering in slots and staircases

I will describe some recent work on convection in a two-dimensional porous media due to double diffusion, asking the question whether this configuration forms layers. Numerical computations suggest that layers are not straightforward to generate in this system, due to the presence of elevator-type modes that are not destroyed by secondary instabilities. I will then describe some work on shear instabilities in stratified fluid in which the undisturbed density field has the form of a staircase. For this problem, a combination of linear stability theory and numerical computation is used to examine how shear instability might influence the evolution of a density staircase.

Bill Young
University of California San Diego

Layering in single-component, stably stratified, mechanically driven turbulence

Abstract: Based on a phenomenological nonlinear diffusion equation Owen Phillips suggested that homogenous turbulence in a single-component stably stratified fluid is unstable to the spontaneous formation of layers. This "Phillips model" hypothesizes that there is a regime in which the flux of density is decreased if the mean density gradient is increased (because strong stable stratification suppresses turbulence). Incorporating this mechanism into a nonlinear equation for the mean density shows that this "negative differential diffusivity" results in an instability. But the Phillips model is ill-posed because the growth rate of high-wavenumber disturbances is unbounded. Nonetheless, experiments with turbulence driven by stirring rods provides compelling evidence in support of the Phillips mechanism.

I'll discuss a model that retains the basic Phillips mechanism but circumvents the high-wavenumber problem by taking account of the production and dissipation of turbulent kinetic energy. Close to the onset of the instability, this model reduces to the Cahn-Hilliard equation. This provides evidence that the Cahn-Hilliard equation is likely a universal model of layering in both single-component and double-diffusive systems.

Dick Peltier
University of Toronto

Stratified turbulent layers, diapycnal diffusivity, and the low frequency variability of the MOC

A model problem for wave breaking induced stratified turbulent mixing in the oceans is provided by a parallel flow susceptible to the KH mechanism. At sufficiently high Reynolds number DNS-based anlyses demonstrate that the fully developed turbulent flows produced by this mechanism may be employed to infer the diapycnal diffusivities supported by the irreversible component of turbulent mixing. These match the values needed to support the vertical flux of mass required to close the abyssal component of the meridional overturning circulation. The DNS data may be employed as basis for an appropriate parameterization scheme for "kappa" and this may be tested partially using Argo float-based measurements.. An important characteristic of the MOC is that it is known to "fibrillate" on the millenium timescale under cold climate conditions. A large scale coupled climate model is employed to investigate the detailed dependence of the characteristics of this oscillatory behavior upon models of the diapycnal diffusivity.

Karan Venayagamoorthy
Colorado State University

On the prediction of turbulent diapycnal mixing in stably stratified geophysical flows

Two key quantities that are essential for estimating turbulent mixing in stably stratified flows are: the dissipation rate of the turbulent kinetic energy and the mixing efficiency, which is a measure of the amount of turbulent kinetic energy that is irreversibly converted into background potential energy. A linear relationship between the Thorpe (vertical overturn) length scale and the Ozmidov scale is widely assumed in oceanography to infer the dissipation rate of turbulent kinetic energy. This approach is particularly attractive since the vertical scales of overturns are easily calculable using a sorting algorithm from inversions in standard density profiles obtained from Conductivity-Temperature-Depth (CTD) measurements in the ocean. Hence the Thorpe scale is essentially a kinematic scale that provides a description of the turbulence at a given sampling location and instant in time. On the other hand, The Ozmidov scale is obtained from dimensional reasoning based on the assumption that there is a balance between inertial and buoyancy forces. In other words, the Ozmidov scale is a representative dynamic length scale of the largest eddy that is unaffected by buoyancy. A review of a number of recent studies will be presented in this talk to highlight the lack of a linear relationship between the Thorpe length scale and the Ozmidov scale. These studies indicate that inferred estimates of the dissipation rate of turbulent kinetic energy may be biased high by up to an order of magnitude or more especially for large overturns in the ocean. An alternative framework using a two-dimensional parameter space based on a buoyancy strength parameter (i.e. an inverse Froude number) and a shear strength parameter will be presented to characterize the scaling correspondence of the overturning scale with pertinent turbulent length scales. A discussion on the mixing efficiency and implications for estimates of diapycnal mixing in the ocean will be presented.

Paul Linden
University of Cambridge

Interface dynamics in stratified shear flow

I will describe experiments on stratified shear flow generated in an inclined duct joining two reservoirs containing fluids of different densities. The flow shows a number of different regimes including Holmboe modes, intermittent flow and a fully turbulent, highly dissipative flow. The state space of these flows will be discussed and a criterion for the transition to the fully turbulent state will be derived. Finally, some effects of Prandtl number will be mentioned.

