Theory and Computational Methods for SPDEs (Online) (22w5172)

David Cohen (Chalmers University of Technology and University of Gothenburg),
Annika Lang (Chalmers University of Technology and University of Gothenburg),
Marta Sanz-Solé (University of Barcelona),
Samy Tindel (Purdue University)

September 12 – September 16, 2022

The online BIRS-CMO workshop on the theory and computational methods for SPDEs brought together a number of world experts in the theoretical analysis of stochastic partial differential equations, which model dynamical systems disturbed by noise, together with specialists in scientific computing, in a first meeting of its kind.

1 Overview of the Field

Mathematical modeling is a key tool in modern science, society, and industry. The past decades have seen a rapidly growing interest in mathematical models with stochastic terms and random components. These stochastic terms are needed to model various uncertainties such as:

- errors in the measurement of input variables, errors/unknowns in the model parameters, or more general model uncertainties – e.g., weather prediction with limited knowledge of the initial conditions, ground water flow modeling with imprecise characterization of subsurface properties;
- microscopic fluctuations in a macroscopic model, i.e., unresolved scales – e.g., thermal fluctuations, molecular dynamics effects, cellular level phenomena, statistical description of turbulence.

The main focus of the workshop was the study of a large class of mathematical models with stochastic terms, namely Stochastic Partial Differential Equations (SPDEs). This exciting research area has recently expanded in the following directions:

- At a theoretical level, a growing attention has been devoted to track physically relevant phenomena displayed by certain classes of SPDEs. Among the most exciting and challenging situations, one can mention the parabolic Anderson model (describing nontrivial effects of inhomogeneous media in the simple heat equation context), as well as the Φ^4_3 model for quantum interactions or the KPZ equation describing growth models. This line of investigation has been spurred by the introduction of the regularity structure theory by Martin Hairer (Fields medal 2014 and Breakthrough Prize in Mathematics 2021), which can be seen as a way to properly define the previously inextricable aforementioned systems.
- One should also highlight the recent advances in SPDEs related to applications of modern stochastic analysis techniques (such as Dirichlet forms, Kolmogorov equations in infinite dimensional spaces,

large deviations for self intersection local times or Malliavin calculus) to the realm of SPDEs. This has lead to significant improvements in terms of scope of the theory, as well as a deeper understanding of the behavior of our stochastic systems.

- As far as the computational aspects of SPDEs are concerned, a huge effort has been made in order to develop new simulation methods and analyze their convergence rates. Those numerical methods take their roots in the existing numerical analysis for deterministic partial differential equations, but nontrivial adaptations have to be designed in order to handle stochastic perturbations.

As one can see, SPDEs give raise to a vibrant mathematical activity, arguably one of the most active in probability theory.

It is also worth highlighting that SPDEs are considered as a meaningful model in numerous relevant applications. Among the ones which are particularly appealing to us, we include a list of known examples:

- biology: e.g., the motion of a strand of DNA floating in a fluid [**?**];
- climate and weather forecast models, see the seminal work [**?**];
- industry and manufacturing: e.g. optimization of an airfoil in compressible aerodynamics, accounting for uncertain operating conditions (Mach number or angle of attack);
- neurology: e.g., models for the initiation and propagation of action potentials in neurons [**?**];
- physics: e.g., phase separation models [**?**] and random surface grow models [**?**];
- finance: e.g., portfolio optimization [**?**] and forward dynamics in energy markets [**?**];
- image analysis: e.g., segmentation of images [**?**, **?**].

Such complex extensions of deterministic mathematical models are sometimes inevitable. Furthermore, the stochastic equations related to these models are significantly more difficult to theoretically understand and also more expensive to numerically simulate than their deterministic counterparts. On top of that, in order to apply numerical methods and in particular to develop efficient, robust, and reliable numerical algorithms, it is essential to get a solid grasp on some deep aspects of the mathematical objects at stake. At the other end of the mathematical spectrum, it is equally crucial to master the implementation techniques for the most relevant numerical methods related to SPDEs.

