Latin American and Caribbean Workshop on Mathematics and Gender (22w5053)

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1 Overview of the Field

The workshop "Latin American and Caribbean Workshop on Mathematics and Gender " took place on 15-20 May 2022 at Casa Matemática Oaxaca, the Mexican affiliate of the Banff International Research Station. The event was supported by BIRS-Oaxaca, the Committee of Women in Mathematics of the International Mathematical Union (CWM-IMU), and the Mexican Mathematical Society (SMM). The workshop was organized for three members of the Gender and Diversity Committee of the Mathematical Union of Latin America and Caribbean (UMALCA), Carolina Araujo, Gabriela Araujo-Pardo, and Andrea Vera and Silvia Fernández-Merchant, a Mexican Mathematician that works in California State University.

This group of mathematicians proposed a multidisciplinary research workshop in mathematics with a strong scientific framework on Gender and Mathematics functioning as its hub. One of our main goals was the creation and enhancement of collaboration networks among Latin American women working in high-level mathematics.

Aligned with this plan, the workshop consisted of two general components. The first one seeks to jump-start high-level mathematical research collaborations among members of the Latin American community through the creation of research groups on various areas of mathematics. We will describe in this document the research that any of these groups had done before, during, and after this workshop.

The scientific framework of the Gender and Mathematics component of this workshop will be based on the work carried out by the Chilean research Project PIA Anillo SOC180025 "Women in mathematics in Chile, sociology of a scientific field from an interdisciplinary and gender perspective". (From now on: "Anillo de Matemáticas y Género", www.anillomatematicasygenero.cl/.)

All the groups will continue with the research and probably we publish a special number in some journal with the research results obtained from the workshop. We are working on that, but unfortunately at this moment of all, we are very busy with a lot of different things. We are evaluating the possibility, but it is a fact that this workshop gives many of us the possibility to start working with new people and create important collaboration networks of research between Latin American women mathematicians.

Moreover, all the components related to Gender and Mathematics were also really wonderful, we have one interesting online workshop "Workshop on the Impostor Syndrome" given by Karina Battistelli and Julia Plavnik (mathematics), with many people connected from many countries in Latin America and even from Europe, the discussions and participation in this workshop were really interesting for all of us.

We are sure that this workshop was an exit.

2 Presentation Highlights

We consider that it is important to share the outreach activities that were carried out in order to strengthen links with the territory and the local community of Oaxaca, taking advantage of the presence of workshop attendees:

1. A round table that took place on Wednesday 18th at the Public Library ”Andres Henestrosa”. The organizers were Sara Carrillo (Universidad Autónoma Benito Juárez de Oaxaca) and Luis Miguel García (UNAM and CGD Umalca). The panelists were: Eugenia Ellis, Natalia Garcia-Colin, Irma Leon, Gabriela Ovando, and Andrea Vera. The round table was titled ”Possible futures: experiences of Latin American women in mathematics”, and was held in hybrid modality [1], attended by about 40 people in person.

2. A children's workshop at a Public Library called ”Caminando Sobre fichas de dominó” taught by the Dr. Loiret Alejandria Dosal Trujillo

3. A talk at the Universidad Benito Juaárez del Estado de Oaxaca (UABJO) entitled ”Gráficas y Diseñnos de Bloques: Una Buena Amistad” the speaker was Dr. Gabriela Araujo Pardo, actual President of the Mexican Mathematical Society and organizer of this workshop, we think that, as the position, she is a right model for the young student girls in mathematics.

3 Scientific Progress Made

As we said before we will describe all the advances and theoretical results that the different research groups obtained.

3.1 Research Group on Algebraic Geometry

Carolina Araujo, IMPA, Brasil *(Organizer and Senior Researcher)*
Michela Artebani, Universidad de Concepción, Chile. *(Senior Researcher)*
Paola Comparin, Universidad de La Frontera, Chile. *(Senior Researcher)*
Alice Garbagnati, Università degli Studi di Milano Statale, Italy. *(Senior Researcher)*
Cecilia Salgado, University of Groningen, Netherlands. *(Senior Researcher)*
Claudia Correa Deisler, Universidad de Tarapacá, Arica, Chile. *(Postdoc/Young Researcher)*
María Elisa Valdés, Universidad de la Frontera, Temuco, Chile. *(Postdoc/Young Researcher)*
Eduardo Alves da Silva, IMPA, Brasil. *(PhD Student)*
Luíze D'Urso, IMPA, Brasil. *(PhD Student)*
Ana Quedo, IMPA, Brasil. *(PhD Student)*
Daniela Paiva Peñuela, IMPA, Brasil. *(PhD Student)*
Crislaine Kuster, IMPA, Brasil. *(PhD Student)*
Felipe Zingali Meira, University of Groningen, Netherlands. *(PhD Student)*
Renato Dias da Costa, UFRJ, Brasil. *(PhD Student)*
Benedetta Piroddi, Università degli studi di Milano, Italy. *(PhD Student)*
Martina Monti, Università degli studi di Milano, Italy. *(PhD Student)*
Pablo Quezada, Universidad de La Frontera, Chile. *(PhD Student)*