Cristobal Arratia (with Jean-Marc Chomaz)
Universidad de Chile

Transient mechanisms of vertical scale selection for the layering process in unstationary stratified flows.

We consider different types of vertically invariant and strongly stratified base flows and study the transient growth of linear optimal perturbations as a function of vertical wavelength. This allows us to quantitatively compare the capacity of linear perturbations to extract energy from each of the base flows and potentially determine the vertical scale selection. We can thus, for example, study the competition between the zigzag instability and Lilly's mechanism of vertical decorrelation through independently evolving horizontal layers. We observe that the zigzag instability is more relevant than Lilly's mechanism only when the base flow is composed of well separated vortices, but the zigzag scaling laws remain valid when that's not the case. We also perform a toroidal-poloidal composition of the optimal perturbations to characterize the energy associated to vertical vorticity on one side, and to the divergence of horizontal velocity and density perturbations on the other. We consistently observe that as the vertical wavenumber increases, the optimal perturbations are increasingly associated to energy being extracted from the vertical vorticity into vertical velocity and density perturbations.

Colm-Cille Caulfield
University of Cambridge

Spontaneous layer formation and interface dynamics in stratified Taylor-Couette flow

The spontaneous formation of relatively deep horizontal layers of weakly stratified fluid, separated by relatively thin interfaces of substantially stronger density gradient, is a common feature of strongly stably stratified flows and plays a major role in the dynamics of geophysical flows. Dating from the seminal work of O. M. Phillips, such layer/interface formation is to be expected whenever the irreversible vertical buoyancy flux is a non-monotonic function of the overall stratification. However, little is known about the physical mechanisms which drive the layer formation, or indeed what sets their characteristic depth and subsequent evolution. When the mixing is driven by instabilities due to vertical velocity shear, the situation is further complicated by the instabilities themselves having a characteristic vertical length scale, which may or may not set the initial depth of the layers. This uncertainty can be avoided by considering layer formation in ``stratified Taylor--Couette flow'' in the annulus between a rotating inner cylinder and a fixed outer cylinder, initially filled with stably, axially or vertically and linearly stratified fluid. Here the dominant velocity shear is in the radial direction, and so the mixing due to turbulence can be considered in isolation from the large scale stirring associated with (vertical) velocity shear. However, such flows are still prone to various primary instabilities, including the inherently stratified strato-rotational instability, and unpicking the relative importance of such linear instabilities, nonlinear dynamics and

disordered turbulent mixing events for the formation and subsequent evolution of layers and interfaces is still challenging. In this talk, I will discuss recent theoretical, experimental and numerical progress on this problem, which has enabled us both to identify the approriate scaling laws relating layer depth to rotation rate, initial stratification, gap width and radius ratio, and also to observe interesting complex yet apparently robust dynamics coupling interfacial evolution to the mixing within individual layers.

Edgar Knobloch
University of California Berkeley

Large scale structure formation in geostrophic turbulence

Low Rossby number convection is studied using an asymptotically reduced system of equations valid in the limit of strong rotation. The equations describe four regimes as the Rayleigh number Ra increases: a disordered cellular regime near threshold, a regime of weakly interacting convective Taylor columns at larger Ra, followed for yet larger Ra by a breakdown of the convective Taylor columns into a disordered plume regime characterized by reduced heat transport efficiency, and finally by geostrophic turbulence. The Nusselt number--Rayleigh number scaling in the "ultimate" regime geostrophic turbulence is predicted and confirmed using direct numerical simulations of the reduced equations. These simulations reveal that geostrophic turbulence is unstable to the formation of large scale barotropic vortices, via a process known as spectral condensation. The details of this process are quantified and its implications explored.

M.-Pascale Lelong, Pascale Bouruet-Aubertot , Yannis Cuypers
Northwest Research Associates

Near-inertial energy propagation inside a Mediterranean anticyclonic eddy

The present study was motivated by observations of intense near-inertial wave activity above and below the core of the semi-permanent anticyclonic Cyprus eddy in the Eastern Mediterranean during the BOUM field campaign.