2 Aims of the workshop

As a result of the previous considerations, we feel that a deeper communication as well as a strengthening of collaboration between the theoretical and applied SPDE communities would benefit the whole research area. To give a concrete example, it seems that modern theoretical tools open new possibilities in terms of analysis of existing algorithms for SPDEs. The physical models, Anderson model, Φ^4_3, KPZ, alluded to above also require specific and challenging numerical methods. We thus wish to see a better interplay between numerics and theoretical aspects of SPDEs in the next future.

With this global aim in mind, the goal of our workshop was to initiate new collaborations between people from our two research communities (in addition to the usual aim of communicating current investigations and discussing challenging open problems).

3 Presentation Highlights

Due to the limited online format of our workshop, we decided to focus on state-of-the-art presentations by leading as well as early-career researchers. Each 45 minute presentation were followed by a 15 minute discussion. Further time and space for discussions were offered in zoom breakout rooms and on gather town.

We now list the invited speakers (in presentation order) with the titles of their presentations:

1. Carsten Chong: *A landscape of peaks: The intermittency islands of the stochastic heat equation with Lévy noise*
2. Raluca Balan: *Stochastic wave equation with Lévy white noise*
3. Charles-Edouard Bréhier: *Analysis of a modified Euler scheme for SPDEs*
4. Leonid Mytnik: *On the speed of a front for stochastic reaction-diffusion equations*
5. Konstantinos Dareiotis: *Approximation of Stochastic PDEs with measurable reaction term*
6. Jingyu Huang: *Stochastic heat equation with super-linear drift and multiplicative noise on* $\mathbb{R}^d$
7. Hakima Bessaih: *Various numerical schemes for Hydrodynamic models*
8. Xu Wang: *Inverse random potential scattering for elastic waves*
9. Istvan Gyöngy: *On solvability of degenerate parabolic SPDEs in* L_p*-spaces*
10. Carlo Marinelli: *Singular perturbations and asymptotic expansions for SPDEs with an application to term structure model*
11. Carl Mueller: *Valleys for the Stochastic Heat Equation*
12. Arnaud Debussche: *Transport noise models from two-scale systems with additive noise in fluid dynamics*
13. Davar Khoshnevisan: *Optimal regularity of SPDEs with additive noise*.

Further information (abstract, videos) is available on the workshop's homepage
https://www.birs.ca/events/2022/5-day-workshops/22w5172.

4 List of registered participants

1. Andersson, Adam (Saab AB and Chalmers University of Technology)
2. Balan, Raluca (University of Ottawa)
3. Banas, Lubomir (Bielefeld University)
4. Barth, Andrea (University of Stuttgart)
5. Benth, Fred Espen (University of Oslo)
6. Berg, André (Umeå University)
7. Bessaih, Hakima (Florida International University)
8. Brzezniak, Zdzislaw (York University)
9. Bréhier, Charles-Edouard (Université de Pau et des Pays de l'Adour)
10. Buckwar, Evelyn (Johannes Kepler University Linz)
11. Campbell, Stuart (Heriot Watt University)
12. Candil, David (EPF Lausanne)
13. Cerrai, Sandra (University of Maryland)
14. Chaparro Jaquez, Luis Mario (University of Leeds)
15. Chen, Ziheng (Yunnan University)