3.1.1 Overview

The proposal of the group was to work on the following problem, originally posed by Gizatullin:

Problem: Given a smooth quartic K3 surface $S \subset \mathbb{P}^3$, which automorphisms of S are induced by a Cremona transformation of $\mathbb{P}^3$?

This question was first addressed by Oguiso in the papers [5] and [6], where interesting examples and counter-examples were constructed. In order to attack the problem, one must have a good understanding of K3 surfaces and their automorphisms. Recently, a new set of techniques from birational geometry has been developed in this context, the so-called “Sarkisov Program”.

The goal of the working group was to gather specialists in the theory of K3 surfaces and specialists in birational geometry to exchange ideas and make progress on this problem, as we believe that Gizatullin's question should be approached by a combination of tools from birational geometry and the theory of automorphisms groups of K3 surfaces. The working group included a good number of graduate students and

[1] You can see the video here https://www.youtube.com/watch?v=vAXJ7IMQH0A

young researchers. So it started with a few general introductory lectures about the topic of the project, aimed at building common knowledge among the members of the group, in particular Ph.D. students and postdocs. There was also time dedicated to discussing the problem itself, possible strategies, and special cases that could be further investigated by the team. In the end, some small projects were designed for future work, some of which are suitable for thesis projects of Ph.D. students.

3.1.2 Developments and Future Projects

In the last meeting of the working group during the workshop, we analyzed special quartic surfaces $S \subset \mathbb{P}^3$, and discussed possible strategies and projects for the future. In what follows, we describe the main problem that has been addressed, concerning K3 surfaces with a small Picard number.

Let $S \subset \mathbb{P}^3$ be a quartic K3 surface. The Picard number of S, denoted by $\rho(S)$, is a positive integer measuring the homological complexity of S. It is known from the general theory of K3 surfaces that $1 \leq \rho(S) \leq 20$. Given a Cremona transformation φ of $\mathbb{P}^3$, the Sarkisov program provides a factorization of φ as a composition of simple maps, called *elementary links*. When $\rho(S) = 1$, one can analyze the possible elementary links in the Sarkisov program to show that every Cremona transformation of $\mathbb{P}^3$ preserving S is an automorphism. So Guizatullin's problem is completely settled in this case.

We are interested in the next case, namely when $\rho(S) = 2$. Answering Guizatullin's question, in this case, seems an interesting and feasible project. There are only three possibilities for the automorphism group of a K3 surface S when $\rho(S) = 2$:

1. $Aut(S) \cong \mathbb{Z}_2$;
2. $Aut(S) \cong \mathbb{Z}$; or
3. $Aut(S) \cong D_\infty$ (the infinite dihedral group).

We have looked at a few concrete examples of the case (1), and in all these cases the nontrivial automorphism of S is induced by a Cremona transformation of $\mathbb{P}^3$. This is consistent with a conjecture of Oguiso that predicts that every automorphism of finite order of S is induced by a Cremona transformation of $\mathbb{P}^3$. There are examples by Oguiso of surfaces in case (2) where no nontrivial automorphism of S is induced by a Cremona transformation of $\mathbb{P}^3$. It would be interesting to construct an example of a quartic K3 surface, together with nontrivial automorphisms φ_1 and φ_2, such that φ_1 is induced by a Cremona transformation of $\mathbb{P}^3$ but φ_2 is not. A natural candidate to look at is a surface in case (3).

Other topics were also discussed. For example: how can one use the structure of elliptic fibrations to study the Guizatullin problem for quartics with elliptic fibrations? A list of interesting examples was assembled for future study.