To explain the observations, we have conducted a general numerical study of eddy/near-inertial interactions with a Boussinesq (nonhydrostatic) model. The problem is posed as an initial-value problem, with initial conditions consisting of an isolated shielded vortex in geostrophic equilibrium and surface-intensified near-inertial oscillations designed to simulate a uniform wind impulse. When ambient parameters (eddy strength and extent, mean stratification profile) are matched to the BOUM observations, our numerical results replicate quite well the wavelengths and intensity of the observed near-inertial wave field inside and below the Cyprus eddy core. The nature of near-inertial wave propagation inside an eddy is further explored as a function of initial near-inertial amplitudes at the surface and ambient stratification. Downward near-inertial propagation is most pronounced in the center of the eddy where the vortex velocity is zero (anticyclonic vorticity a maximum), and can be explained by means of a simple theoretical formulation based on transmission coefficients in a piecewise-linear approximation to the density profile.

Claudia Cenedese
Woods Hole Oceanographic Institute

Mixing and entraining at the interface of lock-release gravity currents over a sparse and dense rough bottom.

Dense oceanic overflows mix with surrounding waters along the descent down the continental slope. The amount of mixing and entrainment at the interface of the dense currents dictates the final properties of these overflows, and thus is of fundamental importance to the understanding of the formation of deep water masses. The existing parameterizations for entrainment in dense currents account primarily for the shear-induced entrainment at the interface between the dense flow and the ambient fluid. However, the turbulence generated by roughness elements at the bottom boundary, which produces an enhanced drag, can be intense and must be considered. One may expect that for dense currents having a height much larger than the roughness height, the turbulent eddies generated by the bottom roughness will play a role in homogenizing the dense current but will not contribute to entrainment of ambient waters within the dense current (i.e. will not contribute to changes in the water properties). Conversely, one may expect that for dense currents having a height comparable to, or smaller than, the roughness height, more complex regimes may occur where the turbulent eddies generated near the bottom may impact the entrainment of ambient waters and may significantly influence the dense water properties. We will discuss laboratory experiments investigating the mechanisms by which bottom roughness enhances or inhibits entrainment and dilution in a lock-release dense gravity current. The bottom roughness has been idealized by an array of cylinders. Both spacing (sparse vs. dense configuration) and height of the roughness elements compared with the height of the current have been varied. Both density and velocity fields have been obtained. Experimental results suggest that enhancement of the entrainment /dilution of the current can occur due to two different mechanisms. For a sparse configuration the dense current propagates between the cylinders and the entrainment is enhanced by the vortices generated in the wake of the cylindrical obstacles. For a dense configuration the dense current rides on top of the cylinders and the dilution is enhanced by the onset of convective instability between the dense current above the cylinders and the ambient lighter water between the cylinders. As expected, for large values of the ratio of the lock height to the cylinder height, H/h_c, the dense current behavior approaches that of a current over a smooth bottom, while the largest deviations from the smooth bottom case are observed for small values of H/h_c.

Eckart Meiburg (with Nasr-Azadani)
University of California Santa Barbara.

Vorticity-based Models of Gravity Currents Propagating into Ambients with Arbitrary Shear and Density Stratification

We develop a vorticity-based approach for modeling quasisteady gravity currents propagating into arbitrary density and velocity stratification. The model enforces the conservation of mass, horizontal and vertical momentum, and in contrast to previous approaches it does not rely on empirical, energy-based closure assumptions. Instead, the effective energy loss of the flow can be calculated a posteriori. The present model results in the formulation of a second order, nonlinear ODE that can be solved in a straightforward fashion to determine the gravity current velocity, along with the downstream ambient velocity and density profiles. Comparisons between model predictions and DNS simulations show excellent agreement. They furthermore indicate that for high Reynolds numbers the gravity current height adjusts itself so as to maximize the loss of energy.

Thursday

Eliot Fried
Okinawa Institute of Science and Technology

Sharp-interface limits of diffuse-interface theories for phase transitions

We will consider a model for phase transitions that gives rise to a scalar Ginzburg–Landau equation. In the context of this model, an interface between two phases is a three-dimensional transition layer. Granted a particular scaling, we will briefly present the most salient steps of a matched asymptotic analysis leading to a description in which an interface between two phases is a sharp surface. The partial-differential equation that arises from this limiting process generalizes the well-known equation for surface motion by mean curvature to account for orientation dependence of the interfacial energy density and mobility and dependence of the interfacial mobility on the interfacial normal velocity. Aside from explaining how the described approach can be extended to account for fluid flow and the transport of heat and different chemical species, we will provide a physically unambiguous basis for the Gibbs– Thomson equation commonly encountered in theories for diffusional phase transitions and solidification.