16. Chen, Chuchu (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)
17. Cheung, Hang (University of Calgary)
18. Chong, Carsten (Columbia University)
19. Cohen, David (Chalmers University of Technology and University of Gothenburg)
20. Cui, Jianbo (Hong Kong Polytechnic University)
21. Dai, Xinjie (Chinese Academy of Sciences)
22. Dang, Tonghe (Chinese Academy of Sciences)
23. Dareiotis, Konstantinos (University of Leeds)
24. de Bouard, Anne (CNRS/Ecole Polytechnique)
25. Debussche, Arnaud (ENS Rennes)
26. Delgado, Francisco (National Autonomous University of Mexico)
27. Di Nunno, Giulia (University of Oslo)
28. Djurdjevac, Ana (Freie Universität Berlin)
29. Eisenmann, Monika (Lund University)
30. Erdoğan, Utku (Eskişehir Technical University)
31. Feng, Xiaoli (Xidian Univeristy)
32. Geldhauser, Carina (Lund University)
33. Gerencsér, Máté (TU Wien)
34. Gess, Benjamin (Universität Bielefeld & MPI MiS Leipzig)
35. Glubokov, Andrey (Purdue University)
36. Goudenège, Ludovic (Centre National de la Recherche Scientifique)
37. Gubinelli, Max (University of Bonn)
38. Gyongy, Istvan (University of Edinburgh)
39. Hausenblas, Erika (Montanuniversität Leoben)
40. Hong, Jialin (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)
41. Hou, Baohui (Chinese Academy of Sciences)
42. Hu, Yaozhong (University of Alberta)
43. Huang, Jingyu (University of Birmingham)
44. Huang, Chuying (Fujian Normal University)
45. Hutzenthaler, Martin (University of Duisburg-Essen)
46. Jentzen, Arnulf (Chinese University of Hong Kong, Shenzhen & University of Münster)
47. Jin, Diancong (Huazhong University of Science and Technology)
48. Khoshnevisan, Davar (University of Utah)

49. Kloeden, Peter (University of Tübingen)
50. Kovács, Mihály (Pázmány Péter Catholic University and Budapest University of Technology and Economics and Chalmers University)
51. Kruse, Raphael (Martin-Luther-Universität Halle-Wittenberg)
52. Krylov, Nicolai (University of Minnesota)
53. Lang, Annika (Chalmers University of Technology and University of Gothenburg)
54. Larsson, Stig (Chalmers University of Technology and University of Gothenburg)
55. Laurent, Adrien (University of Bergen)
56. León, Jorge A. (CINVESTAV-IPN)
57. Li, Peijun (Purdue University)
58. Li, Jianliang (Yunnan University)
59. Li, Qiang (Chinese Academy of Sciences)
60. Li, Peijun (Purdue)
61. Li, Buyang (The Hong Kong Polytechnic University)
62. Liang, Ying (Purdue University)
63. Liang, Ge (Chinese Academy of Sciences)
64. Lin, Guang (Purdue University)
65. Lindner, Felix (University of Kassel)
66. Liu, Wei (Shanghai Normal University)
67. Lord, Gabriel (Radboud University)
68. Marinelli, Carlo (University College London)
69. Meddouni, Khadija (Radboud University)
70. Millet, Annie (University of Paris 1 Pantheon-Sorbonne)
71. Motschan-Armen, Ioanna (Chalmers University of Technology and University of Gothenburg)
72. Mueller, Carl (University of Rochester)
73. Mytnik, Leonid (Technion)
74. Nualart, Eulalia (University Pompeu Fabra)
75. Nualart, David (University of Kansas)
76. Oberhauser, Harald (University of Oxford)
77. Oh, Tadahiro (Choonghong) (The University of Edinburgh)
78. Ortiz-Latorre, Salvador (University of Oslo)
79. Pardoux, Etienne (Universite d'Aix-Marselle)
80. Peszat, Szymon (Jagiellonian University)