3.2 Research Group on Algebraic Quantum Groups

Gisela Tartaglia, Universidad Nacional de la Plata, Argentina.
Ana González, Universidad de la República, Uruguay.
Eugenia Ellis, Universidad de la República, Uruguay. *(Organizer)*

3.2.1 The problem: Induced actions for algebraic quantum groups

Let $\mathcal{H}$ be an algebraic quantum group [16], and $\mathcal{U}$ an algebraic quantum subgroup. Given an $\mathcal{U}$-module algebra A, our main goal is to define an $\mathcal{H}$-module algebra $\operatorname{Ind}_{\mathcal{U}}^{\mathcal{H}}(A)$ such that $\mathcal{U}\#A$ and $\mathcal{H}\#\operatorname{Ind}_{\mathcal{U}}^{\mathcal{H}}(A)$ are Morita equivalent. This problem was explained by Gisela Tartaglia in her talk at the conference in Oaxaca.

3.2.2 Results obtained

We are studying this problem at different levels.

1. Discrete case: $\mathcal{H} = C_0(G)$ (i.e. $\mathcal{H}$ is a discrete algebraic quantum group) and $\mathcal{U} = C_0(H)$ with H finite subgroup of G.

- Trivial coefficients: $A = \mathbb{C}$.
- General coefficients A any $\mathbb{C}$-algebra.

2. Compact case: $\mathcal{H}$ is a Hopf algebra with finite dimension (i.e. a compact algebraic quantum group) and $\mathcal{U}$ is a semisimple subalgebra of $\mathcal{H}$.
   - Trivial coefficients: $A = \mathbb{C}$.
   - General coefficients A any $\mathbb{C}$-algebra.
3. General case: $\mathcal{H}$ be an algebraic quantum group and $\mathcal{U}$ be an algebraic quantum subgroup of $\mathcal{H}$
   - Trivial coefficients: $A = \mathbb{C}$.
   - General coefficients A any $\mathbb{C}$-algebra.

We have finished the first step. We are working at the moment on the second step.

3.2.3 Relevance of the problem

Let $\mathcal{G}$ be a $\mathbb{C}$-algebra with nondegenerate product, and write $M(\mathcal{G} \otimes \mathcal{G})$ for the multiplier algebra of $\mathcal{G} \otimes \mathcal{G}$. We say that $\mathcal{G}$ is a *regular multiplier Hopf algebra* if it is equipped with two maps $\Delta : \mathcal{G} \to M(\mathcal{G} \otimes \mathcal{G})$, $S : \mathcal{G} \to \mathcal{G}$, where Δ is a comultiplication and S is an invertible antihomomorphism (called the *antipode*). An *algebraic quantum group* (a.q.g.) is a regular multiplier Hopf algebra with invariant functionals [16]. Every a.q.g. $\mathcal{G}$ has a canonical dual $\hat{\mathcal{G}}$ which is again an a.q.g., and verifies $\hat{\hat{\mathcal{G}}} \cong \mathcal{G}$ [16]*Thm. 4.12. Algebraic kk-theory for $\mathcal{G}$-module algebras was defined in [8].

Let G be a second countable locally compact group and let A be a separable G-C^*-algebra. The aim of the Baum-Connes conjecture is to compute the K-theory of the reduced crossed product $A \rtimes_r G$. One defines a graded abelian group $K_*^{top}(G; A)$, called the topological K-theory of G with coefficients in A, and a morphism

$$\mu_A : K_*^{top}(G; A) \to K_*^{top}(A \rtimes_r G),$$

which is called the *assembly map*. The Baum-Connes conjecture for G with coefficients A asserts that the assembly map is an isomorphism. It has important applications in topology, geometry and representation theory, and is known to hold in many cases, for instance, for amenable groups [10].

Meyer and Nest have reformulated this conjecture using the language of triangulated categories and derived functors [13]. In this approach, the left-hand side of the assembly map is identified with the localization $\mathbb{L}F$ of the functor $F(A) = K_*^{top}(A \rtimes_r G)$ on the equivariant Kasparov category KK^G. This is a natural starting point to study an analog of the Baum-Connes conjecture for locally compact quantum groups. The usual definition of the left-hand side of the assembly map is based on the universal space for proper actions, a concept that does not translate to the quantum setting in an obvious way. According to [13], we have to specify instead an appropriate subcategory of KK^G corresponding to *compactly induced actions* in the group case. This approach is implemented in [12] where an assembly map for torsion-free quantum groups are defined. If we want to translate these techniques to the (quantum) algebraic setting, an important step is to have an appropriate definition of induced actions. We will follow the ideas of Vaes ([15]) for induced coactions of locally compact quantum groups.