Ryan Moll
University of California Santa Cruz

A review of layer formation in oscillatory double-diffusive convection in astrophysics

Fluids with an unstable thermal stratification and a stabilizing mean molecular weight stratification are common in a variety of geophysical and astrophysical systems. Double-diffusive fluids stratified in this way have a tendency to be organized into convective thermo-compositional layers which enhance the transport of both temperature and chemical species (compared to thermal and compositional diffusion). This layering phenomenon was studied first in the geophysical context with laboratory experiments where a fluid was initialized in a layered state (Turner 1965; Shirtcliffe 1973), and the fluxes of temperature and salt (or sometimes salt and sugar) were measured through the diffusive, stably stratified layer interfaces. Results from early geophysical studies which assumed a pre-existing layered state were later applied to similarly stratified fluids in the astrophysical context (Stevenson 1985; Spruit 1992). However, in the astrophysical context, such fluids are likely to be unstable to a form of oscillatory double-diffusive convection (ODDC) (commonly referred to as \semi-convection") which can lead to the spontaneous formation of layers (through the so-called gamma-instability) (Radko 2003; Rosenblum et al. 2011). Recent studies of direct numerical simulations of ODDC at astrophysical parameters (Rosenblum et al. 2011; Mirouh et al. 2012; Wood et al. 2013) suggest that spontaneous layer formation is the most relevant layering mechanism known in stars and giant planets. Furthermore, there are significant differences between layers that form naturally and those that are imposed as an initial condition, both qualitatively and in terms thermal and compositional transport. This calls into question the validity of earlier studies which assumed that astrophysical layering was analogous to layering studied in the geophysical case. Presented will be a review of early layering studies at geophysical parameters, as well as a discussion astrophysical literature which adopted the geophysical results. Also presented is the case for the importance of spontaneous layer formation in astrophysics, and the quantitative and qualitative differences between layers that form spontaneously from ODDC and layers that are assumed as an initial condition. Finally, current research on the effects of global rotation on spontaneous layer formation will be discussed. It will be shown that despite some differences in dynamics and thermal and compositional transport, layers still form naturally in rotating ODDC at rotation rates relevant to actual stars and giant planets.

Gerardo Hernandez-Duenas
National University of Mexico

Dissection of Boussinesq non-linear interactions using intermediate models

Nonlinear coupling among wave modes and vortical modes is dissected in order to probe the question: Can we distinguish the wave-vortical interactions largely responsible for formation versus evolution of coherent, balanced structures? It is well known that the quasi-geostrophic (QG) equations can be derived from the Boussinesq system in a non-perturbative way by ignoring wave interactions and considering vortical modes only. One qualitative difference between those two models is the lack of skewness in the QG dynamics. In this talk, non-perturbative intermediate models that include more and more classes of non-linear interactions will be used to identify their role in different qualitative properties of the Boussinesq system. Numerical results will be shown to describe the effect of each class in the transfer of energy between vortical modes and waves, transfer of energy (or the lack of it) between scales, formation layers and vortices, and skewness.

Brad Marston
Brown University

Direct Statistical Simulation of Anisotropic and Inhomogenous Flows

Statistics of models of anisotropic and inhomogeneous fluid flows may be directly accessed by solving the equations of motion for the statistics themselves as proposed by Lorenz nearly 50 years ago. Motivated by the desire to capture seamlessly multiscale physics we introduce a new approach to such Direct Statistical Simulation (DSS) based upon separating eddies by length scale. Discarding triads that involve only small-scale waves, the equations of motion generalize the quasi-linear approximation (GQL) and are able to accurately reproduce the low-order statistics of a stochastically-driven barotropic jet. Furthermore the two-point statistics of high wavenumber modes close and thus generalize second-order cumulant expansions (CE2) that employ zonal averaging. This GCE2 approach is tested on a variety of models. Comparison to statistics accumulated from numerical simulation finds GCE2 to often be quantitatively accurate. But to be practically useful as a sub-grid model within full-scale simulations, the "curse of dimensionality" must be addressed: The statistical description of huge numbers of degrees of freedom needs to be reduced to a smaller, more manageable set of effective modes. Coherent structures (eddies, jets, and plumes) frequently dominate boundary layer transport, suggesting that such a huge reduction in dimensionality may be feasible.

Guilhelm Dif-Pradalier
CEA

Plasma ExB Staircase.