81. Petersson, Andreas (University of Oslo)
82. Prohl, Andreas (Universität Tübingen)
83. Quer-Sardanyons, Lluís (Universitat Autònoma de Barcelona)
84. Reiß, Markus (Universität zu Berlin)
85. Röckner, Michael (Bielefeld University)
86. Salimova, Diyora (Albert-Ludwigs-University of Freiburg)
87. Sanz-Solé, Marta (University of Barcelona)
88. Shardlow, Tony (University of Bath)
89. Sheng, Derui (Chinese Academy of Sciences)
90. Siska, David (University of Edinburgh)
91. Sun, Liying (Chinese Academy of Sciences, Beijing)
92. Tindel, Samy (Purdue University)
93. Tretyakov, Michael (University of Nottingham)
94. Ulander, Johan (Chalmers University of Technology and University of Gothenburg)
95. van Neerven, Jan (Delft University of Techology)
96. von Hallern, Claudine (Universität Hamburg)
97. Vilmart, Gilles (Université de Genève)
98. Wang, Xiaojie (Central South University)
99. Wang, Xu (Chinese Academy of Sciences)
100. Wang, Mengchao (Radboud University)
101. Yan, Yubin (University of Chester)
102. Zambotti, Lorenzo (Sorbonne Université)
103. Zhang, Liying (China University of Mining and Technology (Beijing))
104. Zheng, Guangqu (The University of Liverpool)
105. Zhou, Tau (Chinese Academy of Sciences)
106. Zimmermann, Aleksandra (University of Duisburg-Essen)
107. Zine, Younes (University of Edinburgh)

5 Comments on the online format of our workshop

The online format of our workshop was very well supported by the BIRS-CMO staff. Several questions were asked during the online presentations. The talks were followed by interesting discussions and scientific exchange between our two sub-communities.

However, the offered meeting platform *www.gather.town* was hardly used. We are not aware of the exact reasons, but would suspect that most of us are tired of such digital meetings which are not appropriate on the long run.

It was good to have an online option for our workshop instead of no meeting at all. Nevertheless, we strongly believe that having an on-site meeting is the right platform for having a successful research workshop.

6 Outcome of the Meeting

Our workshop was a success in terms of participation and diversity with more than 110 confirmed participants.

We received very positive feedbacks on the scientific quality of the programme.

In conclusion, our workshop has gathered participants coming from two sub-communities. We have brought together worldwide researchers specialized in the abstract analysis of SPDEs, as well as outstanding experts in the design and study of computational methods for SPDEs. By establishing or consolidating existing interactions between theoretical researchers and computational scientists, our workshop has reached two important goals:

- further develop the mathematical foundations and analysis of SPDEs and
- produce effective and efficient computational methods achieving a satisfying numerical simulation of our stochastic systems.

We firmly believe that such an interaction will have a significant impact on the global development of the theory and numerics of SPDEs.

7 Acknowledgement

We would like to thank BIRS for the selection of this workshop, BIRS-CMO for the fantastic support, as well as all speakers and participants for making this workshop a success.

References

[1] R. Dalang and D. Khoshnevisan and C. Mueller and D. Nualart and Y. Xiao, *A minicourse on stochastic partial differential equations*, Lecture Notes in Mathematics, Springer-Verlag, 2009.

[2] K. Hasselmann, Stochastic climate models Part I. Theory, *Tellus* **28** 1976, 473-485.

[3] J.H. Goldwyn and E. Shea-Brown, The What and Where of Adding Channel Noise to the Hodgkin–Huxley Equations, *PLOS Computational Biology* **7** (2011), 1–9.

[4] H.E. Cook, Brownian motion in spinodal decomposition, *Acta Metallurgica* **18** (1970), 297–306.

[5] M. Kardar and G. Parisi and Y.-C. Zhang, Dynamic Scaling of Growing Interfaces, *Phys. Rev. Lett.* **56** (1986), 889–892.

[6] M. Musiela and T. Zariphopoulou, Stochastic partial differential equations and portfolio choice in *Contemporary quantitative finance*, 195–216, Springer, Berlin, 2010.

[7] A. Barth and F.E. Benth, Forward dynamics in energy markets—Infinite dimensional modeling and simulation, *Stochastics* 86 (2014), 932–966.

[8] O. Juan and R. Keriven and G. Postelnicu, Stochastic Motion and the Level Set Method in Computer Vision: Stochastic Active Contours, *International Journal of Computer Vision* **69** (2006), 7–25.

[9] A. Lang, Simulation of stochastic partial differential equations and stochastic active contours, PhD thesis, 2007.