3.3 Research Group on Discrete Mathematics and Combinatorics

Silvia Fernández-Merchant, California State University, Northridge, USA. *(Organizer)*
Dolores Lara, Departamento de Computación, CINVESTAV, Instituto Politéctico Nacional, México.
Déborah Oliveros, Instituto de Matemáticas, Campus Juriquilla, UNAM, Mexico.

3.3.1 Overview

In the 1980s, Goodman and Pollack published a series of influential papers [19, 20, 21] where they introduced the concept of *circular sequences of points* as a tool to study combinatorial properties of finite sets of points in the plane. Since then, allowable sequences have been used to advance important classical open problems in combinatorial geometry. One example that is key to this project is a beautiful proof by Ungar [29] of a conjecture from 1970 by Scott [28]. It states that any finite set P of n points in the plane, not all collinear, determines at least $\lfloor n/2 \rfloor$ directions. In other words, the lines connecting pairs of points in P generate at least $\lfloor n/2 \rfloor$ different slopes. Any configurations of n points in the plane that determines exactly $\lfloor n/2 \rfloor$ directions is called *directions-critical*. The search for a full classification of all directions-critical configurations became the object of study in this area and even though there are several partial results and a few widely believe conjectures [18, 23, 24, 25, 26], the quest still continues.

3.3.2 Recent Developments and Open Problems

In 2008, Goodman and Pollack [22] generalized circular sequences of points to *circular sequences of intervals*. These new sequences codify the combinatorial properties of finite families of compact convex sets in the plane. In 2015, Novick [27] used this new tool to study the directions problem for families of convex sets. Let F be a family of disjoint compact convex sets in the plane with nonempty interior, then $d(F)$ is defined as the number of directions determined by the tangent lines to all pairs of sets in F, and d_n is the minimum of $d(F)$ over all families F of size n. Novick proved that $d_n \geq n - 1$, provided examples of such families that determine $2n-1$ directions, and conjectured that these families are direction-critical, that is $d_n = 2n-1$. He proposed restricting the minimum to families of sets that satisfy certain geometric properties. In this project, we considered different such restrictions and studied the general problem removing the condition that the sets have nonempty interior.

3.3.3 Scientific Progress Made and Future Plans

In order to deal with the hybrid format of the workshop, which meant we were not meeting in person in May, our group started weekly online meetings in early March. We have worked with different restricted families and obtained several partial results. We have particularly focused on removing the condition that the convex sets have nonempty interior, extending the problem to families of disjoint closed intervals. We developed a middle-ground tool, between circular sequences of points and intervals, to attack this problem. We have proved that the minimum number of directions determined by the tangents of any nontrivial family of n disjoint closed intervals, denoted by d_n^-, is bounded by $n \leq d_n^- \leq 2n - 1$, which improves on the lower bound and is consistent with previous results and conjectures for general families of convex sets. In contrast to Novick's Conjecture, when n is divisible by 4, we have found families of n disjoint intervals whose tangents determine exactly $2n - 2$ directions. We are currently working on improving the lower bound. We actually believe that $d_n^- = 2n - 2$ or $2n - 1$ depends on the class of $n \bmod 4$. We will continue our weekly meetings for the rest of the year and possibly meet in person for a week to complete this project. We are hopeful that our work for families of intervals will allow us to improve on the work for families of convex sets with nonempty interiors.

3.4 Research Group on Differential Geometry and Differential Equations

Romina Arroyo, Universidad Nacional de Córdoba and CONICET, Argentina.
Laura Barberis, National University of Cordoba, Argentina.
Veronica Diaz, Universidad Nacional de Mar de Plata, Argentina.
Anahita Eslami Rad, FaMAF, Universidad Nacional de Córdoba, Argentina.
Yamile Godoy, FaMAF, Universidad Nacional de Córdoba, Argentina.
Estela L. González, Mexico.
Ma. Isabel Hernández, CIMAT-Mérida, Mexico.
Gabriela P. Ovando, Universidad Nacional de Rosario, Argentina. *(Organizer)*
Raquel Perales, Instituto de Matemáticas, UNAM, Mexico.

Karen A. Serna Tello, Matemática Educativa del Instituto Politécnico Nacional (CICATA), Mexico.
Carolina Rey, Universidad Técnica Federico Santa María, Chile *(Posdoc)*
Mariel Saez, P.Universidad Católica de Chile, Chile.

