Knots, Surfaces, 3-manifols

Mario Eudave-Muñoz (UNAM Querétaro),
Jennifer Schultens (UC Davis)

June 22, 2021 - June 25, 2021

CMO Workshop 21w5094 delved into the inner workings of 3-manifolds and how knots and surfaces can be positioned in 3-manifolds. Specifically, participants conferred on recent research about structures, especially geometric structures, exhibited by 3-manifolds and how distinct such structures interact and inform us about each other. An unexpected recurring theme came via covering spaces: Inspired by Etienne Ghys' ICM plenary lecture, Tali Pinsky discussed Anosov flows, paying particular attention to the complement of the trefoil knot. The trefoil knot complement admits a degree 6 cover that illustrates Anosov flows and shows how dynamics and topology interact. On a different note, Emily Hamilton discussed hyperbolic 3-manifolds with infinitely many virtual geometric triangulations. The structures involved, geometric triangulations, live in covers of the actual 3-manifold and can only exist when certain algebraic conditions (subgroup separability) on the fundamental group of the 3-manifold are met. Inkang Kim computed degrees of virtual coverings maps using harmonic maps and their relation to Gromov's dihedral rigidity. Finally, Jesus Rodríguez-Viorato discussed the relationship between contact structures and universality of knots. A knot K is universal if every 3-manifold can be constructed as a branched cover of K.

Several perennial problems were discussed, such as Kakimizu complexes of certain knot complements (Luis Valdez-Sanchez), Kauffman Skein modules (José Román Aranda Cuevas), concordance of knots (Homayun Karimi), questions related to Dehn surgery on knots (David Futer, Kenneth Baker, Masakazu Teragaito), the Neuwirth conjecture and spanning surfaces of knots (Joshua Howie), $SU(2)$ representations of 3-manifold groups (Juanita Pinzón-Caidedo) and the meridional rank conjecture (Puttipong Pongtanapaisan). Moreover, several clever bookkeeping tools made their appearance, such as the sphere complex in the context of Heegaard splittings (Sebastian Hensel), Taylor and Tomova's equivariant Heegaard splittings (Scott Taylor), geometric triangulations of 3-manifolds (Jessica Purcell), and (g,b)-decompositions of knots, especially $(1,1)$-decompositions (Fabiola Manjarrez-Gutiérrez). Other related topics were also discussed, such as hyperbolic groups (Yo'av Rieck), dynamically defined wild knots (Gabriela Hinojosa), norms on the cohomology of 3-manifolds (Xiaolong Hans Han), and embeddings of surfaces in $\mathbf{R}^3$ (Margaret Nichols).

Since the work of Wolfgang Haken in the 1960s, low-dimensional topology has boasted interesting computational problems. For instance, Nathan Dunfield is using cutting edge

technology to compute all essential surfaces in a given 3-manifold. His work builds on Haken's algorithm now implemented on a computational platform named SnapPy. Given a hyperbolic 3-manifold M, he gives a quasi-polynomial bound on the number of surfaces in M of a given Euler characteristic. A rather different algorithmic question discussed concerns group theory, relevant to low-dimensional topologists by virtue of the algebraic tools used in the discipline. In this vein, Richard Webb discussed the conjugacy problem. Algorithms to recognize whether two elements of a group lie in the same conjugacy class have been around for a few decades, but Mark Bell and Richard Webb have discovered an algorithm that does so in polynomial time. In the spirit of finding efficient algorithms, Kristof Huszar discussed thin triangulations of 3-manifolds.

Our workshop was diverse in many aspects. Of the 24 talks presented, 12 were by speakers based in the United States, 3 from Mexico, 2 from Canada, and 1 from each from Australia, England, France, Germany, Israel, Japan and Korea. In all talks we had more than 30 attendants, in many more than 40. In addition, participants joined from several countries, including some with a big time difference. We also had gender diversity, of the 24 speakers, 7 were women.

In summary, the workshop served to highlight several distinct streams of research currently taking place in low-dimensional topology. Several interesting conversations arose during the intervals between talks. We applaud the organizers for setting up the continuous Zoom session for the conference. This format served speakers and audience members well and proved amenable to the interactions between talks. We hope to run a follow-up in-person workshop.