A puzzling result in the recent years in plasma turbulence has arguably been the discovery of the quasi-regular pattern of ExB flows and interacting avalanches that we have come to call the “plasma staircase”. This structure is a spontaneously formed, self-organising pattern of quasiregular, long-lived and localised shear layers that organise the turbulent transport. Called “jets” in the context of planetary flows they are pervasive features of planetary atmospheres and critically influence the transport properties of heat, momentum, chemicals or even biota. An acute proximity exists between fusion plasmas and geophysical fluid dynamics. The plasma staircase is investigated through flux-driven gyrokinetic computations using the GYSELA code. Extensive parameter scans show that this flow pattern is a robust feature of plasma size, collisionality, turbulence drive, safety factor profile and poses new challenges in understanding profile stiffness, distance to criticality and nonlocal effects. The plasma staircase, in contrast with its geophysical counterpart, is indeed intimately linked with heat and momentum avalanching. The staircase step size is mesoscale, it modulates the outer scale of the avalanche distribution and is beneficial for confinement. The plasma staircase also displays a dynamics of its own and meanders, riverlike. The overall transport within the plasma is really that of a self-organised state in which the staircase is a key dynamical player. Observed and characterised theoretically, we went on hunting for its existence in actual experiments, using fast-sweeping reflectometry turbulence correlation measurements in ToreSupra. This is an instance where prediction from a numerical model leading to discovery in observations. Many of its features (meandering, scaling with the plasma size,…) agree well with the theoretical predictions. This observation may have far-reaching consequences for the understanding of turbulent organisation and for the validation of models of plasma turbulence.

P.H. Diamond (with M. Malkov, A. Ashourvan and D. Hughes)
University of California San Diego

Models of Staircase Formation in Drift-Rossby Turbulence: Constraints and What We Learn From Them

Reduced models of staircase formation in drift-Rossby wave turbulence were developed and explored. Drift-Rossby turbulence has the crucial feature of potential vorticity flux roll-over with increasing potential vorticity gradient, i.e. $\delta_{Q}\Gamma\ \delta \nabla Q < 0$. In the simplest one field system (i.e. Quasi-geostrophic or Hasegawa-Mima), the negative flux increment results from increased Rossby wave memory, for steeper PV gradients. This allows negative viscosity, the formation of PV transport barriers and so the development of PV staircases. A simple closure model has been developed and analyzed. The model consists of coupled equations for mean PV and fluctuation potential enstrophy, and conserves total potential enstrophy. In this system, the mixing length is regulated by the Rhines scale. Studies have elucidated dependencies of system states upon mean PV gradient, viscosity and system size. A variety of staircase-like states have been obtained. Of particular interest are the dependencies of step width and the step height (i.e. barrier width) on system parameters.

Studies of the richer, two-field Hasegawa-Wakatani system have formulated a reduced 3-field model for mean density, mean vorticity and fluctuation potential enstrophy. Note that here, density and fluid vorticity combine to define a conserved potential vorticity. Key points here are that $P_r = \nu_0 \ D_0 >>1$, the existence of vorticity drag and turbulence spreading, and that there exist dynamical constraints on the relation between particle and vorticity fluxes. Most notable here is the precise relation between the residual (nondiffusive) vorticity flux (and thus Reynolds stress) and the particle flux. Ongoing studies are concerned with multi-field staircase structures. Particular emphasis focused on tradeoffs between particle and vorticity fluxes, and their implication for density profile corrugations and shear layers in staircases.

James Cho
Queen Mary University of London

PV staircases and jet formation on giant planets

I will review the dynamics of potential vorticity and the formation of staircases and jets on giant planets, including on extrasolar giant planets. The discussion will center on one-layer turbulent dynamics: homogenous, inhomogeneous, and magnetized.

Stephan Stellmach
University of Munster

Spontaneous Formation of Buoyancy and Potential Vorticity Staircases in Rotating, Compressible Convection

Turbulent convection that is strongly affected by the fluid's compressibility is common in geo- and astrophysical systems. Using numerical simulations, we show that in the presence of sufficiently strong rotation, the convective flow in such systems spontaneously organizes itself into a multitude of alternating horizontal jets. This process is accompanied by the formation of a pronounced buoyancy staircase, in which layers of well-mixed entropy are separated by thin, high-gradient interfaces. The average potential density increases with height from layer to layer, resulting in an unstable buoyancy stratification that is maintained across the interface regions. The dynamics is rich, and involves relaxation oscillations as well as layer merging.

We show that in a certain limit, the jet formation process can be interpreted as a consequence of a "compressional" beta effect, in which rising or sinking fluid particles change their vorticity by expanding and contracting as they move within the hydrostatic pressure field. The width of the jets is demonstrated to follow a Rhines-type scaling law, and is controlled by the density scale height, rotation rate and jet velocity. The dynamics can be described in terms of inhomogeneous potential vorticity (PV) mixing that generates a PV staircase. However, the self-sharpening of the PV interfaces also generates sharp, unstable buoyancy jumps across these interfaces, and therefore has to be strong enough to maintain the interfaces from collapsing.