3.4.1 Overview

Since the possible topics and tools for working in the area are very diverse, we divided our work into to groups:

Group I: R. Arroyo, E. L. González, G. Ovando, R. Perales, C. Rey, M. Saez, and K. A. Serna Tello. This group is working with the mean curvature flow, by trying to apply some symmetries, which could be read off through a Lie group acting on the Riemannian manifold. Until now, different approaches were considered. During the time in Oaxaca, several discussions took place with the goal of finding suitable problems on which the group could focus.

Group II: R. Arroyo, L. Barberis, A. Eslami Rad, Y. Godoy, M. I. Hernández, and V. Diaz. This group is studying invariant generalized complex structures on almost abelian Lie groups (that is, the corresponding Lie algebra admits an abelian ideal of codimension one). The idea is to find examples and to describe which of those groups admit generalized complex structures which do not come from a complex or symplectic structure.

3.4.2 Other Activities

During the meeting in Oaxaca, the group presented a plenary talk: *Un paseo por los fibrados de Higgs, hyper polígonos y dualidad espejo* by Laura Shaposnik from University of Illinois, Chicago. Besides the mathematical discussions, all members of the group had the opportunity to have interchanging experiences about working in Mathematics, as well as difficulties, encountered in their Mathematics careers. Information about research opportunities in different countries was shared.

3.5 Research Group on Dynamical Systems

Patricia Cirilo, UNIFESP, Brazil.
Adriana da Luz, Universidade Federal Fluminense, Rio de Janeiro, Brazil.
Gabriela Estevez, Universidade Federal Fluminense, Rio de Janeiro, Brazil.
Luna Lomonaco, IMPA, Rio de Janeiro, Brazil. *(Organizer)*
Enrique Pujals, The City University of New York (CUNY), USA.

3.5.1 The problem

In 1983, Mañé, Sad, and Sullivan [31] introduced holomorphic motion as a powerful tool with countless applications in holomorphic dynamics. They prove that, if all the periodic points of a holomorphic map in a holomorphic family have analytic continuation, then there exists a conjugacy between this map and all the neighboring maps in the holomorphic family on their Julia set (the map is stable on the Julia set), where the Julia set is the closure of repelling periodic points. The first step for this work is to show that, if all the periodic points of a holomorphic map have analytic continuation, which implies that there exists a conjugacy between the periodic points of a holomorphic map and its neighbors in the holomorphic family, then this conjugacy extends to the closure of the set of periodic points.

There are similar results in some settings for real dynamics, in which the fact that all periodic orbits have a continuation in a neighborhood of the map implies stability in the closure of the periodic points (analogous in this setting to the Julia set). But contrary to the techniques in [31], the tool used to prove this kind of result is a structure induced by the differential of our map in the tangent space of the manifold called hyperbolicity. However, attempts to recover these techniques to other settings of real dynamics have been enormous and not very successful.

The goal of the project is to explore to what extent we can adapt the mind frame of [31] to the study of real maps, with the intention to understand stability from a different point of view than that of hyperbolicity. We plan to study the relationship between the stability in the closure of the periodic points and the existence of continuations for the periodic points, as well as possible weaker notions of stability (semiconjugacies for instance) and continuations of objects other than periodic orbits (as for instance measures).

3.5.2 Developments

After BIRS we had several online meetings and a week of meetings in person, between June 30 and July 3, in Rio de Janeiro. Following the above-described goals, we explored smooth real families in dimension 2, with the intention to understand stability from a different point of view than that of hyperbolicity. More precisely, we are studying the relation between the stability in the closure of the periodic points, and the existence of continuations for the periodic points as well as possible weaker notions of stability (semiconjugacies or correspondences). In that direction, first defined the setting we are interested in. We are interested in smooth one parameter families $(f_t)_{t\in I}$ of surface maps, where I is an interval, such that:

1. for all t, the periodic points are hyperbolics of saddle type;
2. all the homoclinic intersections of periodic points are transversal for all parameters.

With those hypotheses and using some known facts, we obtain that for any $t \in I$ there is a bijection

$$h_t : Per(f_0) \to Per(f_t)$$

defined as $h_t(p_0) = p_t$, where $Per(f_i)$ is the set of periodic points of f_i.