References

[1] C. Adams, The knot book. An elementary introduction to the theory of knots. American Mathematical Society, Providence, RI, 2004.

[2] Banks, Jessica, The Kakimizu complex of a connected sum of links, Trans. Amer. Math. Soc. 365 (2013), 6017–6036.

[3] J. R. Eisner: Knots with infinitely many minimal spanning surfaces, Trans. Amer. Math. Soc. 229 (1977) 329–349.

[4] M. Eudave-Muñoz, A. Guzmán-Tristán, Toroidal surgeries and the genus of a knot. Osaka J. Math. 56 (2019), no. 3, 549–575.

[5] U. Hamenstädt, Geometry of the complex of curves and of Teichmüller space in Handbook of Teichmüller theory, IRMA Lect. Math. Theor. Phys., Volume 11, 447–467. Eur. Math. Soc., Zürich, 2007.

[6] W. J. Harvey, W. J., Boundary structure of the modular group, In Riemann surfaces and related topics: Proceedings of the 1978 Stony Brook Conference (State Univ. New York, Stony Brook, N.Y., 1978), Ann. of Math. Stud. 97, 245–251, Princeton Univ. Press, Princeton, N.J., 1981.

[7] Hempel, John, 3-manifolds, Reprint of the 1976 original. AMS Chelsea Publishing, Providence, RI, 2004. xii+195 pp.

[8] Hempel, John, 3-manifolds as viewed from the curve complex, Topology 40 (2001), no. 3, 631–657.

[9] Jaco, William, Lectures on three-manifold topology, CBMS Regional Conference Series in Mathematics 43 (1980), American Mathematical Society, Providence, R.I., xii+251 pp.

[10] W.B.R. Lickorish: An introduction to knot theory. Graduate Texts in Mathematics 175. Springer-Verlag, New York, 1997.

[11] I. Irmer, Geometry of the Homology Curve complex, J. Topol. Anal. 4 (2012), no. 3, 335–359.

[12] J. Johnson, R. Pelayo, R. Wilson, The coarse geometry of the Kakimizu complex, Algebr. Geom. Topol. 14 (2014), no. 5, 2549–2560.

[13] O. Kakimizu, Finding disjoint incompressible spanning surfaces for a link, Hiroshima Math. J. 22 (1992) 225–236.

[14] W. B. R. Lickorish, A representation of orientable combinatorial 3-manifolds. Ann. of Math. (2) 76 (1962), 531–540.

[15] H. A. Masur, Y. N. Minsky, Geometry of the complex of curves. I. Hyperbolicity. Invent. Math. 138 (1999), no. 1, 103–149.

[16] H. A. Masur, Y. N. Minsky, Geometry of the complex of curves. II. Hierarchical structure. Geom. Funct. Anal. 10 (2000), no. 4, 902–974.

[17] P. Ozsváth, Z. Szabó, Absolutely graded Floer homologies and intersection forms for four-manifolds with boundary. Adv. Math. 173 (2003), no. 2, 179–261.

[18] P. Przytycki, J. Schultens, Contractibility of the Kakimizu complex and symmetric Seifert surfaces, Trans. Amer. Math. Soc. 364, no. 3 (2012) 1489–1508.

[19] D. Rolfsen: Knots and links. Mathematics Lecture Series 7. Publish or Perish, Inc., Huston, TX, 1990.

[20] M. Sakuma, K. J. Shackleton, K. J., On the distance between two Seifert surfaces of a knot, Osaka J. Math. 46 (2009), no. 1, 203–221.

[21] Schultens, Jennifer, Introduction to 3-manifolds, Graduate Studies in Mathematics 151. American Mathematical Society, Providence, RI, 2014. x+286 pp.

[22] Schultens, Jennifer, Kakimizu complexes of Seifert fibered spaces, Algebr. Geom. Topol. 18 (2018), no. 5, 2897–2918.

[23] Seifert, Herbert, Topologie dreidimensionaler gefaserter Räume, Acta Math. 60 (1932) 147–238.