The first question we are interested in answering is the following:

Question 1. *Can the function h_t be extended to the closure in a meaningful way?*

During our meetings, we approach the problem in 3 ways:

1. we studied the state of the art,
2. we studied the possibility to answer question 1 by using bi-Pliss points.
3. we consider whether we could show in our setting continuity of hyperbolic measures,

3.5.3 State of the Art

In what follows, we describe the scenarios where an extension of the results in [31] has been obtained and Question 1 has been answered:

a) **Polynomials in $\mathbb{C}$ (also rational maps in the Riemann sphere):** In [31] h_t was extended to the closure as a holomorphic map.

b) **One-dimensional smooth real interval maps:** It is possible (using a result in [34]) to extend the map to the closure as a monotone map. In C^3 class the extension is a conjugacy.

c) **Polynomials in $\mathbb{C}^2$:** In [32] the bijection was extended to a correspondence. In [33] the correspondence is in fact a conjugacy restricted to a set of total measure for any hyperbolic measure.

3.5.4 Developed Strategies

I. bi-Pliss Points Given $\lambda < 1 < \sigma$ one denotes with $BP^{\lambda,\sigma}$ the set of periodic points p such that

$$|D_p f^j_{|E^s}| < \lambda^j \quad \text{and} \quad |D_p f^{-j}_{|E^u}| < \sigma^{-j},$$

where E^s and E^u are the eigenspaces of $D_p f^{per(p)}$. Observe that Bi-Pliss points are not invariant. Bi-Pliss points have good properties:

1. For any hyperbolic measure, regular points (a set of total measures where they are defined as both non-zero exponents) are well approximated by Bi-Pliss periodic points.
2. Bi-Pliss periodic points (and their closure, or said in other words, hyperbolic points of the measure) has "large" local stable and unstable manifolds; moreover,
   (a) those manifolds are **locally totally ordered**;
   (b) those stable/unstable manifolds induce a local stable/unstable manifold on the accumulation points (disregarding if the accumulation point is regular for some measure).
   (c) even if the continuations of the Bi-Pliss points are not Bi-Pliss, their local manifolds have continuations and they are locally totally ordered.

II. Hyperbolic Measures It is said that an ergodic measure is hyperbolic if all its Lyapunov exponents are non-zero. In the case of a surface, any non-atomic hyperbolic measure has a positive Lyapunov exponent, and a negative one: $\lambda^- < 0 < \lambda^+$. Associated to these exponents, one has the Oseledets splitting, $E^- \oplus E^+$. A hyperbolic block, is a compact set Γ (not necessary invariant) satisfying that there exists $\gamma < 0$ (such that $\lambda^- < \gamma < 0 < -\gamma < \lambda^+$) and a positive integer n_0 such that for any $x \in \Gamma$ and any $n \geq n_0$ it holds that for

$$|D_x f^n_{|E^-}| < e^{(n\gamma)}, \quad |D_p f^{-n}_{|E^+}| < e^{(n\gamma)}.$$

For a hyperbolic block one can prove:

– There exists a hyperbolic block with positive measure and therefore, almost every point has a forward and backward iterate in a hyperbolic block.

– Any point in a hyperbolic block is accumulated by Bi-Pliss periodic points with exponents $(e^{\gamma}, e^{-\gamma})$.

– All points in a hyperbolic block has a local stable and unstable manifold with uniform size; moreover, the local stable and unstable manifold form a C^1-lamination.

– Hyperbolic blocks are accumulated by horseshoes.

– Relaxing the constant γ (or increasing n_0) one has a hyperbolic block with larger and larger measures.

3.6 Research Group on Extremal Graph Theory

Gabriela Araujo-Pardo, Instituto de Matemáticas, UNAM, México. *(Organizer)*
Silvia Fernández-Merchant, California State University, Northridge, USA. *(Organizer)*
Adriana Hangsberg,Instituto de Matemáticas, UNAM-Juriquilla, México.
Amanda Montejano, Instituto de Matemáticas, UNAM-Juriquilla, México.

3.6.1 Overview

A graph is *planar* if it can be drawn on the plane without edge-crossings. A natural extension of this definition involves allowing a certain number of crossings per edge. For any nonnegative integer k, we say that a graph is k-planar if it can be drawn on the plane with at most k crossings per edge. Planar graphs have been extensively studied. while several results are known for 1-planar graphs, a lot less is known for k-planar graphs when $k \geq 2$. One reason is that deciding whether a graph is planar can be done in linear time [37], but testing for k-planarity is NP-complete for any $k \geq 1$ (see [39, 40] for 1-planarity and [41] for any k).

3.6.2 Recent Developments and Open Problems

It is known that any 1-planar graph with n vertices has at most $4n - 8$ edges [36]. Graphs achieving this bound are called *optimal*. In 2013, Czap and Hudaák proved that any drawing of a 1-planar optimal graph can be decomposed into two plane drawings, one of them the drawing of a forest [38]. In 2014, Ackerman extended this result to any 1-planar drawing of a graph [35]. This structural result is key in the development of 1-planar graphs but nothing similar is known for larger planarity. Our group is looking for structural results for k-planar graphs for $k \geq 2$.

3.6.3 Scientific Progress Made and Future Plans

Building on Ackerman's result for 1-planar graphs, we have proved that any simple k-planar drawing of the graph can be decomposed into $k+1$ plane drawings, at least $\lceil k/2 \rceil$ of which are forests. We believe that this result can be improved so that a larger number of forests can be guaranteed among the $k+1$ plane drawings. In particular, for $k=2$, we conjecture that any 2-planar drawing of a graph can be decomposed into 3 plane sub drawings, at least 2 of which are forest (our current result only guarantees that one of the 3 plane drawings in the decomposition is a forest).

3.7 Research Group on Probability Theory

Inés Arizmendi, Universidad de Buenos Aires, Argentina).
Laura Eslava, Universidad Nacional Autónoma de México, CDMX, México. *(Organizer)*
Alejandra Fonseca, Universidad de Sonora, Hermosillo, México.
Saraí Hernández-Torres, Technion - Israel Institute of Technology, Haifa, Israel.
Florencia Leonardi, Instituto de Matemática e Estatística, Universidade de São Paulo, Brasil.
Lizbeth Peñaloza, Universidad del Mar, Huatulco, México.

3.7.1 Overview

Florencia Leonardi presented the plenary talk *Modelo estocástico de bloques, algunos resultados recientes y problemas abiertos*. Inés Armendariz, being an elected Committee member of the SLAPEM (Sociedad Latinoamericana de Probabilidad y Estadística Matemática, participated as an advisor in two of the first probability-focused sessions of the workshop. Laura Eslava coordinated the activities of the rest of the group. The main purpose was

1. To provide a diagnosis of the state of the organic networks among probabilist women who are currently working in Latin America and the Caribbean or that identify as part of such communities.

2. To propose strategies to create or strengthen networks of women interested in pursuing research in the fields of probability and statistics.

During the meeting, we worked on daily sessions from 9:30 am to 12:30 pm (breaking for coffee between 10:30-11:00 am). We held two 1-hour sessions of Q&A for female students interested in pursuing a Ph.D. in areas related to probability and statistics, two 1-hour sessions with Inés Armendariz as a senior advisor on the state of the current networks among probabilists in Latin America and the Caribbean. The rest of the sessions were devoted to defining specific goals and timelines to the second item of our main purpose. We reached a consensus on specific objectives, which are the following.

1. To create a webpage for women in probability and statistics focused on the Latin-American and Caribbean community.

2. To organize a research seminar to encourage the collaboration of Mexican women with interests in probability and statistics.

3. To organize a reading program for math students with a target community as wide as the financial resources and student participation may allow for.

3.8 Future plans

In what follows we present the motivation and details defined so far for each of these objectives. We provide a tentative schedule for the rest of the collaboration which is expected to conclude this initial stage in March 2023.

3.8.1 Website

We claim that an initial barrier for young female researchers is access to information and soft-skills advice. Therefore, the creation of a website (in a similar spirit to www.womeninprobability.org) will cover several objectives. First, to provide a database of the academic profiles of probabilists and statisticians, highlighting those profiles from people that identify as women. Second, gather selected essays on strategies to cope with common challenges of graduate studies and young researchers' development. Finally, the website will serve as a point of contact to be added to a mailing list that will broadcast relevant announcements such as registration to conferences in related areas, opportunities for funding to graduate programs, and available positions in academia. We are considering promoting our website, hosted at an institution-independent server with sister organizations such as Sociedad 'Latinoamericana de Probabilidad y Estadística Matematica' (SLAPEM) and 'Women in probability'.

3.9 Research seminar

We reckon that it is important to foster research groups where women participate as main collaborators. We will start a group with female Mexican probabilists and statisticians. In hybrid mode, we plan on having a monthly seminar where we will have the opportunity to share our research interests and promote the creation of research groups among ourselves. We hope that the group will eventually strengthen its connections as we continue looking for and contacting other female researchers in Mexico and other countries.

3.10 Reading program

We are aware and would like to initiate one of the directed reading programs, whose objective is to link bachelor students with graduate students so that they are able to support each other through dialogue and the goal of reading an advanced book (as promoted in https://math.mit.edu/research/undergraduate/drp/index.php).

Two of the main challenges for our community are funding and communication. First, the graduate communities in either mathematics or specifically in probability and statistics are still small and scattered in Latin America and the Caribbean; as such, it is difficult to find enough community participation. Second, the directed reading program would need to be established among universities and most likely, in several countries and this presents logistical issues to be resolved. Nevertheless, we will watch out for opportunities to finance the purchase of mathematics books that will be proposed in the readings and to host a few activities where the students may interact and form lasting support networks.

3.11 Timeline

The previous objectives and activities will be developed according to the following.

- Monthly meetings to follow up on the general progress of the objectives.
- June to December 2022: creating the database for the website.
- January to March 2023: setting up the website and the mailing lists that will be used to broadcast relevant announcements; e.g. on available positions in academia (both for interested graduate students and young researchers).

4 Overview of the Gender Axis

We had originally proposed that one of the research groups would be on gender and mathematics. However, this was intended to be a fully face-to-face workshop. Given that the format of the workshop was hybrid and that only 15 people could attend in person, we decided to make the gender axis a transversal axis for the whole workshop, i.e. to dedicate every afternoon of the week to a talk or workshop on gender, facilitated by specialists in the area. This, among other reasons, gives priority to mathematicians attending the workshop in person. The research area called "science and gender" has been developing as part of the feminist epistemological critique of science for some decades. However, mathematics has usually been left out of this critique for various reasons. Moreover, according to Harding (1986), the discipline of mathematics requires special mention, primarily substantiated by abstract reasoning, where the production of knowledge is not based on the interpretation of data, and, therefore, it is difficult to demonstrate there is a gender bias when choosing to study certain topics or in demonstrating a theorem. On the other hand, Damarin (2008) suggests that the invisibility of mathematics in the feminist critique of science may be due, among other things, to the unawareness of the existence of this discipline by those who do not practice it. Thus, the mere fact of having brought together 15 mathematicians in person and more than 50 in online format to discuss topics of science and gender from a theoretical point of view, constitutes in itself an innovative and novel event of tremendous power to generate knowledge. Broadly speaking, the activities developed in this axis were:

1. Introductory talk by mathematician Andrea Vera-Gajardo: ”Mathematics and gender: a map and a plan”.
2. Talk by social science researcher Carla Fardella: ”The importance of gender in the scientific profession”.
3. Workshop on the Impostor Syndrome, by Karina Battistelli and Julia Plavnik (mathematics).
4. Talk by Uzuri Albizu, a researcher in mathematics education and gender: ”Science and gender: crossroads and interstices. Reflections on experiences, optics, and practices of feminist scientists”.
5. Talk on concrete proposals to close gaps by mathematician María Isabel Cortez: ”Gender Gap in Mathematics: from experience, research, and action”.
6. Talk by researcher and philosopher Eulalia Pérez Sedeño: ”Feminist epistemology and epistemic justice”.

It should be noted that there was great interest from all participants in discussing these topics, each talk and workshop gave rise to very interesting discussions that lay the groundwork for an emerging line of research in mathematics and gender.

5 Outcome of the Meeting

As we said in the first section, all the groups will continue with the research and probably we publish a special number in some journal with the research results obtained from the workshop. We are working on that, but unfortunately at this moment of all, we are very busy with a lot of different things. We are evaluating the possibility, but it is a fact that this workshop gives many of us the possibility to start working with new people and create important collaboration networks or research between Latin American women mathematicians. We are very satisfied with it.

All the groups were interested in the creation of collaboration networks, as a result of this workshop, collaboration networks have been created between the different research groups and in some of them, specific groups of Latin American and Mexican women mathematicians have been created in the different research areas. As two examples we have the group on Discrete Mathematics called “*Matemáticas inDiscretas*” and the seminar in Mexico called “*Aleatorias y Normales*” that aims to create a network among Mexican female mathematicians who work in probability and statistics and is organized by the participants of this workshop in this area. And also, probably other groups that we don’t know about and that are in formation and consolidation.

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